

In this Maple worksheet we shall prove that the JMU differentials is related to the evaluation of the Hamiltonian differential at $\hbar=0$.

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> restart :
with (LinearAlgebra) :
t10:=-t20-t30;

P1:=unapply( (t12+t22+t32)*lambda+t11+t21+t31,lambda);
P2:=unapply( (t12*t22+t12*t32+t22*t32)*lambda^2+((t21+t31)*t12+
(t11+t31)*t22+t32*(t11+t21))*lambda-t10*t12-t20*t22-t30*t32+t21*
t11 +t31*t11+t21*t31,lambda);
P3:=unapply(t12*t22*t32*lambda^3+(t12*t22*t31+t12*t32*t21+t22*
t32*t11)*lambda^2+(t12*t22*t30+t12*t32*t20+t22*t32*t10 +t12*t21*
t31+t22*t11*t31+t32*t11*t21)*lambda,lambda);

taufunction:=unapply( ((t21-t31)*t12+(t31-t11)*t22+(t11-t21)*
t32)/(sqrt((t22-t12)*(t12-t32)*(t32-t22))),t11,t21,t31,t12,t22,
t32);

checkqfunction:=unapply( sqrt((t12-t32)/((t22-t12)*(t32-t22)))*(-
p+t22*q+t21),q,p);
checkpfunction:=unapply( sqrt((t32-t22)/((t12-t32)*(t22-t12)))*
(p-t12*q-t11),q,p);

HamtauchekCoordinatesTheo:=unapply(-(checkq*checkp^2+checkq^2*
checkp-tau*checkq*checkp-t20*checkp+(t10+h)*checkq),checkq,
checkp);

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$$\begin{aligned}
 t10 &:= -t20 - t30 & (1) \\
 P1 &:= \lambda \rightarrow (t12 + t22 + t32) \lambda + t11 + t21 + t31 \\
 P2 &:= \lambda \rightarrow (t12 t22 + t12 t32 + t22 t32) \lambda^2 + ((t21 + t31) t12 + (t11 + t31) t22 + t32 (t11 \\
 &\quad + t21)) \lambda - (-t20 - t30) t12 - t20 t22 - t30 t32 + t21 t11 + t31 t11 + t21 t31 \\
 P3 &:= \lambda \rightarrow t12 t22 t32 \lambda^3 + (t11 t22 t32 + t12 t21 t32 + t12 t22 t31) \lambda^2 + ((-t20 - t30) t22 t32 \\
 &\quad + t32 t11 t21 + t22 t11 t31 + t12 t32 t20 + t12 t21 t31 + t12 t22 t30) \lambda \\
 \text{taufunction} &:= (t11, t21, t31, t12, t22, t32) \\
 &\rightarrow \frac{(t21 - t31) t12 + (t31 - t11) t22 + (t11 - t21) t32}{\sqrt{(t22 - t12) (t12 - t32) (t32 - t22)}} \\
 \text{checkqfunction} &:= (q, p) \rightarrow \sqrt{\frac{t12 - t32}{(t22 - t12) (t32 - t22)}} (q t22 - p + t21) \\
 \text{checkpfunction} &:= (q, p) \rightarrow \sqrt{\frac{t32 - t22}{(t22 - t12) (t12 - t32)}} (-q t12 + p - t11) \\
 \text{HamtauchekCoordinatesTheo} &:= (checkq, checkp) \rightarrow -checkq checkp^2 - checkq^2 checkp \\
 &\quad + \tau checkq checkp + t20 checkp - (-t20 - t30 + h) checkq
 \end{aligned}$$

Loading the results on the JMU differentials

> omegaJMUdt11:=1/((t12-t22)*(-t12+t32))*(-p^3+(t11+t21+t31+(t12+t22+t32)*q)*p^2+((-t22-t32)*t12-t22*t32)*q^2+((-t31-t21)*t12+(-t11-t31)*t22-t32*(t11+t21))*q+t12*(-t20-t30)+t22*t20+t30*t32+(-t31-t21)*t11-t21*t31)*p+q^3*t12*t22*t32+(t21*t32+t22*t31)*t12+t22*t32*t11)*q^2+(t20*t32+t21*t31+t22*t30)*t12+((-t20-t30)*t32+t11*t31)*t22+t32*t11*t21)*q+(t20+t30)*(t31-t11+t21)*t12+(t11*t30-t31*(t20+t30))*t22+(t11*t20-t21*(t20+t30))*t32+t11*t21*t31);

