

We load the Hamiltonians from the general Hamiltonian evolutions including the purely time-dependent terms

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> restart :
with(LinearAlgebra) :
with(DifferentialGeometry) :
with(Tools) :

t10:=-t20-t30:
P1:=unapply( (t12+t22+t32)*lambda+t11+t21+t31,lambda) ;
P2:=unapply( (t12*t22+t12*t32+t22*t32)*lambda^2+((t21+t31)*t12+
(t11+t31)*t22+t32*(t11+t21))*lambda-t10*t12-t20*t22-t30*t32+
t21*t11 +t31*t11+t21*t31,lambda) ;
P3:=unapply(t12*t22*t32*lambda^3+(t12*t22*t31+t12*t32*t21+t22*
t32*t11)*lambda^2+(t12*t22*t30+t12*t32*t20+t22*t32*t10 +t12*
t21*t31+t22*t11*t31+t32*t11*t21)*lambda,lambda) ;

F22:=unapply(0,tt11, tt21, tt31, tt12, tt22, tt32) ;
F11:=unapply(0,tt11, tt21, tt31, tt12, tt22, tt32) ;
F31:=unapply(h/2*tt21* (-1/(tt22-tt32)-ln(tt22-tt32)/(tt12-
tt32)+ln(tt12-tt22)/(tt12-tt32)),tt11, tt21, tt31, tt12, tt22,
tt32) ;
F12:=unapply( h/2*tt21*tt31*(ln(tt22-tt32)/(tt12-tt32)^2-ln
(tt12-tt22)/(tt12-tt32)^2),tt11, tt21, tt31, tt12, tt22, tt32 )
;
F32:=unapply(h*tt21*tt31/2*((ln(tt12-tt22)-ln(tt22-tt32))/(tt12
-tt32)^2),tt11, tt21, tt31, tt12, tt22, tt32) ;
F21:=unapply(h*tt31/2*(1/(tt22-tt32) +ln(tt12-tt22)/(tt12-
tt32)-ln(tt22-tt32)/(tt12-tt32))
, tt11, tt21, tt31, tt12, tt22, tt32) ;

Hame11 := unapply( (-p^3+((t12+t22+t32)*q+t11+t21+t31)*p^2+((-
t22-t32)*t12-t22*t32)*q^2+((-t31-t21)*t12+(-t11-t31)*t22-t32*
(t11+t21))*q+t12*(-t20-t30)+t22*t20+t32*t30+(-t31-t21)*t11-t21*
t31)*p+q*(q^2*t12*t22*t32+((t22*t31+t32*t21)*t12+t22*t32*t11)*
q+(t22*t30+t32*t20+t21*t31)*t12+((-t20-t30)*t32+t11*t31)*t22+
t32*t11*t21))/((t12-t32)*(t12-t22))
+(((t20+t30)*t12+t31*t11-t20*t32-t30*t32)*t21-t11*t20*(-t32+
t22))/((-t22+t12)*(t12-t32))
+F11(t11,t21,t31,t12,t22,t32)
,q,p) ;

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Hame21:= unapply( (p^3+((-t12-t22-t32)*q-t31-t11-t21)*p^2+((
(t12+t32)*t22+t12*t32)*q^2+(t12*(t21+t31)+t22*(t11+t31)+t32*
(t11+t21))*q+(h-t20)*t22+(t30-h+t20)*t12-t32*t30+t11*(t21+t31)+
t21*t31)*p-q*(q^2*t12*t22*t32+((t12*t31+t32*t11)*t22+t12*t32*
t21)*q+(t12*t30+(-t30+h-t20)*t32+t11*t31)*t22+((-h+t20)*t32+
t21*t31)*t12+t32*t11*t21))/((t22-t32)*(t12-t22))
+(((t12+t32)*t20-t31*t11-t12*t30+t30*t32)*t21+t11*t20*(-t32+
t22))/((-t22+t12)*(-t32+t22))
+F21(t11,t21,t31,t12,t22,t32)
,q,p);

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Hame31:=unapply((-p^3+(t11+t21+t31+(t12+t22+t32)*q)*p^2+((-t12
-t22)*t32-t12*t22)*q^2+((-t21-t11)*t32+(-t31-t21)*t12-t22*(t11+
t31))*q+(-h+t30)*t32+(-t30+h-t20)*t12+t22*t20+(-t31-t21)*t11-
t21*t31)*p+q*(t12*t22*t32*q^2+((t12*t21+t22*t11)*t32+t31*t12*
t22)*q+(t12*t20+(-t30+h-t20)*t22+t11*t21)*t32+((-h+t30)*t22+
t21*t31)*t12+t22*t11*t31))/((t22-t32)*(t12-t32))
+(((t20-t30)*t32+t31*t11+t12*t20+t12*t30)*t21-t11*t20*(-t32+
t22))/((t12-t32)*(-t32+t22))
+F31(t11,t21,t31,t12,t22,t32)
,q,p);

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Hame12:=unapply(((t32*t22*(t21+t31-2*t11)*q^3+(-(t22+t32)*(t21+
t31-2*t11)*p+(-h*t32+t31*(t21+t31-2*t11))*t22+t32*t21*(t21+t31
-2*t11))*q^2+((t21+t31-2*t11)*p^2+(h*t22+h*t32-(t21+t31)*(t21+
t31-2*t11))*p+(-2*t30*t11+t21*t30+(-h+t30)*t31)*t22+(-2*t11*
t20+(-h+t20)*t21+t31*t20)*t32+t31*t21*(t21+t31-2*t11))*q-(p*h+
(-2*t30-2*t20)*t11+(t21+t31)*(t30-h+t20))*p)*t12^2+(-((-t11+
t31)*t22-t32*(t11-t21))*t32*t22*q^3+((-t11+t31)*t22^2-t32^2*
(t11-t21))*p+(h*t32-t31*(-t11+t31))*t22^2+(2*((1/2)*h*t32+(t11-
t21)*(-t11+t31))*t32*t22+t21*t32^2*(t11-t21))*q^2+((-t11+t21)
*t22+t32*(-t11+t31))*p^2+(-h*t22^2+(-2*h*t32+2*t11*(t11-t21))*
t22-h*t32^2-2*t11*(-t11+t31)*t32)*p+(t30*t11-(-h+t30)*t31)*
t22^2+(((3*t20+3*t30)*t11+(h-t20-2*t30)*t21-(-h+2*t20+t30)*t31)
*t32+t31*(t11-t21)*(-2*t11+t31))*t22+(2*((1/2)*t11*t20+(1/2)*
(h-t20)*t21)*t32+(-t11+t31)*t21*(-(1/2)*t21+t11))*t32)*q-p*(
(t21+t31-2*t11)*p^2+(-h*t22-h*t32-(t11+t21+t31)*(t21+t31-2*t11)
)*p+(3*t20+t30)*t11+(h-t20)*t21-(-h+2*t20+t30)*t31)*t22+(t11*
(t20+3*t30)+(h-t20-2*t30)*t21-(-h+t30)*t31)*t32+(t21+t31-2*t11)
*(t11*(t21+t31)+t21*t31))*t12+t32*(((-t11+t31)*t22-t32*(t11-
t21))*p+(-h*t32-t11*(-t11+t31))*t22+t32*t11*(t11-t21))*t22*q^2+
(-((-t11+t31)*t22-t32*(t11-t21))*t22+t32)*p^2+(t32*h-t11^2+

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$$\begin{aligned}
& t_{31}^2) * t_{22}^2 + (h * t_{32}^2 + (-2 * t_{11}^2 + 2 * t_{21} * t_{31}) * t_{32}) * t_{22} + (-t_{11}^2 + \\
& t_{21}^2) * t_{32}^2) * p + ((-t_{20} - t_{30}) * t_{11} + (t_{30} - h + t_{20}) * t_{31}) * t_{32} - t_{11} * t_{31} * \\
& (-t_{11} + t_{31}) * t_{22}^2 - t_{32} * (t_{11} * (t_{20} + t_{30}) - (t_{30} - h + t_{20}) * t_{21}) * t_{32} - \\
& (t_{11} * (t_{21} + t_{31}) - 2 * t_{21} * t_{31}) * t_{11}) * t_{22} + t_{32}^2 * t_{11} * t_{21} * (t_{11} - t_{21}) * q + \\
& p * ((-t_{11} + t_{31}) * t_{22} - t_{32} * (t_{11} - t_{21})) * p^2 + (-h * t_{32} - (-t_{11} + t_{31}) * (t_{11} + \\
& t_{21} + t_{31})) * t_{22} + t_{32} * (t_{11} - t_{21}) * (t_{11} + t_{21} + t_{31}) * p - t_{20} * (-t_{11} + t_{31}) * \\
& t_{22}^2 + ((t_{11} * (t_{20} + t_{30}) + (h - t_{20}) * t_{21} - (-h + t_{30}) * t_{31}) * t_{32} + (-t_{11} + t_{31}) * \\
& (t_{11} * (t_{21} + t_{31}) + t_{21} * t_{31})) * t_{22} + t_{32} * ((-t_{31} - t_{21}) * t_{11} + t_{32} * t_{30} - t_{21} * \\
& t_{31}) * (t_{11} - t_{21})) / (2 * (t_{12} - t_{32})^2 * (t_{12} - t_{22})^2) \\
& + ((-1/2) * t_{12} + (1/2) * t_{32}) * t_{21} + (t_{11} - (1/2) * t_{31}) * t_{12} + (-1/2) * t_{22} - \\
& (1/2) * t_{32}) * t_{11} + (1/2) * t_{22} * t_{31}) * ((-t_{20} - t_{30}) * t_{12} - t_{31} * t_{11} + (t_{20} + \\
& t_{30}) * t_{32}) * t_{21} + t_{11} * t_{20} * (-t_{32} + t_{22})) / ((t_{12} - t_{32})^2 * (-t_{22} + t_{12})^2) \\
& + F_{12}(t_{11}, t_{21}, t_{31}, t_{12}, t_{22}, t_{32}) \\
& , q, p);
\end{aligned}$$