omegaJMUdt21:=1/((t12-t22)*(-t22+t32))*(p^3+((-t12-t22-t32)*q-t31-t11-t21)*p^2+((t12+t32)*t22+t12*t32)*q^2+(t12*(t21+t31)+t22*(t11+t31)+t32*(t11+t21))*q-t22*t20-t30*t32+(t20+t30)*t12+(t11+t31)*t21+t11*t31)*p-q^3*t12*t22*t32+((-t11*t32-t12*t31)*t22-t32*t12*t21)*q^2+((t20+t30)*t32-t11*t31-t12*t30)*t22+(-t11*t21-t12*t20)*t32-t12*t21*t31)*q+t20*(t31+t11-t21)*t22+(t21*(t20+t30)-t11*t20)*t32+(-t20*t31-t21*t30)*t12-t11*t21*t31);

omegaJMUdt31:=1/((-t22+t32)*(-t12+t32))*(p^3+((-t12-t22-t32)*q-t31-t11-t21)*p^2+((t12+t22)*t32+t12*t22)*q^2+(t12*(t21+t31)+t22*(t11+t31)+t32*(t11+t21))*q-t30*t32-t22*t20+(t20+t30)*t12+t31*(t11+t21)+t11*t21)*p-q^3*t12*t22*t32+((-t11*t22-t12*t21)*t32-t31*t12*t22)*q^2+((t20+t30)*t22-t11*t21-t12*t20)*t32+(-t11*t31-t12*t30)*t22-t12*t21*t31)*q-t30*(t31-t11-t21)*t32+(-t11*t30+t31*(t20+t30))*t22+(-t20*t31-t21*t30)*t12-t11*t21*t31);

omegaJMUdt12:=1/(2*(t12-t22)^2*(-t12+t32)^2)*(-(t20+t30)^2*t12^3+((-q^3*(t31-2*t11+t21)*t32+2*q*(-p+t31)*q+t30)*t11-(t21+t31)*(-p+t31)*q^2-t30*(t21+t31)*q+(t20+2*t30)*(t20+t30))*t22+(-2*q*(p-t21)*q-t20)*t11+(p-t21)*(t21+t31)*q^2-t20*(t21+t31)*q+2*t20^2+3*t30*t20+t30^2)*t32+(-t20-t30)*t11^2+(-(2*(p-t21))*(-p+t31)*q+(2*(t20+t30))*(t31-p+t21))*t11+(p-t21)*(t21+t31)*(-p+t31)*q-(t20+t30)*((-t31-t21)*p+t31^2+t21*t31+t21^2))*t12^2+((q^3*(-t11+t31)*t32-q*(-p+t31)*q+t30)*t11+t31*(-p+t31)*q^2+t31*t30*q-t30*(t20+t30))*t22^2+(-q^3*(t11-t21)*t32^2+(2*q^2*t11^2-2*q*(t21+t31)*q+3*t20*(1/2)+3*t30*(1/2))*t11+2*q^2*t21*t31+(t20+2*t30)*t21+2*t31*(t20+(1/2)*t30))*q-2*(t20+t30)^2)*t32+(2*t31-2*p)*q+2*t30)*t11^2+((-p+t31)*(t31+2*t21+p)*q+(t30+3*t20)*p-t21*t30-(3*(t20+4*t30*(1/3))))*t31)*t11+(-p^2*t21+t21*t31^2)*q+(-t21*t20-2*t31*(t20+(1/2)*t30))*p+(2*(t20+t30))*t31*(1/2)*t21+t31))*t22+(q*(p-t21)*q-t20)*t11-(p-t21)*t21*q^2+t21*t20*q-t20*(t20+t30))*t32^2+((-2*p+2*t21)*q+2*t20)*t11^2+(2*(p-t21))*(1/2)*p+t31+(1/2)*t21)*q+(t20+3*t30)*p+(-4*t20-3*t30)*t21-t31*t20)*t11+(-p^2*t31+t21^2*t31)*q+((-t20-2*t30)*t21-t31*t30)*p+t21*(t20+t30)*(t31+2*t21))*

$$\begin{aligned}
& t32 - (p - t21) * (p - t11) * (t31 - 2 * t11 + t21) * (-p + t31) * t12 + ((-q^2 * t11^2 + q * \\
& ((t31 + p) * q + t20 + t30) * t11 - q^2 * p * t31 - (t20 + t30) * t31 * q + t30 * (t20 + t30)) * \\
& t32 - (-t11 + t31) * ((p - t31) * q - t30) * t11 + p * (-p + t31) * q - p * t20 + t31 * (t20 + \\
& t30)) * t22^2 + ((-q^2 * t11^2 + (p + t21) * q + t20 + t30) * q * t11 - q^2 * p * t21 - \\
& t21 * (t20 + t30) * q + t20 * (t20 + t30)) * t32^2 + (-q * (t31 - 2 * p + t21) * t11^2 + (\\
& (-2 * p^2 + 2 * t21 * t31) * q + (-t20 - t30) * p + t31 * t20 + t21 * t30) * t11 + (t21 + t31) \\
& * p - 2 * t21 * t31) * p * q + (t20 * t21 + t30 * t31) * p - t31 * t21 * (t20 + t30) * t32 + (p - \\
& t11) * (-t11 + t31) * (-p + t31) * (p - t21) * t22 - (((-p + t21) * q + t20) * t11 + p * \\
& (p - t21) * q + p * t30 - t21 * (t20 + t30)) * t32 + (p - t21) * (p - t11) * (-p + t31) * (t11 \\
& - t21) * t32);
\end{aligned}$$

$$\begin{aligned}
\text{omegaJMUdt22} := & 1 / (2 * (t12 - t22)^2 * (-t22 + t32)^2) * (-t20^2 * t22^3 + ((- \\
& q^3 * (t31 + t11 - 2 * t21) * t12 - 2 * q * (p - t11) * q + t20 + t30) * t21 + (p - t11) * (t11 + \\
& t31) * q^2 + (t20 + t30) * (t11 + t31) * q + 2 * t20^2 + t30 * t20) * t32 + (2 * q * ((-p + \\
& t31) * q + t30) * t21 - (t11 + t31) * (-p + t31) * q^2 - t30 * (t11 + t31) * q + t20 * (t20 - \\
& t30)) * t12 + t20 * t21^2 + (-2 * (p - t11)) * (-p + t31) * q - 2 * t20 * (t31 - p + t11)) * \\
& t21 + (p - t11) * (t11 + t31) * (-p + t31) * q + t20 * ((-t11 - t31) * p + t31^2 + t11 * t31 + \\
& t11^2)) * t22^2 + (q^3 * (t11 - t21) * t12 + q * (p - t11) * q + t20 + t30) * t21 - (p - \\
& t11) * t11 * q^2 - t11 * (t20 + t30) * q - t20 * (t20 + t30) * t32^2 + (q^3 * (-t21 + t31) \\
& * t12^2 + (2 * q^2 * t21^2 - (2 * ((t11 + t31) * q - 3 * t20 * (1/2)))) * q * t21 + 2 * q^2 * \\
& t11 * t31 + (t11 * (-t20 + t30) - 2 * t31 * (t20 + (1/2) * t30)) * q - 2 * t20^2) * t12 + (\\
& (-2 * p + 2 * t11) * q - 2 * t20 - 2 * t30) * t21^2 + ((2 * ((1/2) * p + t31 + (1/2) * t11)) * \\
& (p - t11) * q + (2 * t30 - t20) * p + (t30 + 4 * t20) * t11 + t31 * (t20 + t30)) * t21 + (-p^2 * \\
& t31 + t11^2 * t31) * q + ((t20 - t30) * t11 - t31 * t30) * p - t11 * t20 * (t31 + 2 * t11)) * \\
& t32 + (-q * ((-p + t31) * q + t30) * t21 + t31 * (-p + t31) * q^2 + t31 * t30 * q + t30 * t20) * \\
& t12^2 + ((2 * t31 - 2 * p) * q + 2 * t30) * t21^2 + (-(-p + t31) * (t31 + 2 * t11 + p) * q + \\
& (-2 * t30 - 3 * t20) * p - t11 * t30 + 3 * t31 * (t20 - (1/3) * t30)) * t21 + (-p^2 * t11 + \\
& t11 * t31^2) * q + ((t20 + t30) * t11 + 2 * t31 * (t20 + (1/2) * t30)) * p - (2 * (t31 + \\
& (1/2) * t11)) * t31 * t20) * t12 - (p - t21) * (p - t11) * (t31 + t11 - 2 * t21) * (-p + t31) \\
&) * t22 + ((-q^2 * t21^2 + (p + t11) * q - t20) * q * t21 - q^2 * p * t11 + t20 * t11 * q + t20 * \\
& (t20 + t30)) * t12 + (t11 - t21) * (((-p + t11) * q - t20 - t30) * t21 + p * (p - t11) * q + p * \\
& t30 + t11 * t20)) * t32^2 + ((-q^2 * t21^2 + q * ((t31 + p) * q - t20) * t21 - q^2 * p * t31 + \\
& t31 * t20 * q - t30 * t20) * t12^2 + (-q * (-2 * p + t31 + t11) * t21^2 + ((-2 * p^2 + 2 * t11 * \\
& t31) * q + p * t20 + t11 * t30 - t31 * (t20 + t30)) * t21 + ((t11 + t31) * p - 2 * t11 * t31) * \\
& p * q + ((-t20 - t30) * t11 + t31 * t30) * p + t31 * t11 * t20) * t12 + (t11 - t21) * (p - t21) \\
& * (p - t11) * (-p + t31) * t32 + t12 * (((-p + t31) * q + t30) * t21 - p * (-p + t31) * q + (- \\
& t20 - t30) * p + t31 * t20) * t12 + (p - t21) * (p - t11) * (-p + t31) * (-t21 + t31));
\end{aligned}$$