$$\begin{aligned}
\text{Hame}_{22} := & \text{unapply}((t_{12} * t_{32} * (t_{11} - 2 * t_{21} + t_{31}) * q^3 + (-t_{12} + t_{32}) * (t_{11} \\
& - 2 * t_{21} + t_{31}) * p + (-h * t_{12} + t_{11} * (t_{11} - 2 * t_{21} + t_{31})) * t_{32} + t_{12} * t_{31} * (t_{11} - 2 * \\
& t_{21} + t_{31})) * q^2 + (t_{11} - 2 * t_{21} + t_{31}) * p^2 + (h * t_{32} + h * t_{12} - (t_{11} + t_{31}) * (t_{11} \\
& - 2 * t_{21} + t_{31})) * p + (2 * t_{30} - 2 * h + 2 * t_{20}) * t_{21} + (-t_{30} + h - t_{20}) * t_{31} - t_{11} * \\
& (t_{20} + t_{30})) * t_{32} + (-2 * t_{21} * t_{30} + (-h + t_{30}) * t_{31} + t_{30} * t_{11}) * t_{12} + t_{11} * t_{31} * \\
& (t_{11} - 2 * t_{21} + t_{31}) * q - (p * h + (-2 * h + 2 * t_{20}) * t_{21} - t_{20} * (t_{11} + t_{31})) * p) * \\
& t_{22}^2 + (-t_{12} * t_{32} * (t_{32} * (t_{11} - t_{21}) + t_{12} * (-t_{21} + t_{31})) * q^3 + (t_{32}^2 * (t_{11} \\
& - t_{21}) + t_{12}^2 * (-t_{21} + t_{31})) * p + (h * t_{12} - t_{11} * (t_{11} - t_{21})) * t_{32}^2 - 2 * t_{12} * (- \\
& (1/2) * h * t_{12} + (t_{11} - t_{21}) * (-t_{21} + t_{31})) * t_{32} - t_{31} * (-t_{21} + t_{31}) * t_{12}^2) * \\
& q^2 + ((-t_{21} + t_{31}) * t_{32} + t_{12} * (t_{11} - t_{21})) * p^2 + (-h * t_{32}^2 + (-2 * h * t_{12} - 2 * \\
& t_{21} * (-t_{21} + t_{31})) * t_{32} - t_{12} * (h * t_{12} + 2 * t_{21} * (t_{11} - t_{21}))) * p + ((-t_{30} + h - \\
& t_{20}) * t_{21} + t_{11} * (t_{20} + t_{30})) * t_{32}^2 + ((3 * h - 3 * t_{20}) * t_{21} + (-h + 2 * t_{20} + t_{30}) * \\
& t_{31} - (-t_{20} + t_{30}) * t_{11}) * t_{12} - t_{11} * (-2 * t_{21} + t_{11}) * (-t_{21} + t_{31})) * t_{32} - t_{12} * (\\
& (-t_{21} * t_{30} + (-h + t_{30}) * t_{31}) * t_{12} + t_{31} * (t_{11} - t_{21}) * (t_{31} - 2 * t_{21})) * q - (t_{11} \\
& - 2 * t_{21} + t_{31}) * p^2 + (-h * t_{32} - h * t_{12} - (t_{11} + t_{21} + t_{31}) * (t_{11} - 2 * t_{21} + t_{31})) * p + \\
& (2 * t_{30} + h - t_{20}) * t_{21} + (h - t_{30}) * t_{31} - (-t_{20} + t_{30}) * t_{11}) * t_{32} + ((-2 * t_{30} + 3 * \\
& h - 3 * t_{20}) * t_{21} + (-h + 2 * t_{20} + t_{30}) * t_{31} + t_{11} * (t_{20} + t_{30})) * t_{12} + (t_{11} - 2 * t_{21} + \\
& t_{31}) * (t_{11} + t_{31}) * t_{21} + t_{11} * t_{31}) * p) * t_{22} + t_{12} * (t_{32} * (t_{11} - t_{21}) + t_{12} * (- \\
& t_{21} + t_{31})) * p + (-h * t_{12} - t_{21} * (t_{11} - t_{21})) * t_{32} - t_{12} * t_{21} * (-t_{21} + t_{31})) * t_{32} * \\
& q^2 + (-t_{12} + t_{32}) * (t_{32} * (t_{11} - t_{21}) + t_{12} * (-t_{21} + t_{31})) * p^2 + (t_{12} * h + \\
& t_{11}^2 - t_{21}^2) * t_{32}^2 + (h * t_{12}^2 + (2 * t_{11} * t_{31} - 2 * t_{21}^2) * t_{12}) * t_{32} + t_{12}^2 * \\
& (-t_{21}^2 + t_{31}^2)) * p + ((-h + t_{20}) * t_{21} - t_{11} * t_{20}) * t_{12} - t_{11} * t_{21} * (t_{11} - t_{21}) \\
&) * t_{32}^2 - t_{12} * ((h - t_{20}) * t_{21} + t_{31} * t_{20}) * t_{12} + (-t_{11} - t_{31}) * t_{21}^2 + 2 * t_{11} * \\
& t_{21} * t_{31}) * t_{32} - t_{31} * t_{12}^2 * t_{21} * (-t_{21} + t_{31})) * q + (t_{32} * (t_{11} - t_{21}) + t_{12} * (- \\
& t_{21} + t_{31})) * p^2 + ((-h * t_{12} - (t_{11} - t_{21}) * (t_{11} + t_{21} + t_{31})) * t_{32} - t_{12} * (-t_{21} + \\
& t_{31}) * (t_{11} + t_{21} + t_{31})) * p - t_{30} * (t_{11} - t_{21}) * t_{32}^2 + ((h - t_{20}) * t_{21} + (h - t_{30}) \\
& * t_{31} + t_{11} * (t_{20} + t_{30})) * t_{12} + (t_{11} - t_{21}) * (t_{11} + t_{31}) * t_{21} + t_{11} * t_{31}) * t_{32} +
\end{aligned}$$

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t12*((t30-h+t20)*t12+(t11+t31)*t21+t11*t31)*(-t21+t31))*p)/(2*
(t22-t32)^2*(t12-t22)^2)
-((-t31*t11-(t20+t30)*(t12-t32))*t21+t11*t20*(-t32+t22))*((t12
-2*t22+t32)*t21+(t11+t31)*t22-t11*t32-t12*t31)/(2*(-t32+t22)^2*
(-t22+t12)^2)
+F22(t11,t21,t31,t12,t22,t32)
,q,p);

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Hame32:=unapply( ((-2*(t31-(1/2)*t11-(1/2)*t21))*t12*t22*q^3+
(2*(t31-(1/2)*t11-(1/2)*t21))*(t12+t22)*p+(-h*t12-(2*(t31-
(1/2)*t11-(1/2)*t21))*t11)*t22-(2*(t31-(1/2)*t11-(1/2)*t21))*
t12*t21)*q^2+((t11+t21-2*t31)*p^2+(h*t22+h*t12+(2*(t31-(1/2)*
t11-(1/2)*t21))*(t11+t21))*p+(2*t30-2*h+2*t20)*t31+(-t30+h-
t20)*t21-t11*(t20+t30))*t22+(-2*t31*t20+(-h+t20)*t21+t11*t20)*
t12-2*t21*(t31-(1/2)*t11-(1/2)*t21)*t11)*q-p*(p*h+(2*t30-2*h)*
t31-t30*(t11+t21))*t32^2+(t12*((-t11+t31)*t22+t12*(-t21+t31))*
t22*q^3+(((t11-t31)*t22^2-t12^2*(-t21+t31))*p+(h*t12+t11*(-t11+
t31))*t22^2-2*t12*(-(1/2)*h*t12+(-t21+t31)*(-t11+t31))*t22+t21*
t12^2*(-t21+t31))*q^2+(((t21-t31)*t22-t12*(-t11+t31))*p^2+(-h*
t22^2+(-2*h*t12+2*t31*(-t21+t31))*t22-h*t12^2+2*t31*(-t11+t31)*
t12)*p+((-t30+h-t20)*t31+t11*(t20+t30))*t22^2+((-3*t30+3*h)*
t31+(-h+t20+2*t30)*t21+(-t20+t30)*t11)*t12-(2*(-t21+t31))*(-
(1/2)*t11+t31)*t11)*t22-2*t12*((-1/2)*t31*t20-(1/2)*(h-t20)*
t21)*t12+(-t11+t31)*t21*(t31-(1/2)*t21))*q+(2*((t31-(1/2)*t11-
(1/2)*t21)*p^2+(1/2)*h*t22+(1/2)*h*t12-(t31-(1/2)*t11-(1/2)*
t21)*(t11+t21+t31))*p+((-1/2)*h-t20+(1/2)*t30)*t31+t21*((1/2)*
t20-(1/2)*h)-(1/2)*(-t20+t30)*t11)*t22+((t20+3*t30*(1/2)-3*h*
(1/2))*t31+(-t30+(1/2)*h-(1/2)*t20)*t21-(1/2)*t11*(t20+t30))*
t12+(t31-(1/2)*t11-(1/2)*t21)*((t11+t21)*t31+t11*t21))*p)*t32-
t12*t22*(((-t11+t31)*t22+t12*(-t21+t31))*p+(h*t12-t31*(-t11+
t31))*t22-t12*t31*(-t21+t31))*q^2+((t12+t22)*((-t11+t31)*t22+
t12*(-t21+t31))*p^2+((t12*h+t11^2-t31^2)*t22^2+(h*t12^2+(2*t11*
t21-2*t31^2)*t12)*t22+(t21^2-t31^2)*t12^2)*p+((-h+t30)*t31-
t30*t11)*t12+t11*t31*(-t11+t31))*t22^2+((-t21*t30+(-h+t30)*t31)
*t12+t31*(t11+t21)*t31-2*t11*t21))*t12*t22+t31*t12^2*t21*(-
t21+t31))*q-(((t11+t31)*t22+t12*(-t21+t31))*p^2+(h*t12-(-t11+
t31)*(t11+t21+t31))*t22-t12*(-t21+t31)*(t11+t21+t31))*p-t20*(-
t11+t31)*t22^2+((-h+t30)*t31+(-h+t20)*t21-t11*(t20+t30))*t12+
(-t11+t31)*(t11+t21)*t31+t11*t21))*t22+t12*((t30-h+t20)*t12+
(t11+t21)*t31+t11*t21)*(-t21+t31))*p)/(2*(t22-t32)^2*(t12-t32)
^2)
+(((t20+t30)*t32-t31*t11-(t20+t30)*t12)*t21+t11*t20*(-t32+t22))

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$$\begin{aligned}
& * ((t12-t32)*t21+(-t11+2*t31)*t32+(-t12-t22)*t31+t11*t22) / (2*(- \\
& t32+t22)^2*(t12-t32)^2) \\
& +F32(t11, t21, t31, t12, t22, t32) \\
& , q, p);
\end{aligned}$$

$$\begin{aligned}
P1 & := \lambda \rightarrow (t12 + t22 + t32) \lambda + t11 + t21 + t31 & (1.1) \\
P2 & := \lambda \rightarrow (t12 t22 + t12 t32 + t22 t32) \lambda^2 + ((t21 + t31) t12 + (t11 + t31) t22 + t32 (t11 \\
& + t21)) \lambda - (-t20 - t30) t12 - t20 t22 - t30 t32 + t21 t11 + t31 t11 + t21 t31 \\
P3 & := \lambda \rightarrow t12 t22 t32 \lambda^3 + (t11 t22 t32 + t12 t21 t32 + t12 t22 t31) \lambda^2 + ((-t20 \\
& - t30) t22 t32 + t32 t11 t21 + t22 t11 t31 + t12 t32 t20 + t12 t21 t31 + t12 t22 t30) \lambda \\
F22 & := (t11, t21, t31, t12, t22, t32) \rightarrow 0 \\
F11 & := (t11, t21, t31, t12, t22, t32) \rightarrow 0 \\
F31 & := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t21 \left(-\frac{1}{t22 - t32} - \frac{\ln(t22 - t32)}{t12 - t32} \right. \\
& \left. + \frac{\ln(t12 - t22)}{t12 - t32} \right) \\
F12 & := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t21 t31 \left(\frac{\ln(t22 - t32)}{(t12 - t32)^2} \right. \\
& \left. - \frac{\ln(t12 - t22)}{(t12 - t32)^2} \right) \\
F32 & := (t11, t21, t31, t12, t22, t32) \\
& \rightarrow \frac{1}{2} \frac{h t21 t31 (\ln(t12 - t22) - \ln(t22 - t32))}{(t12 - t32)^2} \\
F21 & := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t31 \left(\frac{1}{t22 - t32} + \frac{\ln(t12 - t22)}{t12 - t32} \right. \\
& \left. - \frac{\ln(t22 - t32)}{t12 - t32} \right) \\
Hame11 & := (q, p) \rightarrow \frac{1}{(t12 - t32) (t12 - t22)} \left(-p^3 + ((t12 + t22 + t32) q + t11 + t21 \right. \\
& + t31) p^2 + (((-t22 - t32) t12 - t22 t32) q^2 + ((-t31 - t21) t12 + (-t11 - t31) t22 \\
& - t32 (t11 + t21)) q + (-t20 - t30) t12 + t20 t22 + t30 t32 + (-t31 - t21) t11 \\
& - t21 t31) p + q (q^2 t12 t22 t32 + ((t21 t32 + t22 t31) t12 + t22 t32 t11) q + (t20 t32 \\
& + t21 t31 + t22 t30) t12 + ((-t20 - t30) t32 + t31 t11) t22 + t32 t11 t21) \\
& \left. + \frac{((t20 + t30) t12 + t31 t11 - t32 t20 - t30 t32) t21 - t11 t20 (-t32 + t22)}{(t12 - t32) (t12 - t22)} \right) \\
Hame21 & := (q, p) \rightarrow \frac{1}{(-t32 + t22) (t12 - t22)} \left(p^3 + ((-t12 - t22 - t32) q - t31 - t11 \right. \\
& - t21) p^2 + (((t12 + t32) t22 + t12 t32) q^2 + ((t21 + t31) t12 + (t11 + t31) t22 \\
& + t32 (t11 + t21)) q + (h - t20) t22 + (t30 - h + t20) t12 - t30 t32 + t11 (t21 \\
& + t31) + t21 t31) p - q (q^2 t12 t22 t32 + ((t11 t32 + t12 t31) t22 + t12 t32 t21) q \\
& + (t12 t30 + (-t30 + h - t20) t32 + t31 t11) t22 + ((-h + t20) t32 + t21 t31) t12 \\
& + t32 t11 t21) \\
& \left. + \frac{((-t12 + t32) t20 - t31 t11 - t12 t30 + t30 t32) t21 + t11 t20 (-t32 + t22)}{(-t32 + t22) (t12 - t22)} \right)
\end{aligned}$$