$$\begin{aligned}
\text{omegaJMUdt32} := & 1 / (2 * (-t12 + t32)^2 * (-t22 + t32)^2) * (-t30^2 * t32^3 + (2 * \\
& q^3 * (t31 - (1/2) * t11 - (1/2) * t21) * t12 - 2 * q * (p - t11) * q + t20 + t30) * t31 + \\
& (t11 + t21) * (p - t11) * q^2 + (t20 + t30) * (t11 + t21) * q + t30 * (t20 + 2 * t30)) * t22 + \\
& (-2 * q * (p - t21) * q - t20) * t31 + (t11 + t21) * (p - t21) * q^2 - t20 * (t11 + t21) * q -
\end{aligned}$$

$$\begin{aligned}
& t_{30} * (t_{20} - t_{30}) * t_{12} + t_{30} * t_{31}^2 + ((2 * (p - t_{21})) * (p - t_{11}) * q + 2 * t_{30} * (p - t_{11} - \\
& t_{21})) * t_{31} - (t_{11} + t_{21}) * (p - t_{21}) * (p - t_{11}) * q - t_{30} * ((t_{11} + t_{21}) * p - t_{11}^2 - t_{11} * \\
& t_{21} - t_{21}^2) * t_{32}^2 + ((-q^3 * (-t_{11} + t_{31}) * t_{12} + q * ((p - t_{11}) * q + t_{20} + t_{30}) * t_{31} \\
& - (p - t_{11}) * t_{11} * q^2 - t_{11} * (t_{20} + t_{30}) * q - t_{30} * (t_{20} + t_{30})) * t_{22}^2 + (-q^3 * (- \\
& t_{21} + t_{31}) * t_{12}^2 + (2 * q^2 * t_{31}^2 - 2 * q * ((t_{11} + t_{21}) * q - 3 * t_{30} * (1/2))) * t_{31} + 2 * \\
& q^2 * t_{11} * t_{21} + ((t_{20} - t_{30}) * t_{11} - (t_{20} + 2 * t_{30}) * t_{21}) * q - 2 * t_{30}^2 * t_{12} + ((-2 * \\
& p + 2 * t_{11}) * q - 2 * t_{20} - 2 * t_{30}) * t_{31}^2 + ((p - t_{11}) * (p + t_{11} + 2 * t_{21}) * q + (-t_{30} + 2 * \\
& t_{20}) * p + t_{11} * (4 * t_{30} + t_{20}) + t_{21} * (t_{20} + t_{30})) * t_{31} + (-p^2 * t_{21} + t_{11}^2 * t_{21}) * q + \\
& (t_{11} * (-t_{20} + t_{30}) - t_{21} * t_{20}) * p - (2 * (t_{11} + (1/2) * t_{21})) * t_{11} * t_{30} * t_{22} + (q * (\\
& (p - t_{21}) * q - t_{20}) * t_{31} - (p - t_{21}) * t_{21} * q^2 + t_{21} * t_{20} * q + t_{30} * t_{20}) * t_{12}^2 + (\\
& (-2 * p + 2 * t_{21}) * q + 2 * t_{20}) * t_{31}^2 + ((p - t_{21}) * (p + 2 * t_{11} + t_{21}) * q + (-2 * t_{20} - 3 * \\
& t_{30}) * p - t_{11} * t_{20} - t_{21} * (t_{20} - 3 * t_{30})) * t_{31} + (-p^2 * t_{11} + t_{11} * t_{21}^2) * q + ((t_{20} + \\
& t_{30}) * t_{11} + (t_{20} + 2 * t_{30}) * t_{21}) * p - t_{30} * t_{21} * (t_{11} + 2 * t_{21})) * t_{12} + (2 * (t_{31} - \\
& (1/2) * t_{11} - (1/2) * t_{21})) * (-p + t_{31}) * (p - t_{21}) * (p - t_{11})) * t_{32} + ((-q^2 * t_{31}^2 + \\
& q * ((p + t_{11}) * q - t_{30}) * t_{31} - q^2 * p * t_{11} + t_{30} * t_{11} * q + t_{30} * (t_{20} + t_{30})) * t_{12} + (- \\
& t_{11} + t_{31}) * ((p - t_{11}) * q + t_{20} + t_{30}) * t_{31} - p * (p - t_{11}) * q - p * t_{20} - t_{11} * t_{30})) * \\
& t_{22}^2 + ((-q^2 * t_{31}^2 + q * ((p + t_{21}) * q - t_{30}) * t_{31} - q^2 * p * t_{21} + t_{30} * t_{21} * q - t_{30} * \\
& t_{20}) * t_{12}^2 + (2 * q * (p - (1/2) * t_{11} - (1/2) * t_{21})) * t_{31}^2 + ((-2 * p^2 + 2 * t_{11} * t_{21}) \\
& * q + p * t_{30} + t_{11} * t_{20} - t_{21} * (t_{20} + t_{30})) * t_{31} + ((t_{11} + t_{21}) * p - 2 * t_{11} * t_{21}) * p * q + (\\
& (-t_{20} - t_{30}) * t_{11} + t_{21} * t_{20}) * p + t_{21} * t_{11} * t_{30} * t_{12} - (p - t_{11}) * (-t_{11} + t_{31}) * (- \\
& p + t_{31}) * (p - t_{21}) * t_{22} - t_{12} * (((-p + t_{21}) * q + t_{20}) * t_{31} + p * (p - t_{21}) * q + (-t_{20} - \\
& t_{30}) * p + t_{21} * t_{30}) * t_{12} + (p - t_{21}) * (p - t_{11}) * (-p + t_{31})) * (-t_{21} + t_{31});
\end{aligned}$$