$$+ \frac{1}{2} h t_{31} \left(\frac{1}{-t_{32} + t_{22}} + \frac{\ln(t_{12} - t_{22})}{t_{12} - t_{32}} - \frac{\ln(-t_{32} + t_{22})}{t_{12} - t_{32}} \right)$$

$$\begin{aligned} \text{Hame31} := (q, p) \rightarrow & \frac{1}{(-t_{32} + t_{22})(t_{12} - t_{32})} \left(-p^3 + ((t_{12} + t_{22} + t_{32})q + t_{11} + t_{21} \right. \\ & + t_{31})p^2 + (((-t_{12} - t_{22})t_{32} - t_{12}t_{22})q^2 + ((-t_{21} - t_{11})t_{32} + (-t_{31} - t_{21})t_{12} \\ & - (t_{11} + t_{31})t_{22})q + (-h + t_{30})t_{32} + (-t_{30} + h - t_{20})t_{12} + t_{20}t_{22} + (-t_{31} \\ & - t_{21})t_{11} - t_{21}t_{31})p + q \left(q^2 t_{12} t_{22} t_{32} + ((t_{11} t_{22} + t_{12} t_{21}) t_{32} + t_{12} t_{22} t_{31}) q \right. \\ & + (t_{12} t_{20} + (-t_{30} + h - t_{20}) t_{22} + t_{21} t_{11}) t_{32} + ((-h + t_{30}) t_{22} + t_{21} t_{31}) t_{12} \\ & \left. + t_{22} t_{11} t_{31} \right) \\ & + \frac{((-t_{20} - t_{30}) t_{32} + t_{31} t_{11} + t_{12} t_{20} + t_{12} t_{30}) t_{21} - t_{11} t_{20} (-t_{32} + t_{22})}{(-t_{32} + t_{22})(t_{12} - t_{32})} \\ & + \frac{1}{2} h t_{21} \left(-\frac{1}{-t_{32} + t_{22}} - \frac{\ln(-t_{32} + t_{22})}{t_{12} - t_{32}} + \frac{\ln(t_{12} - t_{22})}{t_{12} - t_{32}} \right) \end{aligned}$$

$$\begin{aligned} \text{Hame12} := (q, p) \rightarrow & \frac{1}{2} \frac{1}{(t_{12} - t_{32})^2 (t_{12} - t_{22})^2} \left((t_{32} t_{22} (t_{21} + t_{31} - 2 t_{11}) q^3 + (\right. \\ & - (t_{22} + t_{32}) (t_{21} + t_{31} - 2 t_{11}) p + (-h t_{32} + t_{31} (t_{21} + t_{31} - 2 t_{11})) t_{22} \\ & + t_{32} t_{21} (t_{21} + t_{31} - 2 t_{11})) q^2 + ((t_{21} + t_{31} - 2 t_{11}) p^2 + (h t_{22} + h t_{32} - (t_{21} \\ & + t_{31}) (t_{21} + t_{31} - 2 t_{11})) p + (-2 t_{30} t_{11} + t_{21} t_{30} + (-h + t_{30}) t_{31}) t_{22} + (\\ & -2 t_{11} t_{20} + (-h + t_{20}) t_{21} + t_{31} t_{20}) t_{32} + t_{31} t_{21} (t_{21} + t_{31} - 2 t_{11})) q - (p h \\ & + (-2 t_{30} - 2 t_{20}) t_{11} + (t_{21} + t_{31}) (t_{30} - h + t_{20})) p) t_{12}^2 + \left(-((-t_{11} \right. \\ & + t_{31}) t_{22} - t_{32} (t_{11} - t_{21})) t_{32} t_{22} q^3 + \left(((-t_{11} + t_{31}) t_{22}^2 - t_{32}^2 (t_{11} - t_{21})) p \right. \\ & + (h t_{32} - t_{31} (-t_{11} + t_{31})) t_{22}^2 + 2 \left(\frac{1}{2} h t_{32} + (t_{11} - t_{21}) (-t_{11} + t_{31}) \right) t_{32} t_{22} \\ & \left. + t_{21} t_{32}^2 (t_{11} - t_{21}) \right) q^2 + \left(((-t_{11} + t_{21}) t_{22} + t_{32} (-t_{11} + t_{31})) p^2 + (-h t_{22}^2 \right. \\ & + (-2 h t_{32} + 2 t_{11} (t_{11} - t_{21})) t_{22} - h t_{32}^2 - 2 t_{11} (-t_{11} + t_{31}) t_{32}) p + (t_{30} t_{11} \\ & - (-h + t_{30}) t_{31}) t_{22}^2 + ((3 t_{20} + 3 t_{30}) t_{11} + (h - t_{20} - 2 t_{30}) t_{21} - (-h \\ & + 2 t_{20} + t_{30}) t_{31}) t_{32} + t_{31} (t_{11} - t_{21}) (-2 t_{11} + t_{31})) t_{22} + 2 \left(\left(\frac{1}{2} t_{11} t_{20} \right. \right. \\ & \left. \left. + \frac{1}{2} (h - t_{20}) t_{21} \right) t_{32} + (-t_{11} + t_{31}) t_{21} \left(-\frac{1}{2} t_{21} + t_{11} \right) \right) t_{32} \Big) q - p \left((t_{21} \right. \\ & + t_{31} - 2 t_{11}) p^2 + (-h t_{22} - h t_{32} - (t_{11} + t_{21} + t_{31}) (t_{21} + t_{31} - 2 t_{11})) p \\ & + ((3 t_{20} + t_{30}) t_{11} + (h - t_{20}) t_{21} - (-h + 2 t_{20} + t_{30}) t_{31}) t_{22} + (t_{11} (t_{20} \\ & + 3 t_{30}) + (h - t_{20} - 2 t_{30}) t_{21} - (-h + t_{30}) t_{31}) t_{32} + (t_{21} + t_{31} \\ & - 2 t_{11}) (t_{11} (t_{21} + t_{31}) + t_{21} t_{31})) t_{12} + t_{32} (((-t_{11} + t_{31}) t_{22} - t_{32} (t_{11} \\ & - t_{21})) p + (-h t_{32} - t_{11} (-t_{11} + t_{31})) t_{22} + t_{32} t_{11} (t_{11} - t_{21})) t_{22} q^2 + \left(-((- \right. \\ & - t_{11} + t_{31}) t_{22} - t_{32} (t_{11} - t_{21})) (t_{22} + t_{32}) p^2 + ((h t_{32} - t_{11}^2 + t_{31}^2) t_{22}^2 \\ & + (h t_{32}^2 + (-2 t_{11}^2 + 2 t_{21} t_{31}) t_{32}) t_{22} + (-t_{11}^2 + t_{21}^2) t_{32}^2) p + (((-t_{20} \end{aligned}$$

$$\begin{aligned}
& -t30) t11 + (t30 - h + t20) t31) t32 - t11 t31 (-t11 + t31)) t22^2 - t32 ((t11 (t20 \\
& + t30) - (t30 - h + t20) t21) t32 - (t11 (t21 + t31) - 2 t21 t31) t11) t22 \\
& + t32^2 t11 t21 (t11 - t21)) q + p ((-t11 + t31) t22 - t32 (t11 - t21)) p^2 + ((\\
& -h t32 - (-t11 + t31) (t11 + t21 + t31)) t22 + t32 (t11 - t21) (t11 + t21 + t31)) p \\
& - t20 (-t11 + t31) t22^2 + ((t11 (t20 + t30) + (h - t20) t21 - (-h + t30) t31) t32 \\
& + (-t11 + t31) (t11 (t21 + t31) + t21 t31)) t22 + t32 ((-t31 - t21) t11 + t30 t32 \\
& - t21 t31) (t11 - t21))) + \frac{1}{(t12 - t32)^2 (t12 - t22)^2} \left(\left(\left(-\frac{1}{2} t12 \right. \right. \right. \\
& \left. \left. + \frac{1}{2} t32 \right) t21 + \left(t11 - \frac{1}{2} t31 \right) t12 + \left(-\frac{1}{2} t22 - \frac{1}{2} t32 \right) t11 + \frac{1}{2} t22 t31 \right) (((\\
& -t20 - t30) t12 - t31 t11 + (t20 + t30) t32) t21 + t11 t20 (-t32 + t22)) \Big) \\
& + \frac{1}{2} h t21 t31 \left(\frac{\ln(-t32 + t22)}{(t12 - t32)^2} - \frac{\ln(t12 - t22)}{(t12 - t32)^2} \right) \\
Hame22 := (q, p) \rightarrow & \frac{1}{2} \frac{1}{(-t32 + t22)^2 (t12 - t22)^2} \left((t12 t32 (t11 - 2 t21 + t31) q^3 + (\right. \\
& - (t12 + t32) (t11 - 2 t21 + t31) p + (-h t12 + t11 (t11 - 2 t21 + t31)) t32 \\
& + t12 t31 (t11 - 2 t21 + t31)) q^2 + ((t11 - 2 t21 + t31) p^2 + (h t32 + h t12 - (t11 \\
& + t31) (t11 - 2 t21 + t31)) p + ((2 t30 - 2 h + 2 t20) t21 + (-t30 + h - t20) t31 \\
& - t11 (t20 + t30)) t32 + (-2 t21 t30 + (-h + t30) t31 + t30 t11) t12 + t11 t31 (t11 \\
& - 2 t21 + t31)) q - (p h + (-2 h + 2 t20) t21 - t20 (t11 + t31)) p) t22^2 + \left(\right. \\
& -t12 t32 (t32 (t11 - t21) + t12 (-t21 + t31)) q^3 + \left((t32^2 (t11 - t21) + t12^2 (-t21 \\
& + t31)) p + (h t12 - t11 (t11 - t21)) t32^2 - 2 t12 \left(-\frac{1}{2} h t12 + (t11 - t21) (-t21 \\
& + t31) \right) t32 - t31 (-t21 + t31) t12^2 \right) q^2 + (((-t21 + t31) t32 + t12 (t11 - t21)) p^2 \\
& + (-h t32^2 + (-2 h t12 - 2 t21 (-t21 + t31)) t32 - t12 (h t12 + 2 t21 (t11 \\
& - t21))) p + ((-t30 + h - t20) t21 + t11 (t20 + t30)) t32^2 + ((3 h - 3 t20) t21 \\
& + (-h + 2 t20 + t30) t31 - (-t20 + t30) t11) t12 - t11 (-2 t21 + t11) (-t21 \\
& + t31)) t32 - t12 ((-t21 t30 + (-h + t30) t31) t12 + t31 (t11 - t21) (t31 \\
& - 2 t21)) q - ((t11 - 2 t21 + t31) p^2 + (-h t32 - h t12 - (t11 + t21 + t31) (t11 \\
& - 2 t21 + t31)) p + ((2 t30 + h - t20) t21 + (h - t30) t31 - (-t20 + t30) t11) t32 \\
& + ((-2 t30 + 3 h - 3 t20) t21 + (-h + 2 t20 + t30) t31 + t11 (t20 + t30)) t12 + (t11 \\
& - 2 t21 + t31) ((t11 + t31) t21 + t31 t11)) p) t22 + t12 ((t32 (t11 - t21) + t12 (- \\
& -t21 + t31)) p + (-h t12 - t21 (t11 - t21)) t32 - t12 t21 (-t21 + t31)) t32 q^2 + \left(\right. \\
& - (t12 + t32) (t32 (t11 - t21) + t12 (-t21 + t31)) p^2 + (h t12 + t11^2 - t21^2) t32^2 \\
& + (h t12^2 + (2 t11 t31 - 2 t21^2) t12) t32 + t12^2 (-t21^2 + t31^2)) p + (((-h \\
& + t20) t21 - t11 t20) t12 - t11 t21 (t11 - t21)) t32^2 - t12 ((h - t20) t21 \\
& + t31 t20) t12 + (-t11 - t31) t21^2 + 2 t11 t21 t31) t32 - t31 t12^2 t21 (-t21 + t31)) q
\end{aligned}$$