$$\omega_{JM Udt11} := \frac{1}{(-t_{22} + t_{12})(-t_{12} + t_{32})} (-p^3 + (t_{11} + t_{21} + t_{31} + (t_{12} + t_{22} + t_{32}) q) p^2 + \dots) \quad (2)$$

$$\begin{aligned}
& + ((-t_{22} - t_{32}) t_{12} - t_{22} t_{32}) q^2 + ((-t_{31} - t_{21}) t_{12} + (-t_{11} - t_{31}) t_{22} \\
& - t_{32} (t_{11} + t_{21})) q + (-t_{20} - t_{30}) t_{12} + t_{20} t_{22} + t_{30} t_{32} + (-t_{31} - t_{21}) t_{11} \\
& - t_{21} t_{31}) p + q^3 t_{12} t_{22} t_{32} + ((t_{21} t_{32} + t_{22} t_{31}) t_{12} + t_{22} t_{32} t_{11}) q^2 + ((t_{20} t_{32} \\
& + t_{21} t_{31} + t_{22} t_{30}) t_{12} + ((-t_{20} - t_{30}) t_{32} + t_{31} t_{11}) t_{22} + t_{32} t_{11} t_{21}) q + (t_{20} \\
& + t_{30}) (t_{31} - t_{11} + t_{21}) t_{12} + (t_{11} t_{30} - t_{31} (t_{20} + t_{30})) t_{22} + (t_{11} t_{20} - t_{21} (t_{20} \\
& + t_{30})) t_{32} + t_{11} t_{21} t_{31})
\end{aligned}$$

$$\omega_{JM Udt21} := \frac{1}{(-t_{22} + t_{12})(t_{32} - t_{22})} (p^3 + ((-t_{12} - t_{22} - t_{32}) q - t_{31} - t_{11} - t_{21}) p^2 + \dots)$$

$$\begin{aligned}
& + ((t_{12} + t_{32}) t_{22} + t_{12} t_{32}) q^2 + ((t_{21} + t_{31}) t_{12} + (t_{11} + t_{31}) t_{22} \\
& + t_{32} (t_{11} + t_{21})) q - t_{20} t_{22} - t_{30} t_{32} + (t_{20} + t_{30}) t_{12} + (t_{11} + t_{31}) t_{21} + t_{31} t_{11}) p \\
& - q^3 t_{12} t_{22} t_{32} + ((-t_{11} t_{32} - t_{12} t_{31}) t_{22} - t_{12} t_{32} t_{21}) q^2 + ((t_{20} + t_{30}) t_{32} \\
& - t_{31} t_{11} - t_{12} t_{30}) t_{22} + (-t_{11} t_{21} - t_{12} t_{20}) t_{32} - t_{12} t_{21} t_{31}) q + t_{20} (t_{31} + t_{11} \\
& - t_{21}) t_{22} + (t_{21} (t_{20} + t_{30}) - t_{11} t_{20}) t_{32} + (-t_{20} t_{31} - t_{21} t_{30}) t_{12} - t_{11} t_{21} t_{31})
\end{aligned}$$

$$\omega_{JM Udt31} := \frac{1}{(t_{32} - t_{22})(-t_{12} + t_{32})} (p^3 + ((-t_{12} - t_{22} - t_{32}) q - t_{31} - t_{11} - t_{21}) p^2 + \dots)$$

$$\begin{aligned}
& + ((t_{12} + t_{22}) t_{32} + t_{12} t_{22}) q^2 + ((t_{21} + t_{31}) t_{12} + (t_{11} + t_{31}) t_{22} \\
& + t_{32} (t_{11} + t_{21})) q - t_{30} t_{32} - t_{20} t_{22} + (t_{20} + t_{30}) t_{12} + t_{31} (t_{11} + t_{21}) + t_{21} t_{11}) p \\
& - q^3 t_{12} t_{22} t_{32} + ((-t_{11} t_{22} - t_{12} t_{21}) t_{32} - t_{12} t_{22} t_{31}) q^2 + ((t_{20} + t_{30}) t_{22} \\
& - t_{21} t_{11} - t_{12} t_{20}) t_{32} + (-t_{11} t_{31} - t_{12} t_{30}) t_{22} - t_{12} t_{21} t_{31}) q - t_{30} (t_{31} - t_{11} \\
& - t_{21}) t_{32} + (-t_{11} t_{30} + t_{31} (t_{20} + t_{30})) t_{22} + (-t_{20} t_{31} - t_{21} t_{30}) t_{12} - t_{11} t_{21} t_{31})
\end{aligned}$$

$$\begin{aligned}
\omega_{JMUdt12} := & \frac{1}{2} \frac{1}{(-t_{22} + t_{12})^2 (-t_{12} + t_{32})^2} \left(-(t_{20} + t_{30})^2 t_{12}^3 + ((-q^3 (t_{31} \right. \\
& - 2 t_{11} + t_{21}) t_{32} + 2 q ((-p + t_{31}) q + t_{30}) t_{11} - (t_{21} + t_{31}) (-p + t_{31}) q^2 \\
& - t_{30} (t_{21} + t_{31}) q + (t_{20} + 2 t_{30}) (t_{20} + t_{30})) t_{22} + (-2 q ((p - t_{21}) q - t_{20}) t_{11} \\
& + (p - t_{21}) (t_{21} + t_{31}) q^2 - t_{20} (t_{21} + t_{31}) q + 2 t_{20}^2 + 3 t_{30} t_{20} + t_{30}^2) t_{32} + (-t_{20} \\
& - t_{30}) t_{11}^2 + (-2 (p - t_{21}) (-p + t_{31}) q + 2 (t_{20} + t_{30}) (t_{31} - p + t_{21})) t_{11} + (p \\
& - t_{21}) (t_{21} + t_{31}) (-p + t_{31}) q - (t_{20} + t_{30}) ((-t_{31} - t_{21}) p + t_{31}^2 + t_{21} t_{31} \\
& + t_{21}^2)) t_{12}^2 + \left((q^3 (t_{31} - t_{11}) t_{32} - q ((-p + t_{31}) q + t_{30}) t_{11} + t_{31} (-p + t_{31}) q^2 \right. \\
& + t_{31} t_{30} q - t_{30} (t_{20} + t_{30})) t_{22}^2 + \left(-q^3 (t_{11} - t_{21}) t_{32}^2 + \left(2 q^2 t_{11}^2 - 2 q \left((t_{21} \right. \right. \right. \\
& + t_{31}) q + \frac{3}{2} t_{20} + \frac{3}{2} t_{30}) t_{11} + 2 q^2 t_{21} t_{31} + \left. \left. \left. (t_{20} + 2 t_{30}) t_{21} + 2 t_{31} \left(t_{20} \right. \right. \right. \right. \\
& + \frac{1}{2} t_{30}) \left. \left. \left. \right) \right) q - 2 (t_{20} + t_{30})^2 \right) t_{32} + ((2 t_{31} - 2 p) q + 2 t_{30}) t_{11}^2 + \left(-(-p \right. \\
& + t_{31}) (t_{31} + 2 t_{21} + p) q + (t_{30} + 3 t_{20}) p - t_{21} t_{30} - 3 \left(t_{20} + \frac{4}{3} t_{30}) t_{31} \right) t_{11} + (\\
& -p^2 t_{21} + t_{21} t_{31}^2) q + \left(-t_{21} t_{20} - 2 t_{31} \left(t_{20} + \frac{1}{2} t_{30} \right) \right) p + 2 (t_{20} \\
& + t_{30}) t_{31} \left(\frac{1}{2} t_{21} + t_{31} \right) \right) t_{22} + (q ((p - t_{21}) q - t_{20}) t_{11} - (p - t_{21}) t_{21} q^2 \\
& + t_{21} t_{20} q - t_{20} (t_{20} + t_{30})) t_{32}^2 + \left(((-2 p + 2 t_{21}) q + 2 t_{20}) t_{11}^2 + \left(2 (p \right. \right. \\
& - t_{21}) \left(\frac{1}{2} p + t_{31} + \frac{1}{2} t_{21} \right) q + (t_{20} + 3 t_{30}) p + (-4 t_{20} - 3 t_{30}) t_{21} - t_{20} t_{31} \right) t_{11} \\
& + (-p^2 t_{31} + t_{21}^2 t_{31}) q + ((-t_{20} - 2 t_{30}) t_{21} - t_{31} t_{30}) p + t_{21} (t_{20} + t_{30}) (t_{31} \\
& + 2 t_{21}) t_{32} - (p - t_{21}) (p - t_{11}) (t_{31} - 2 t_{11} + t_{21}) (-p + t_{31}) t_{12} + ((-q^2 t_{11}^2 \\
& + q ((t_{31} + p) q + t_{20} + t_{30}) t_{11} - q^2 p t_{31} - (t_{20} + t_{30}) t_{31} q + t_{30} (t_{20} + t_{30})) t_{32} \\
& - (t_{31} - t_{11}) ((p - t_{31}) q - t_{30}) t_{11} + p (-p + t_{31}) q - p t_{20} + t_{31} (t_{20} + t_{30})) \\
& t_{22}^2 + ((-q^2 t_{11}^2 + ((p + t_{21}) q + t_{20} + t_{30}) q t_{11} - q^2 p t_{21} - t_{21} (t_{20} + t_{30}) q \\
& + t_{20} (t_{20} + t_{30})) t_{32}^2 + (-q (t_{31} - 2 p + t_{21}) t_{11}^2 + ((-2 p^2 + 2 t_{21} t_{31}) q + (-t_{20} \\
& - t_{30}) p + t_{20} t_{31} + t_{21} t_{30}) t_{11} + ((t_{21} + t_{31}) p - 2 t_{21} t_{31}) p q + (t_{20} t_{21} \\
& + t_{30} t_{31}) p - t_{31} t_{21} (t_{20} + t_{30})) t_{32} + (p - t_{11}) (t_{31} - t_{11}) (-p + t_{31}) (p - t_{21}) \\
& t_{22} - (((-p + t_{21}) q + t_{20}) t_{11} + p (p - t_{21}) q + p t_{30} - t_{21} (t_{20} + t_{30})) t_{32} + (p \\
& - t_{21}) (p - t_{11}) (-p + t_{31}) (t_{11} - t_{21}) t_{32} \left. \right)
\end{aligned}$$