$$\begin{aligned}
& + \left((t32 (t11 - t21) + t12 (-t21 + t31)) p^2 + ((-h t12 - (t11 - t21) (t11 + t21 \right. \\
& + t31)) t32 - t12 (-t21 + t31) (t11 + t21 + t31)) p - t30 (t11 - t21) t32^2 + ((h \\
& - t20) t21 + (h - t30) t31 + t11 (t20 + t30)) t12 + (t11 - t21) ((t11 + t31) t21 \\
& + t31 t11)) t32 + t12 ((t30 - h + t20) t12 + (t11 + t31) t21 + t31 t11) (-t21 \\
& + t31)) p - \frac{1}{2} \frac{1}{(-t32 + t22)^2 (t12 - t22)^2} (((-t31 t11 - (t20 + t30) (t12 \\
& - t32)) t21 + t11 t20 (-t32 + t22)) ((t12 - 2 t22 + t32) t21 + (t11 + t31) t22 \\
& - t32 t11 - t12 t31))
\end{aligned}$$

$$\begin{aligned}
\text{Hame32} := (q, p) \rightarrow & \frac{1}{2} \frac{1}{(-t32 + t22)^2 (t12 - t32)^2} \left(\left(-2 \left(t31 - \frac{1}{2} t11 \right. \right. \right. \\
& \left. \left. - \frac{1}{2} t21 \right) t12 t22 q^3 + \left(2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) (t12 + t22) p + \left(-h t12 \right. \right. \right. \\
& \left. \left. - 2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) t11 \right) t22 - 2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) t12 t21 \right) q^2 \\
& + \left((t11 + t21 - 2 t31) p^2 + \left(h t22 + h t12 + 2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) (t11 \right. \right. \\
& \left. \left. + t21) \right) p + ((2 t30 - 2 h + 2 t20) t31 + (-t30 + h - t20) t21 - t11 (t20 + t30)) t22 \right. \\
& \left. + (-2 t31 t20 + (-h + t20) t21 + t11 t20) t12 - 2 t21 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) t11 \right) \\
& q - p (p h + (2 t30 - 2 h) t31 - t30 (t11 + t21)) t32^2 + \left(t12 ((-t11 + t31) t22 \right. \\
& \left. + t12 (-t21 + t31)) t22 q^3 + \left(((t11 - t31) t22^2 - t12^2 (-t21 + t31)) p + (h t12 \right. \right. \\
& \left. \left. + t11 (-t11 + t31)) t22^2 - 2 t12 \left(-\frac{1}{2} h t12 + (-t21 + t31) (-t11 + t31) \right) t22 \right. \right. \\
& \left. \left. + t21 t12^2 (-t21 + t31) \right) q^2 + \left(((t21 - t31) t22 - t12 (-t11 + t31)) p^2 + (-h t22^2 \right. \right. \\
& \left. \left. + (-2 h t12 + 2 t31 (-t21 + t31)) t22 - h t12^2 + 2 t31 (-t11 + t31) t12) p + ((-t30 \right. \right. \\
& \left. \left. + h - t20) t31 + t11 (t20 + t30)) t22^2 + \left(((-3 t30 + 3 h) t31 + (-h + t20 \right. \right. \right. \\
& \left. \left. + 2 t30) t21 + (-t20 + t30) t11) t12 - 2 (-t21 + t31) \left(-\frac{1}{2} t11 + t31 \right) t11 \right) t22 \right. \\
& \left. - 2 t12 \left(\left(-\frac{1}{2} t31 t20 - \frac{1}{2} (h - t20) t21 \right) t12 + (-t11 + t31) t21 \left(t31 \right. \right. \right. \\
& \left. \left. - \frac{1}{2} t21 \right) \right) \right) q + 2 \left(\left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) p^2 + \left(\frac{1}{2} h t22 + \frac{1}{2} h t12 - \left(t31 \right. \right. \right. \\
& \left. \left. - \frac{1}{2} t11 - \frac{1}{2} t21 \right) (t11 + t21 + t31) \right) p + \left(\left(-\frac{1}{2} h - t20 + \frac{1}{2} t30 \right) t31 \right. \\
& \left. + t21 \left(\frac{1}{2} t20 - \frac{1}{2} h \right) - \frac{1}{2} (-t20 + t30) t11 \right) t22 + \left(\left(t20 + \frac{3}{2} t30 - \frac{3}{2} h \right) t31 \right. \\
& \left. + \left(-t30 + \frac{1}{2} h - \frac{1}{2} t20 \right) t21 - \frac{1}{2} t11 (t20 + t30) \right) t12 + \left(t31 - \frac{1}{2} t11 \right. \\
& \left. - \frac{1}{2} t21 \right) ((t11 + t21) t31 + t21 t11) p \Big) t32 - t12 t22 (((-t11 + t31) t22 + t12 (- \\
& -t21 + t31)) p + (h t12 - t31 (-t11 + t31)) t22 - t12 t31 (-t21 + t31)) q^2 + ((t12 \\
& + t22) ((-t11 + t31) t22 + t12 (-t21 + t31)) p^2 + ((h t12 + t11^2 - t31^2) t22^2
\end{aligned}$$

$$\begin{aligned}
& + (h t_{12}^2 + (2 t_{11} t_{21} - 2 t_{31}^2) t_{12}) t_{22} + (t_{21}^2 - t_{31}^2) t_{12}^2) p + (((-h + t_{30}) t_{31} \\
& - t_{30} t_{11}) t_{12} + t_{11} t_{31} (-t_{11} + t_{31})) t_{22}^2 + ((-t_{21} t_{30} + (-h + t_{30}) t_{31}) t_{12} \\
& + t_{31} ((t_{11} + t_{21}) t_{31} - 2 t_{21} t_{11})) t_{12} t_{22} + t_{31} t_{12}^2 t_{21} (-t_{21} + t_{31}) q - (((-t_{11} \\
& + t_{31}) t_{22} + t_{12} (-t_{21} + t_{31})) p^2 + ((h t_{12} - (-t_{11} + t_{31}) (t_{11} + t_{21} + t_{31})) t_{22} \\
& - t_{12} (-t_{21} + t_{31}) (t_{11} + t_{21} + t_{31})) p - t_{20} (-t_{11} + t_{31}) t_{22}^2 + (((-h + t_{30}) t_{31} \\
& + (-h + t_{20}) t_{21} - t_{11} (t_{20} + t_{30})) t_{12} + (-t_{11} + t_{31}) ((t_{11} + t_{21}) t_{31} + t_{21} t_{11})) \\
& t_{22} + t_{12} ((t_{30} - h + t_{20}) t_{12} + (t_{11} + t_{21}) t_{31} + t_{21} t_{11}) (-t_{21} + t_{31})) p) \\
& + \frac{1}{2} \frac{1}{(-t_{32} + t_{22})^2 (t_{12} - t_{32})^2} (((t_{20} + t_{30}) t_{32} - t_{31} t_{11} - (t_{20} + t_{30}) t_{12}) t_{21} \\
& + t_{11} t_{20} (-t_{32} + t_{22})) ((t_{12} - t_{32}) t_{21} + (-t_{11} + 2 t_{31}) t_{32} + (-t_{12} - t_{22}) t_{31} \\
& + t_{22} t_{11})) + \frac{1}{2} \frac{h t_{21} t_{31} (\ln(t_{12} - t_{22}) - \ln(-t_{32} + t_{22}))}{(t_{12} - t_{32})^2}
\end{aligned}$$

> **Hame11bis:=unapply(1/((t12-t32)*(t12-t22))*(-p^3+ P1(q)*p^2-P2(q)*p+P3(q)) +t11*t21*t31/((t12-t22)*(t12-t32)) -t11*t20*(t22-t32)/((t12-t22)*(t12-t32)) -t10*t21/(t12-t22),q,p):**

t11*t21*t31/((t12-t22)*(t12-t32))-t11*t20*(t22-t32)/((t12-t22)*(t12-t32)) -t10*t21/(t12-t22);

series(simplify(Hame11(q,p)-Hame11bis(q,p)),t21);

Hame21bis:=unapply(-1/((t22-t32)*(t12-t22))*(-p^3+ P1(q)*p^2-P2(q)*p+P3(q)) +(p-q*t32)*h/(t32-t22),q,p):

series(simplify(Hame21(q,p)-(Hame21bis(q,p)-t11*t21*t31/((t12-t22)*(t22-t32))

+t11*t20/(t12-t22)+t10*(t12-t32)*t21/((t22-t32)*(t12-t22))+F21(t11,t21,t31,t12,t22,t32)),t21);

-t11*t21*t31/((t12-t22)*(t22-t32))

+t11*t20/(t12-t22)+t10*(t12-t32)*t21/((t22-t32)*(t12-t22))+F21(t11,t21,t31,t12,t22,t32);

Hame31bis:=unapply(1/((t22-t32)*(t12-t32))*(-p^3+ P1(q)*p^2-P2(q)*p+P3(q)) -(p-q*t22)*h/(t32-t22),q,p):

series(simplify(Hame31(q,p)-(Hame31bis(q,p)+t11*t21*t31/((t22-t32)*(t12-t32))-t11*t20/(t12-t32)-t10*t21/(t22-t32) +F31(t11,t21,t31,t12,t22,t32))),t31);

t11*t21*t31/((t22-t32)*(t12-t32))-t11*t20/(t12-t32)-t10*t21/(t22-t32) +F31(t11,t21,t31,t12,t22,t32);

$$\begin{aligned}
& \frac{t_{11} t_{21} t_{31}}{(t_{12} - t_{32}) (t_{12} - t_{22})} - \frac{t_{11} t_{20} (-t_{32} + t_{22})}{(t_{12} - t_{32}) (t_{12} - t_{22})} - \frac{(-t_{20} - t_{30}) t_{21}}{t_{12} - t_{22}} \\
& - \frac{t_{11} t_{21} t_{31}}{(-t_{32} + t_{22}) (t_{12} - t_{22})} + \frac{t_{11} t_{20}}{t_{12} - t_{22}} + \frac{(-t_{20} - t_{30}) (t_{12} - t_{32}) t_{21}}{(-t_{32} + t_{22}) (t_{12} - t_{22})} \\
& + \frac{1}{2} h t_{31} \left(\frac{1}{-t_{32} + t_{22}} + \frac{\ln(t_{12} - t_{22})}{t_{12} - t_{32}} - \frac{\ln(-t_{32} + t_{22})}{t_{12} - t_{32}} \right)
\end{aligned} \tag{1.2}$$

$$\frac{t11 t21 t31}{(-t32 + t22) (t12 - t32)} - \frac{t11 t20}{t12 - t32} - \frac{0}{(-t20 - t30) t21} + \frac{1}{2} h t21 \left(-\frac{1}{-t32 + t22} - \frac{\ln(-t32 + t22)}{t12 - t32} + \frac{\ln(t12 - t22)}{t12 - t32} \right)$$

```
> Hamal:=unapply( simplify(2*(t12-t32)*(t12-t22)*Hame12(q,p)+
(t12*(3*t11-t21-t31)-(t12+t22+t32)*t11+ t22*t31+t32*t21)*Hame11
(q,p)), q,p):
Hamalbis:=unapply( h*( (t22+t32)*q*p-p^2+(t21+t31)*p -t32*
t22*q^2 -(t22*t31+t32*t21)*q), q,p):
```

```
Hama2:=unapply( simplify(2*(t22-t12)*(t22-t32)*Hame22(q,p)+
(t22*(3*t21-t11-t31)- (t12+t22+t32)*t21+t12*t31+t32*t11)*Hame21
(q,p)), q,p):
```

```
Hama2bis:=unapply( h*( (t12+t32)*q*p-p^2+(t11+t31)*p -t32*t12*
q^2 -(t12*t31+t32*t11)*q), q,p):
```

```
Hama3:=unapply( simplify(2*(t32-t12)*(t32-t22)*Hame32(q,p)+
(t32*(3*t31-t11-t21)- (t12+t22+t32)*t31+t12*t21+t22*t11)*Hame31
(q,p)), q,p):
```

```
Hama3bis:=unapply( h*( (t12+t22)*q*p-p^2+(t11+t21)*p -t22*t12*
q^2 -(t12*t21+t22*t11)*q), q,p):
```

```
> Hamv1:=unapply(simplify(Hame11(q,p)+Hame21(q,p)+Hame31(q,p)),
q,p):
Termv1:=h/2*( (t31-t21)/(t22-t32) + (ln(t12-t22)-ln(t22-t32))*
(t21+t31)/(t12-t32));
simplify(Hamv1(q,p) - (-h*q+Termv1));
```

```
Hamv2:=unapply(simplify(Hame12(q,p)+Hame22(q,p)+Hame32(q,p)),
q,p):
simplify(series(Hamv2(q,p)+h*q^2/2, q=0));
```

```
Hamu1:=unapply(simplify(t12*Hame11(q,p)+t22*Hame21(q,p)+t32*
Hame31(q,p)), q,p):
Termu1:=h/2*( (t22*t31-t21*t32)/(t22-t32)+(ln(t12-t22)-ln(t22-
t32))*(t21*t32+t22*t31)/(t12-t32) );
simplify(Hamu1(q,p) - (-h*p+Termu1));
```

```
Hamu2:=unapply(simplify(2*t12*Hame12(q,p)+2*t22*Hame22(q,p)+2*
t32*Hame32(q,p)+t11*Hame11(q,p)+t21*Hame21(q,p)+t31*Hame31(q,p)
), q,p):
simplify(series(Hamu2(q,p)+h*p*q, p=0));
```

$$\text{Termv1} := \frac{1}{2} h \left(\frac{-t21 + t31}{-t32 + t22} + \frac{(\ln(t12 - t22) - \ln(-t32 + t22)) (t21 + t31)}{t12 - t32} \right) \quad (1.3)$$

$$\text{Termu1} := \frac{1}{2} h \left(\frac{-t21 t32 + t22 t31}{-t32 + t22} + \frac{(\ln(t12 - t22) - \ln(-t32 + t22)) (t21 t32 + t22 t31)}{t12 - t32} \right)$$

We load the change of times and coordinates

```

> TT1:=unapply (t12+t22+t32, t11, t21, t31, t12, t22, t32) ;
TT2:=unapply (t11+t21+t31, t11, t21, t31, t12, t22, t32) ;
TT3:=unapply (t22, t11, t21, t31, t12, t22, t32) ;
TT4:=unapply (t11, t11, t21, t31, t12, t22, t32) ;
TT5:=unapply (t32, t11, t21, t31, t12, t22, t32) ;
taufunction:=unapply( ((t21-t31)*t12+(t31-t11)*t22+(t11-t21)*
t32)/(sqrt((t22-t12)*(t12-t32)*(t32-t22))), t11, t21, t31, t12,
t22, t32);
solve({TT1(t11, t21, t31, t12, t22, t32)=T1, TT2(t11, t21, t31, t12, t22,
t32)=T2, TT3(t11, t21, t31, t12, t22, t32)=T3, TT4(t11, t21, t31, t12,
t22, t32)=T4, TT5(t11, t21, t31, t12, t22, t32)=T5, taufunction(t11,
t21, t31, t12, t22, t32)=tau }, {t11, t21, t31, t12, t22, t32});

t11function := T4;
t12function := -T3-T5+T1;
t21function := (tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))+
T1*T2-T1*T4-2*T3*T2-T5*T2+3*T4*T3)/(2*T1-3*T3-3*T5);
t22function := T3;
t31function := -(tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))-
T1*T2+T1*T4+T3*T2+2*T5*T2-3*T4*T5)/(2*T1-3*T3-3*T5);
t32function := T5;
simplify(TT1(t11function, t21function, t31function, t12function,
t22function, t32function));
simplify(TT2(t11function, t21function, t31function, t12function,
t22function, t32function));
simplify(TT3(t11function, t21function, t31function, t12function,
t22function, t32function));
simplify(TT4(t11function, t21function, t31function, t12function,
t22function, t32function));
simplify(TT5(t11function, t21function, t31function, t12function,
t22function, t32function));
simplify(taufunction(t11function, t21function, t31function,

```

```

t12function, t22function, t32function) );

partialtaufunction:=unapply( diff(t11function, tau)*partialt11 +
diff(t21function, tau)*partialt21+diff(t31function, tau)*
partialt31+diff(t12function, tau)*partialt12+diff(t22function,
tau)*partialt22+diff(t32function, tau)*partialt32, T1, T2, T3, T4,
T5, tau) :
partialtau:=partialtaufunction(TT1(t11, t21, t31, t12, t22, t32), TT2
(t11, t21, t31, t12, t22, t32), TT3(t11, t21, t31, t12, t22, t32), TT4(t11,
t21, t31, t12, t22, t32), TT5(t11, t21, t31, t12, t22, t32), taufunction
(t11, t21, t31, t12, t22, t32) ) :

Coefft21:=factor(residue(partialtau/partialt21^2, partialt21=0))
;
Coefft31:=factor(residue(partialtau/partialt31^2, partialt31=0))
;

Hamttau:= unapply( simplify(Coefft21*Hame21(q, p)+Coefft31*Hame31
(q, p)), q, p) :
Ltauq:=diff(Hamttau(q, p), p) :
Ltaup:=-diff(Hamttau(q, p), q) :

checkSolq:=unapply( sqrt((t12-t32)/((t22-t12)*(t32-t22)))*(-p+
t22*q+t21), q, p) ;
checkSolp:=unapply( sqrt((t32-t22)/((t12-t32)*(t22-t12)))*(p-
t12*q-t11), q, p) ;

SOLL:=solve({checkSolq(q, p)=checkq, checkSolp(q, p)=checkp}, {q, p}
) :
Solp:=simplify(rhs(SOLL[1])) :
Solq:=simplify(rhs(SOLL[2])) :
simplify(checkSolq(Solq, Solp)-checkq) ;
simplify(checkSolp(Solq, Solp)-checkp) ;

LtauchekSolq:=unapply( simplify( diff(checkSolq(q, p), q)*Ltauq+
diff(checkSolq(q, p), p)*Ltaup+Coefft21*h*diff(checkSolq(q, p),
t21)+Coefft31*h*diff(checkSolq(q, p), t31)), q, p) :
LtauchekSolp:=unapply(simplify( diff(checkSolp(q, p), q)*Ltauq+
diff(checkSolp(q, p), p)*Ltaup+Coefft21*h*diff(checkSolp(q, p),
t21)+Coefft31*h*diff(checkSolp(q, p), t31)), q, p) :

```

```

LtauchekSolq2:=unapply(simplify(LtauchekSolq(Solq,Solp)),t11,
t21, t31, t12, t22, t32):
LtauchekSolp2:=unapply(simplify(LtauchekSolp(Solq,Solp)),t11,
t21, t31, t12, t22, t32):
Ltauchekq:=simplify(LtauchekSolq2(t11function,t21function,
t31function,t12function,t22function,t32function)):
Ltauchekp:=simplify(LtauchekSolp2(t11function,t21function,
t31function,t12function,t22function,t32function)):
HamtauchekCoordinates:=unapply(simplify(int(Ltauchekq,checkp)
-int(simplify(diff(int(Ltauchekq,checkp),checkq)+Ltauchekp),
checkq)),checkq,checkp);
simplify(Ltauchekq-diff(HamtauchekCoordinates(checkq,checkp),
checkp));
simplify(Ltauchekp+diff(HamtauchekCoordinates(checkq,checkp),
checkq));
solve(taufunction(t11,t21,t31,t12,t22,t32)=tau,t31):
t31function:=- (tau*sqrt((t22-t12)*(t12-t32)*(t32-t22))-t12*t21+
t22*t11-t32*t11+t32*t21)/(t12-t22):
simplify(taufunction(t11,t21,t31function,t12,t22,t32));
TT1 := (t11, t21, t31, t12, t22, t32) → t12 + t22 + t32
TT2 := (t11, t21, t31, t12, t22, t32) → t11 + t21 + t31
TT3 := (t11, t21, t31, t12, t22, t32) → t22
TT4 := (t11, t21, t31, t12, t22, t32) → t11
TT5 := (t11, t21, t31, t12, t22, t32) → t32
taufunction := (t11, t21, t31, t12, t22, t32)
→ 
$$\frac{(t21 - t31) t12 + (-t11 + t31) t22 + t32 (t11 - t21)}{\sqrt{(t22 - t12) (t12 - t32) (t32 - t22)}}$$

{ t11 = T4, t12 = -T3 - T5 + T1, t21
= 
$$\frac{1}{2 T1 - 3 T3 - 3 T5} (\tau \sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)}$$

+ T1 T2 - T1 T4 - 2 T3 T2 - T5 T2 + 3 T3 T4), t22 = T3, t31 =
- 
$$\frac{1}{2 T1 - 3 T3 - 3 T5} (\tau \sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)}$$

- T1 T2 + T1 T4 + T3 T2 + 2 T5 T2 - 3 T5 T4), t32 = T5 }
t11function := T4
t12function := -T3 - T5 + T1
t21function := 
$$\frac{1}{2 T1 - 3 T3 - 3 T5} (\tau$$


$$\sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)} + T1 T2 - T1 T4 - 2 T3 T2$$

- T5 T2 + 3 T3 T4)
t22function := T3
t31function :=

```

$$-\frac{1}{2 T1 - 3 T3 - 3 T5} \left(\tau \sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)} \right. \\ \left. - T1 T2 + T1 T4 + T3 T2 + 2 T5 T2 - 3 T5 T4 \right)$$

t32function := T5

T1

T2

T3

T4

T5

τ

$$\text{Coefft21} := \frac{\sqrt{(t12 - t22) (-t32 + t22) (t12 - t32)}}{2 t12 - t22 - t32}$$

$$\text{Coefft31} := -\frac{\sqrt{(t12 - t22) (-t32 + t22) (t12 - t32)}}{2 t12 - t22 - t32}$$

$$\text{checkSolq} := (q, p) \rightarrow \sqrt{\frac{t12 - t32}{(t22 - t12) (t32 - t22)}} (q t22 - p + t21)$$

$$\text{checkSolp} := (q, p) \rightarrow \sqrt{\frac{t32 - t22}{(t12 - t32) (t22 - t12)}} (-q t12 + p - t11)$$