$$\begin{aligned}
\omega_{JMUdt22} := & \frac{1}{2} \frac{1}{(-t_{22} + t_{12})^2 (t_{32} - t_{22})^2} \left(-t_{20}^2 t_{22}^3 + ((-q^3 (t_{31} + t_{11} \right. \\
& - 2 t_{21}) t_{12} - 2 q ((p - t_{11}) q + t_{20} + t_{30}) t_{21} + (p - t_{11}) (t_{11} + t_{31}) q^2 + (t_{20} \\
& + t_{30}) (t_{11} + t_{31}) q + 2 t_{20}^2 + t_{30} t_{20}) t_{32} + (2 q ((-p + t_{31}) q + t_{30}) t_{21} - (t_{11}
\end{aligned}$$

$$\begin{aligned}
& + t31) (-p + t31) q^2 - t30 (t11 + t31) q + t20 (t20 - t30) t12 + t20 t21^2 + (-2 (p \\
& - t11) (-p + t31) q - 2 t20 (t31 - p + t11) t21 + (p - t11) (t11 + t31) (-p + t31) q \\
& + t20 ((-t11 - t31) p + t31^2 + t31 t11 + t11^2)) t22^2 + \left((q^3 (t11 - t21) t12 + q ((p \\
& - t11) q + t20 + t30) t21 - (p - t11) t11 q^2 - t11 (t20 + t30) q - t20 (t20 + t30) t32^2 \right. \\
& + \left. (q^3 (-t21 + t31) t12^2 + \left(2 q^2 t21^2 - 2 \left((t11 + t31) q - \frac{3}{2} t20 \right) q t21 + 2 q^2 t11 t31 \right. \right. \\
& + \left. \left. (t11 (-t20 + t30) - 2 t31 \left(t20 + \frac{1}{2} t30 \right) \right) q - 2 t20^2 \right) t12 + ((-2 p + 2 t11) q \right. \\
& - 2 t20 - 2 t30) t21^2 + \left. \left(2 \left(\frac{1}{2} p + t31 + \frac{1}{2} t11 \right) (p - t11) q + (2 t30 - t20) p + (t30 \right. \right. \\
& + 4 t20) t11 + t31 (t20 + t30) \left. \right) t21 + (-p^2 t31 + t11^2 t31) q + ((t20 - t30) t11 \\
& - t31 t30) p - t11 t20 (t31 + 2 t11) \left. \right) t32 + (-q ((-p + t31) q + t30) t21 + t31 (-p \\
& + t31) q^2 + t31 t30 q + t30 t20) t12^2 + \left(((2 t31 - 2 p) q + 2 t30) t21^2 + \left(-(-p \right. \right. \\
& + t31) (t31 + 2 t11 + p) q + (-2 t30 - 3 t20) p - t11 t30 + 3 t31 \left(t20 - \frac{1}{3} t30 \right) \left. \right) t21 \\
& + (-p^2 t11 + t11 t31^2) q + \left((t20 + t30) t11 + 2 t31 \left(t20 + \frac{1}{2} t30 \right) \right) p - 2 \left(t31 \right. \\
& + \left. \frac{1}{2} t11 \right) t31 t20 \left. \right) t12 - (p - t21) (p - t11) (t31 + t11 - 2 t21) (-p + t31) \left. \right) t22 \\
& + \left((-q^2 t21^2 + ((p + t11) q - t20) q t21 - q^2 p t11 + t20 t11 q + t20 (t20 + t30) \right) t12 \\
& + (t11 - t21) (((-p + t11) q - t20 - t30) t21 + p (p - t11) q + p t30 + t11 t20) \left. \right) t32^2 \\