0

0

HamtauchekCoordinates := (checkq, checkp)

$$\rightarrow \left(\sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)} \left(-\text{checkq} (-T3 - 2 T5 + T1) (-2 T3 - 2 T5 + T1) \right. \right.$$

$$\left. + T1) (\text{checkp}^2 + h - t20 - t30) \sqrt{\frac{-T5 + T3}{(-T3 - 2 T5 + T1) (-2 T3 - T5 + T1)}} \right.$$

$$\left. + \left(\sqrt{(-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1)} \text{checkq} \tau \right. \right.$$

$$\left. + \sqrt{\frac{-T3 - 2 T5 + T1}{(-2 T3 - T5 + T1) (-T5 + T3)}} (-2 T3 - T5 + T1) (-T5 + T3) (-\text{checkq}^2 \right.$$

$$\left. + t20) \right) \text{checkp} \Big) / \left((-2 T3 - T5 + T1) (-T5 + T3) (-T3 - 2 T5 + T1) \right)$$

0

0

τ

▼ We compute the Hamiltonian in direction *t11*, *t21*, *t31* in terms of *check{q}* and *check{p}*. Since the shift is time-dependent, we need to use the evolutions and the shift and integrate them. In particular, we cannot just simply replace *(q,p)* by their expressions in terms of *(check{q}, check{p})*.

> *Lellq := diff (Hame11 (q,p) , p) :*

Lellp := -diff (Hame11 (q,p) , q) :

```

Le21q:=diff (Hame21 (q,p) ,p) :
Le21p:=-diff (Hame21 (q,p) ,q) :
Le31q:=diff (Hame31 (q,p) ,p) :
Le31p:=-diff (Hame31 (q,p) ,q) :

```

```

> Le11checkSolq:=unapply( simplify( h*diff (checkSolq(q,p) ,t12)*0+
h*diff (checkSolq(q,p) ,t22)*0+h*diff (checkSolq(q,p) ,t32)*0+h*
diff (checkSolq(q,p) ,t11)*1+h*diff (checkSolq(q,p) ,t21)*0+h*diff
(checkSolq(q,p) ,t31)*0+diff (checkSolq(q,p) ,q)*Le11q+diff
(checkSolq(q,p) ,p)*Le11p) , q,p) :

```

```

Le11checkSolp:=unapply( simplify( h*diff (checkSolp(q,p) ,t12)*0+
h*diff (checkSolp(q,p) ,t22)*0+h*diff (checkSolp(q,p) ,t32)*0+h*
diff (checkSolp(q,p) ,t11)*1+h*diff (checkSolp(q,p) ,t21)*0+h*diff
(checkSolp(q,p) ,t31)*0+diff (checkSolp(q,p) ,q)*Le11q+diff
(checkSolp(q,p) ,p)*Le11p) , q,p) :

```

```

Le11checkq:=simplify (Le11checkSolq (Solq, Solp) ) :
Le11checkp:=simplify (Le11checkSolp (Solq, Solp) ) :

```

```

Hame11check:=unapply (simplify( int (Le11checkq,checkp) -int
(Le11checkp+diff (int (Le11checkq,checkp) ,checkq) ,checkq) ) ,
checkq, checkp) ;
simplify (Le11checkq-diff (Hame11check (checkq, checkp) ,checkp) ) ;
simplify (Le11checkp+diff (Hame11check (checkq, checkp) ,checkq) ) ;

```

$$\begin{aligned}
Hame11check := (checkq, checkp) \rightarrow & \frac{1}{(t12 - t32) (t12 - t22)} \left(checkq (t12 - t32) (t12 \right. & (2.1) \\
& - t22) (checkp^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32) (t12 - t22)}} + \left((-t32 \right. \\
& + t22) (checkq^2 - t20) (t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22) (-t32 + t22)}} + checkq (t12 (\\
& - t21 + t31) + t22 (t11 - t31) - t32 (t11 - t21)) \left. \right) checkp \left. \right) \\
& 0 \\
& 0
\end{aligned}$$

```

> Le21checkSolq:=unapply( simplify( h*diff (checkSolq(q,p) ,t12)*0+
h*diff (checkSolq(q,p) ,t22)*0+h*diff (checkSolq(q,p) ,t32)*0+h*
diff (checkSolq(q,p) ,t11)*0+h*diff (checkSolq(q,p) ,t21)*1+h*diff
(checkSolq(q,p) ,t31)*0+diff (checkSolq(q,p) ,q)*Le21q+diff
(checkSolq(q,p) ,p)*Le21p) , q,p) :

```

```

Le21checkSolp:=unapply( simplify( h*diff (checkSolp(q,p) ,t12)*0+
h*diff (checkSolp(q,p) ,t22)*0+h*diff (checkSolp(q,p) ,t32)*0+h*

```

```
diff(checkSolp(q,p),t11)*0+h*diff(checkSolp(q,p),t21)*1+h*diff
(checkSolp(q,p),t31)*0+diff(checkSolp(q,p),q)*Le21q+diff
(checkSolp(q,p),p)*Le21p),q,p):
```

```
Le21checkq:=simplify(Le21checkSolq(Solq,Solp)):
```

```
Le21checkp:=simplify(Le21checkSolp(Solq,Solp)):
```

```
Hame21check:=unapply(simplify(int(Le21checkq,checkp)-int
(Le21checkp+diff(int(Le21checkq,checkp),checkq),checkq)),
checkq,checkp);
```

```
simplify(Le21checkq-diff(Hame21check(checkq,checkp),checkp));
```

```
simplify(Le21checkp+diff(Hame21check(checkq,checkp),checkq));
```

$$Hame21check := (checkq, checkp) \rightarrow \frac{1}{(-t32 + t22)(t12 - t22)} \left(-checkq(t12 - t32)(t12 - t22)(checkp^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}} - \left((-t32 + t22)(checkq^2 - t20)(t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} + checkq(t12(-t21 + t31) + t22(t11 - t31) - t32(t11 - t21)) \right) checkp \right) \quad (2.2)$$

```
> Le31checkSolq:=unapply(simplify(h*diff(checkSolq(q,p),t12)*0+
h*diff(checkSolq(q,p),t22)*0+h*diff(checkSolq(q,p),t32)*0+h*
diff(checkSolq(q,p),t11)*0+h*diff(checkSolq(q,p),t21)*0+h*diff
(checkSolq(q,p),t31)*1+diff(checkSolq(q,p),q)*Le31q+diff
(checkSolq(q,p),p)*Le31p),q,p):
```

```
Le31checkSolp:=unapply(simplify(h*diff(checkSolp(q,p),t12)*0+
h*diff(checkSolp(q,p),t22)*0+h*diff(checkSolp(q,p),t32)*0+h*
diff(checkSolp(q,p),t11)*0+h*diff(checkSolp(q,p),t21)*0+h*diff
(checkSolp(q,p),t31)*1+diff(checkSolp(q,p),q)*Le31q+diff
(checkSolp(q,p),p)*Le31p),q,p):
```

```
Le31checkq:=simplify(Le31checkSolq(Solq,Solp)):
```

```
Le31checkp:=simplify(Le31checkSolp(Solq,Solp)):
```

```
Hame31check:=unapply(simplify(int(Le31checkq,checkp)-int
(Le31checkp+diff(int(Le31checkq,checkp),checkq),checkq)),
checkq,checkp);
```

```
simplify(Le31checkq-diff(Hame31check(checkq,checkp),checkp));
```

```
simplify(Le31checkp+diff(Hame31check(checkq,checkp),checkq));
```


$$\begin{aligned}
\text{Hame31check} := (\text{checkq}, \text{checkp}) \rightarrow & \frac{1}{(-t32 + t22)(t12 - t32)} \left(\text{checkq} (t12 - t32) (t12 \right. \\
& - t22) (\text{checkp}^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}} + \left((-t32 \right. \\
& + t22) (\text{checkq}^2 - t20) (t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} + \text{checkq} (t12 (\\
& - t21 + t31) + t22 (t11 - t31) - t32 (t11 - t21)) \left. \right) \text{checkp} \Big) \\
& \begin{matrix} 0 \\ 0 \end{matrix}
\end{aligned} \tag{2.3}$$

> Hamellcheckinter := unapply (Hamellcheck (checkq, checkp) , t31) :
simplify (series (simplify (Hamellcheckinter (t31function)) , checkp
) :
Hamellcheckbis := unapply (simplify (Hamellcheckinter (t31function)
) , checkq, checkp) ;
Hamellcheckter := unapply ((checkq*checkp^2 + (checkq^2 - checkq*tau -
t20)*checkp + (-t30+h-t20)*checkq) *sqrt (t32-t22) / (sqrt (t22-t12) *
sqrt (t12-t32)) , checkq, checkp) ;
simplify (Hamellcheckbis (checkq, checkp) - Hamellcheckter (checkq,
checkp) , symbolic) ;

$$\begin{aligned}
\text{Hamellcheckbis} := (\text{checkq}, \text{checkp}) \rightarrow & \frac{1}{(t12 - t32)(t12 - t22)} \left(\text{checkq} (t12 \right. \\
& - t32) (t12 - t22) (\text{checkp}^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}} - \left((-t32 \right. \\
& + t22) (-\text{checkq}^2 + t20) (t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} \\
& \left. + \sqrt{(t12 - t22)(-t32 + t22)(t12 - t32)} \tau \text{checkq} \right) \text{checkp} \Big) \\
\text{Hamellcheckter} := (\text{checkq}, \text{checkp}) \rightarrow & \frac{1}{\sqrt{t22 - t12} \sqrt{t12 - t32}} \left((\text{checkq} \text{checkp}^2 \right. \\
& \left. + (\text{checkq}^2 - \text{checkq} \tau - t20) \text{checkp} + (-t30 + h - t20) \text{checkq} \right) \sqrt{t32 - t22} \Big) \\
& 0
\end{aligned} \tag{2.4}$$

> Hame21checkinter := unapply (Hame21check (checkq, checkp) , t31) :
factor (simplify (series (simplify (Hame21checkinter (t31function)) ,
checkp) , symbolic)) :
Hame21checkbis := unapply (simplify (Hame21checkinter (t31function)
) , checkq, checkp) ;
Hame21checkter := unapply (-(-t30+h-t20)*checkq*sqrt (t12-t32) /
(sqrt (t12-t22) *sqrt (-t32+t22)) + (-checkq^2+checkq*tau+t20) *sqrt
(t12-t32) *checkp / (sqrt (-t32+t22) *sqrt (t12-t22)) - checkq*sqrt (t12
-t32) *checkp^2 / (sqrt (t12-t22) *sqrt (-t32+t22)) , checkq, checkp) ;
simplify (Hame21checkbis (checkq, checkp) - Hame21checkter (checkq,

checkp) , symbolic) ;

$$Hame21checkbis := (checkq, checkp) \rightarrow \frac{1}{(t12 - t22) (-t32 + t22)} \left(-checkq (t12 - t32) (t12 - t22) (checkp^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32) (t12 - t22)}} + \left((-t32 + t22) (-checkq^2 + t20) (t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22) (-t32 + t22)}} + \sqrt{(t12 - t22) (-t32 + t22) (t12 - t32)} \tau checkq \right) checkp \right) \quad (2.5)$$

$$Hame21checkter := (checkq, checkp) \rightarrow - \frac{(-t30 + h - t20) checkq \sqrt{t12 - t32}}{\sqrt{t12 - t22} \sqrt{-t32 + t22}} + \frac{(-checkq^2 + checkq \tau + t20) \sqrt{t12 - t32} checkp}{\sqrt{t12 - t22} \sqrt{-t32 + t22}} - \frac{checkq \sqrt{t12 - t32} checkp^2}{\sqrt{t12 - t22} \sqrt{-t32 + t22}}$$

**> Hame31checkinter := unapply (Hame31check (checkq, checkp) , t31) :
simplify (series (simplify (Hame31checkinter (t31function)) ,
checkp) , symbolic) :
Hame31checkbis := unapply (simplify (Hame31checkinter (t31function)
) , checkq, checkp) ;
Hame31checkter := unapply (checkq * (-t30 + h - t20) * sqrt (t12 - t22) / (sqrt
(t12 - t32) * sqrt (-t32 + t22)) - sqrt (t12 - t22) * (-checkq^2 + checkq * tau +
t20) * checkp / (sqrt (t12 - t32) * sqrt (-t32 + t22)) + checkq * sqrt (t12 - t22)
* checkp^2 / (sqrt (t12 - t32) * sqrt (-t32 + t22)) , checkq, checkp) ;
simplify (Hame31checkbis (checkq, checkp) - Hame31checkter (checkq,
checkp) , symbolic) ;**

$$Hame31checkbis := (checkq, checkp) \rightarrow \frac{1}{(-t32 + t22) (t12 - t32)} \left(checkq (t12 - t32) (t12 - t22) (checkp^2 + h - t20 - t30) \sqrt{\frac{-t32 + t22}{(t12 - t32) (t12 - t22)}} - \left((-t32 + t22) (-checkq^2 + t20) (t12 - t22) \sqrt{\frac{t12 - t32}{(t12 - t22) (-t32 + t22)}} + \sqrt{(t12 - t22) (-t32 + t22) (t12 - t32)} \tau checkq \right) checkp \right) \quad (2.6)$$

$$Hame31checkter := (checkq, checkp) \rightarrow \frac{checkq (-t30 + h - t20) \sqrt{t12 - t22}}{\sqrt{t12 - t32} \sqrt{-t32 + t22}} - \frac{\sqrt{t12 - t22} (-checkq^2 + checkq \tau + t20) checkp}{\sqrt{t12 - t32} \sqrt{-t32 + t22}} + \frac{checkq \sqrt{t12 - t22} checkp^2}{\sqrt{t12 - t32} \sqrt{-t32 + t22}}$$

We may now obtain the Hamiltonian evolution relatively to tau

**> Hamtauchek := unapply (simplify (Coefft21 * Hame21checkbis (checkq,
checkp) + Coefft31 * Hame31checkbis (checkq, checkp)) , checkq, checkp)**

```

;
Hamtauchekbis:=unapply( -(checkq*checkp^2+checkp*checkq^2-tau*
checkq*checkp -t20*checkp +checkq*(t10+h)) ,checkq,checkp);
simplify(Hamtauchek(checkq,checkp)-Hamtauchekbis(checkq,
checkp),symbolic);
hdcheckqdtau:=diff(Hamtauchek(checkq,checkp),checkp):
hdcheckpdtau:=-diff(Hamtauchek(checkq,checkp),checkq):
HamtauchekOldCoordinatesfunction:=unapply(Hamtauchek(checkq,
checkp),checkq,checkp,tau):
HamtauchekOldCoordinates:=simplify
(HamtauchekOldCoordinatesfunction(checkSolq(q,p),checkSolp(q,
p),taufunction(t11,t21,t31,t12,t22,t32))):
Hamtauchek := (checkq, checkp) →

```

$$\begin{aligned}
& -\frac{1}{(t12-t22)(-t32+t22)(t12-t32)} \left(\left(checkq(t12-t32)(t12-t22)(checkp^2 \right. \right. \\
& + h - t20 - t30) \sqrt{\frac{-t32+t22}{(t12-t32)(t12-t22)}} - \left((-t32+t22)(-checkq^2 \right. \\
& + t20)(t12-t22) \sqrt{\frac{t12-t32}{(t12-t22)(-t32+t22)}} \\
& \left. \left. + \sqrt{(t12-t22)(-t32+t22)(t12-t32)} \tau checkq \right) checkp \right) \\
& \left. \sqrt{(t12-t22)(-t32+t22)(t12-t32)} \right)
\end{aligned}$$

```

Hamtauchekbis := (checkq, checkp) → -checkq checkp^2 - checkp checkq^2
+ checkp checkq tau + checkp t20 - (-t30 + h - t20) checkq
0

```

(2.7)

We now need to compute the symplectic two form Omega using the differential geometry package to compute wedge products.

```

> DGsetup([t12,t22,t32,t11,t21,t31,q,p],B,verbose);

```

The following coordinates have been protected:

[t12,t22,t32,t11,t21,t31,q,p]

The following vector fields have been defined and protected:

```

[_DG(["vector",B,[ ]],[[1],1]),_DG(["vector",B,[ ]],[[2],1]),
_DG(["vector",B,[ ]],[[3],1]),_DG(["vector",B,[ ]],[[4],1]),
_DG(["vector",B,[ ]],[[5],1]),_DG(["vector",B,[ ]],[[6],1]),
_DG(["vector",B,[ ]],[[7],1]),_DG(["vector",B,[ ]],[[8],1])]

```

The following differential 1-forms have been defined and protected:

```

[_DG(["form",B,1],[[1],1]),_DG(["form",B,1],[[2],1]),_DG(["form",
B,1],[[3],1]),_DG(["form",B,1],[[4],1]),_DG(["form",B,1],[[5],
1]),_DG(["form",B,1],[[6],1]),_DG(["form",B,1],[[7],1]),
_DG(["form",B,1],[[8],1])]

```

frame name: B

```

B > dcheckq:=(simplify(diff(checkSolq(q,p),t12)))*(DGform(t12))

```

(3.1)

$(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), t22))) * (\text{DGform}(t22))$
 $(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), t32))) * (\text{DGform}(t32)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), t11))) * (\text{DGform}(t11)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), t21))) * (\text{DGform}(t21)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), t31))) * (\text{DGform}(t31)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolq}(q,p), q))) * (\text{DGform}(q)) + (\text{simplify}$
 $(\text{diff}(\text{checkSolq}(q,p), p))) * (\text{DGform}(p));$

$d\text{checkp} := (\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t12))) * (\text{DGform}(t12))$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t22))) * (\text{DGform}(t22))$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t32))) * (\text{DGform}(t32)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t11))) * (\text{DGform}(t11)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t21))) * (\text{DGform}(t21)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), t31))) * (\text{DGform}(t31)) +$
 $(\text{simplify}(\text{diff}(\text{checkSolp}(q,p), q))) * (\text{DGform}(q)) + (\text{simplify}$
 $(\text{diff}(\text{checkSolp}(q,p), p))) * (\text{DGform}(p));$

$$\begin{aligned}
& \frac{1}{2} \frac{(-q t22 + p - t21) \text{DG}([\text{"form"}, B, 1], [[1], 1])}{\sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} (t12 - t22)^2} + \frac{1}{2} ((q t22 - 2 q t32 + p \\
& - t21) t12 + (q t32 - 2 p + 2 t21) t22 + t32 (p - t21)) (t12 - t32) \text{DG}([\text{"form"}, \\
& B, 1], [[2], 1]) \left/ \left(\sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} (t12 - t22)^2 (-t32 + t22)^2 \right) \right. \\
& + \frac{1}{2} \frac{(q t22 - p + t21) \text{DG}([\text{"form"}, B, 1], [[3], 1])}{\sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} (-t32 + t22)^2} \\
& + \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} \text{DG}([\text{"form"}, B, 1], [[5], 1]) \\
& + \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} t22 \text{DG}([\text{"form"}, B, 1], [[7], 1]) \\
& - \sqrt{\frac{t12 - t32}{(t12 - t22)(-t32 + t22)}} \text{DG}([\text{"form"}, B, 1], [[8], 1]) \\
& - \left((-t32 + t22) \left(\left(-\frac{1}{2} q t22 - \frac{1}{2} q t32 + p - t11 \right) t12 + \left(q t32 - \frac{1}{2} p + \frac{1}{2} t11 \right) t22 \right. \right. \\
& \left. \left. - \frac{1}{2} t32 (p - t11) \right) \text{DG}([\text{"form"}, B, 1], [[1], 1]) \right) \left/ \right. \\
& \left(\sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}} (t12 - t32)^2 (t12 - t22)^2 \right) \\
& + \frac{1}{2} \frac{(-q t12 + p - t11) \text{DG}([\text{"form"}, B, 1], [[2], 1])}{\sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}} (t12 - t22)^2} \\
& + \frac{1}{2} \frac{(q t12 - p + t11) \text{DG}([\text{"form"}, B, 1], [[3], 1])}{(t12 - t32)^2 \sqrt{\frac{-t32 + t22}{(t12 - t32)(t12 - t22)}}}
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
& -\sqrt{\frac{-t_{32} + t_{22}}{(t_{12} - t_{32})(t_{12} - t_{22})}} \text{_}DG([\text{"form"}, B, 1], [[4], 1]]) \\
& -\sqrt{\frac{-t_{32} + t_{22}}{(t_{12} - t_{32})(t_{12} - t_{22})}} t_{12} \text{_}DG([\text{"form"}, B, 1], [[7], 1]]) \\
& +\sqrt{\frac{-t_{32} + t_{22}}{(t_{12} - t_{32})(t_{12} - t_{22})}} \text{_}DG([\text{"form"}, B, 1], [[8], 1]])
\end{aligned}$$

B > dcheckqwedgedcheckp:=simplify((dcheckq) &wedge (dcheckp)) :

B > dtau:=simplify((simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t12))) * (DGform(t12)) + (simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t22))) * (DGform(t22)) + (simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t32))) * (DGform(t32)) + (simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t11))) * (DGform(t11)) + (simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t21))) * (DGform(t21)) + (simplify(diff(taufunction(t11,t21,t31,t12,t22,t32),t31))) * (DGform(t31)))) ;

dHantauchekOldCoordinates:=simplify((simplify(diff(HantauchekOldCoordinates,t12))) * (DGform(t12)) + (simplify(diff(HantauchekOldCoordinates,t22))) * (DGform(t22)) + (simplify(diff(HantauchekOldCoordinates,t32))) * (DGform(t32)) + (simplify(diff(HantauchekOldCoordinates,t11))) * (DGform(t11)) + (simplify(diff(HantauchekOldCoordinates,t21))) * (DGform(t21)) + (simplify(diff(HantauchekOldCoordinates,t31))) * (DGform(t31)) + (simplify(diff(HantauchekOldCoordinates,q))) * (DGform(q)) + (simplify(diff(HantauchekOldCoordinates,p))) * (DGform(p))) :

dtaudwedgedHmtau:= simplify((dtau) &wedge (dHantauchekOldCoordinates)) :

$$\begin{aligned}
& \frac{1}{2} \left(2 \left(\left(t_{11} - \frac{1}{2} t_{21} - \frac{1}{2} t_{31} \right) t_{12} + \left(-\frac{1}{2} t_{11} + \frac{1}{2} t_{31} \right) t_{22} - \frac{1}{2} t_{32} (t_{11} - t_{21}) \right) \right. \\
& \left. - t_{32} + t_{22} \right)^2 \text{_}DG([\text{"form"}, B, 1], [[1], 1]]) - ((t_{21} - t_{31}) t_{12} + (t_{11} - 2 t_{21} \\
& + t_{31}) t_{22} - t_{32} (t_{11} - t_{21})) (t_{12} - t_{32})^2 \text{_}DG([\text{"form"}, B, 1], [[2], 1]]) + (t_{12}
\end{aligned} \tag{3.3}$$

$$\frac{-t22) ((t21 - t31) t12 + t22 (t11 - t31) - t32 (t11 + t21 - 2 t31)) (t12 - t22) _DG([["form", B, 1], [[[3], 1]]]) - 2 (-t32 + t22) (t12 - t32) ((-t32 + t22) _DG([["form", B, 1], [[[4], 1]]]) + (-t12 + t32) _DG([["form", B, 1], [[[5], 1]]]) + _DG([["form", B, 1], [[[6], 1]]]) (t12 - t22))}{(\sqrt{(t12 - t22) (-t32 + t22) (t12 - t32)} (t12 - t22) (-t32 + t22) (t12 - t32))}$$

B > Omega2:=simplify((h&mult(dcheckqwedgedcheckp) &minus (dtaudwedgedHamtau)) :

B > dHame11:=(simplify(diff(Hame11(q,p),t12)))*(DGform(t12))+ (simplify(diff(Hame11(q,p),t22)))*(DGform(t22))+ (simplify(diff(Hame11(q,p),t32)))*(DGform(t32))+ (simplify(diff(Hame11(q,p),t11)))*(DGform(t11))+ (simplify(diff(Hame11(q,p),t21)))*(DGform(t21))+ (simplify(diff(Hame11(q,p),t31)))*(DGform(t31))+ (simplify(diff(Hame11(q,p),q)))*(DGform(q))+ (simplify(diff(Hame11(q,p),p)))*(DGform(p)) :

dHame21:=(simplify(diff(Hame21(q,p),t12)))*(DGform(t12))+ (simplify(diff(Hame21(q,p),t22)))*(DGform(t22))+ (simplify(diff(Hame21(q,p),t32)))*(DGform(t32))+ (simplify(diff(Hame21(q,p),t11)))*(DGform(t11))+ (simplify(diff(Hame21(q,p),t21)))*(DGform(t21))+ (simplify(diff(Hame21(q,p),t31)))*(DGform(t31))+ (simplify(diff(Hame21(q,p),q)))*(DGform(q))+ (simplify(diff(Hame21(q,p),p)))*(DGform(p)) :

dHame31:=(simplify(diff(Hame31(q,p),t12)))*(DGform(t12))+ (simplify(diff(Hame31(q,p),t22)))*(DGform(t22))+ (simplify(diff(Hame31(q,p),t32)))*(DGform(t32))+ (simplify(diff(Hame31(q,p),t11)))*(DGform(t11))+ (simplify(diff(Hame31(q,p),t21)))*(DGform(t21))+ (simplify(diff(Hame31(q,p),t31)))*(DGform(t31))+ (simplify(diff(Hame31(q,p),q)))*(DGform(q))+ (simplify(diff(Hame31(q,p),p)))*(DGform(p)) :

dHame12:=(simplify(diff(Hame12(q,p),t12)))*(DGform(t12))+ (simplify(diff(Hame12(q,p),t22)))*(DGform(t22))+ (simplify(diff(Hame12(q,p),t32)))*(DGform(t32))+ (simplify(diff(Hame12(q,p),t11)))*(DGform(t11))+ (simplify(diff(Hame12(q,p),t21)))*(DGform(t21))+ (simplify(diff(Hame12(q,p),t31)))*(DGform(t31))+ (simplify(diff(Hame12(q,p),q)))*(DGform(q))+ (simplify(diff(Hame12(q,p),p)))*(DGform(p)) :

dHame22:=(simplify(diff(Hame22(q,p),t12)))*(DGform(t12))+ (simplify(diff(Hame22(q,p),t22)))*(DGform(t22))+ (simplify(diff(Hame22(q,p),t32)))*(DGform(t32))+ (simplify(diff

```
(Hame22(q,p),t11))* (DGform(t11))+ (simplify(diff(Hame22(q,
p),t21))* (DGform(t21))+ (simplify(diff(Hame22(q,p),t31))*
(DGform(t31))+ (simplify(diff(Hame22(q,p),q))* (DGform(q))+
(simplify(diff(Hame22(q,p),p))* (DGform(p))):
```

```
dHame32:=(simplify(diff(Hame32(q,p),t12))* (DGform(t12))+
(simplify(diff(Hame32(q,p),t22))* (DGform(t22))+ (simplify
(diff(Hame32(q,p),t32))* (DGform(t32))+ (simplify(diff
(Hame32(q,p),t11))* (DGform(t11))+ (simplify(diff(Hame32(q,
p),t21))* (DGform(t21))+ (simplify(diff(Hame32(q,p),t31))*
(DGform(t31))+ (simplify(diff(Hame32(q,p),q))* (DGform(q))+
(simplify(diff(Hame32(q,p),p))* (DGform(p))):
```

```
B > Omega:=(dHame12) &wedge (DGform(t12)) :
Omega:=( (dHame22) &wedge (DGform(t22)) ) &plus (Omega) :
Omega:=( (dHame32) &wedge (DGform(t32)) ) &plus (Omega) :
Omega:=( (dHame11) &wedge (DGform(t11)) ) &plus (Omega) :
Omega:=( (dHame21) &wedge (DGform(t21)) ) &plus (Omega) :
Omega:=( (dHame31) &wedge (DGform(t31)) ) &plus (Omega) :
Omega:=(h&mult (DGform(q)) &wedge (DGform(p)) ) &plus (Omega) :
Omega:=simplify(Omega) :
```

```
B > DifferenceFundamentalForm:= (Omega) &minus (Omega2) :
```

```
B > DifferenceFundamentalForm:=simplify
(DifferenceFundamentalForm) :
```

Maple has trouble to simplify the expressions due to the presence of square roots that he cannot simplify. In order to help him, we shall make some assumptions on the signs so that he can simplify the expressions.

```
B > ExtractionCoeff:=convert (DifferenceFundamentalForm,list) :
ListCoefficients:=ExtractionCoeff[1][2] :
Eq1:=simplify (ListCoefficients[1][2]) :
Eq2:=simplify (ListCoefficients[2][2]) :
Eq3:=simplify (ListCoefficients[3][2]) :
Eq4:=simplify (ListCoefficients[4][2]) :
Eq5:=simplify (ListCoefficients[5][2]) :
Eq6:=simplify (ListCoefficients[6][2]) :
Eq7:=simplify (ListCoefficients[7][2]) :
Eq8:=simplify (ListCoefficients[8][2]) :
Eq9:=simplify (ListCoefficients[9][2]) :
Eq10:=simplify (ListCoefficients[10][2]) :
Eq11:=simplify (ListCoefficients[11][2]) :
Eq12:=simplify (ListCoefficients[12][2]) :
Eq13:=simplify (ListCoefficients[13][2]) :
Eq14:=simplify (ListCoefficients[14][2]) :
```

```

Eq15:=simplify(ListCoefficients[15][2]):
B > Eq1function:=unapply(simplify(Eq1), t11, t21, t31, t12, t22, t32)
:
Eq2function:=unapply(simplify(Eq2), t11, t21, t31, t12, t22, t32)
:
Eq3function:=unapply(simplify(Eq3), t11, t21, t31, t12, t22, t32)
:
Eq4function:=unapply(simplify(Eq4), t11, t21, t31, t12, t22, t32)
:
Eq5function:=unapply(simplify(Eq5), t11, t21, t31, t12, t22, t32)
:
Eq6function:=unapply(simplify(Eq6), t11, t21, t31, t12, t22, t32)
:
Eq7function:=unapply(simplify(Eq7), t11, t21, t31, t12, t22, t32)
:
Eq8function:=unapply(simplify(Eq8), t11, t21, t31, t12, t22, t32)
:
Eq9function:=unapply(simplify(Eq9), t11, t21, t31, t12, t22, t32)
:
Eq10function:=unapply(simplify(Eq10), t11, t21, t31, t12, t22,
t32):
Eq11function:=unapply(simplify(Eq11), t11, t21, t31, t12, t22,
t32):
Eq12function:=unapply(simplify(Eq12), t11, t21, t31, t12, t22,
t32):
Eq13function:=unapply(simplify(Eq13), t11, t21, t31, t12, t22,
t32):
Eq14function:=unapply(simplify(Eq14), t11, t21, t31, t12, t22,
t32):
Eq15function:=unapply(simplify(Eq15), t11, t21, t31, t12, t22,
t32):

```

```

B > assume(tt22<tt12 and tt22<tt32 and tt12>tt32):
simplify(1+(tt12-tt22)*sqrt((-tt32+tt22)/((tt12-tt22)*(tt12
-tt32)))*sqrt((tt12-tt32)/((tt12-tt22)*(-tt32+tt22)))));

```

0

(3.4)

```

B > Eqq1:=simplify(Eq1function(tt11, tt21, tt31, tt12, tt22, tt32));
Eqq2:=simplify(Eq2function(tt11, tt21, tt31, tt12, tt22, tt32));
Eqq3:=simplify(Eq3function(tt11, tt21, tt31, tt12, tt22, tt32));
Eqq4:=simplify(Eq4function(tt11, tt21, tt31, tt12, tt22, tt32));
Eqq5:=simplify(Eq5function(tt11, tt21, tt31, tt12, tt22, tt32));

```



```

Eqq6:=simplify(Eq6function(tt11,tt21,tt31,tt12,tt22,tt32));
Eqq7:=simplify(Eq7function(tt11,tt21,tt31,tt12,tt22,tt32));
Eqq8:=simplify(Eq8function(tt11,tt21,tt31,tt12,tt22,tt32));
Eqq9:=simplify(Eq9function(tt11,tt21,tt31,tt12,tt22,tt32));
Eqq10:=simplify(Eq10function(tt11,tt21,tt31,tt12,tt22,tt32)
);
Eqq11:=simplify(Eq11function(tt11,tt21,tt31,tt12,tt22,tt32)
);
Eqq12:=simplify(Eq12function(tt11,tt21,tt31,tt12,tt22,tt32)
);
Eqq13:=simplify(Eq13function(tt11,tt21,tt31,tt12,tt22,tt32)
);
Eqq14:=simplify(Eq14function(tt11,tt21,tt31,tt12,tt22,tt32)
);
Eqq15:=simplify(Eq15function(tt11,tt21,tt31,tt12,tt22,tt32)
);

```

```

Eqq1 := 0
Eqq2 := 0
Eqq3 := 0
Eqq4 := 0
Eqq5 := 0
Eqq6 := 0
Eqq7 := 0
Eqq8 := 0
Eqq9 := 0
Eqq10 := 0
Eqq11 := 0
Eqq12 := 0
Eqq13 := 0
Eqq14 := 0
Eqq15 := 0

```

(3.5)

```

B > DifferenceFundamentalFormfunction:=unapply
(DifferenceFundamentalForm,t11,t21,t31,t12,t22,t32):
DifferenceFundamentalFormSimplified:=simplify
(DifferenceFundamentalFormfunction(tt11,tt21,tt31,tt12,
tt22,tt32));

```

```

_DG(["form", B, 2], [[1, 2], 0], [[1, 3], 0], [[1, 4], 0], [[1, 5], 0], [[1, 6], 0], [[1, 7],
0], [[1, 8], 0], [[2, 3], 0], [[2, 4], 0], [[2, 5], 0], [[2, 6], 0], [[2, 7], 0], [[2, 8], 0],
[[3, 4], 0], [[3, 5], 0], [[3, 6], 0], [[3, 7], 0], [[3, 8], 0], [[4, 5], 0], [[4, 6], 0],
[[4, 7], 0], [[4, 8], 0], [[5, 6], 0], [[5, 7], 0], [[5, 8], 0], [[6, 7], 0], [[6, 8], 0],
[[7, 8], 0]])

```

(3.6)