& + \left((-q^2 t21^2 + q ((t31 + p) q - t20) t21 - q^2 p t31 + t31 t20 q - t30 t20) t12^2 + (-q (- \right. \\
& - 2 p + t31 + t11) t21^2 + \left. ((-2 p^2 + 2 t11 t31) q + p t20 + t11 t30 - t31 (t20 + t30) \right) t21 \\
& + ((t11 + t31) p - 2 t31 t11) p q + ((-t20 - t30) t11 + t31 t30) p + t31 t11 t20) t12 \\
& + (t11 - t21) (p - t21) (p - t11) (-p + t31) \left. \right) t32 + t12 ((((-p + t31) q + t30) t21 \\
& - p (-p + t31) q + (-t20 - t30) p + t20 t31) t12 + (p - t21) (p - t11) (-p + t31)) (- \\
& - t21 + t31)
\end{aligned}$$

$$\begin{aligned}
\omega_{JMUdt32} := & \frac{1}{2} \frac{1}{(-t12 + t32)^2 (t32 - t22)^2} \left(-t30^2 t32^3 + \left(\left(2 q^3 \left(t31 - \frac{1}{2} t11 \right. \right. \right. \right. \\
& - \left. \left. \frac{1}{2} t21 \right) t12 - 2 q ((p - t11) q + t20 + t30) t31 + (t11 + t21) (p - t11) q^2 + (t20 \right. \\
& + t30) (t11 + t21) q + t30 (t20 + 2 t30) \left. \right) t22 + (-2 q ((p - t21) q - t20) t31 + (t11 \\
& + t21) (p - t21) q^2 - t20 (t11 + t21) q - t30 (t20 - t30) \left. \right) t12 + t30 t31^2 + (2 (p \\
& - t21) (p - t11) q + 2 t30 (p - t11 - t21)) t31 - (t11 + t21) (p - t21) (p - t11) q \\
& - t30 ((t11 + t21) p - t11^2 - t21 t11 - t21^2) \left. \right) t32^2 + \left((-q^3 (t31 - t11) t12 + q ((p \right.
\end{aligned}$$

$$\begin{aligned}
& -t11) q + t20 + t30) t31 - (p - t11) t11 q^2 - t11 (t20 + t30) q - t30 (t20 + t30) t22^2 \\
& + \left(-q^3 (-t21 + t31) t12^2 + \left(2 q^2 t31^2 - 2 q \left((t11 + t21) q - \frac{3}{2} t30 \right) t31 + 2 q^2 t11 t21 \right. \right. \\
& + \left. \left. ((t20 - t30) t11 - (t20 + 2 t30) t21) q - 2 t30^2 \right) t12 + ((-2 p + 2 t11) q - 2 t20 \right. \\
& - 2 t30) t31^2 + ((p - t11) (p + t11 + 2 t21) q + (-t30 + 2 t20) p + t11 (4 t30 + t20) \\
& + t21 (t20 + t30) t31 + (-p^2 t21 + t11^2 t21) q + (t11 (-t20 + t30) - t21 t20) p \\
& - 2 \left(t11 + \frac{1}{2} t21 \right) t11 t30 \left. \right) t22 + (q ((p - t21) q - t20) t31 - (p - t21) t21 q^2 \\
& + t21 t20 q + t30 t20) t12^2 + (((-2 p + 2 t21) q + 2 t20) t31^2 + ((p - t21) (p + 2 t11 \\
& + t21) q + (-2 t20 - 3 t30) p - t11 t20 - t21 (t20 - 3 t30) t31 + (-p^2 t11 \\
& + t11 t21^2) q + ((t20 + t30) t11 + (t20 + 2 t30) t21) p - t30 t21 (t11 + 2 t21) t12 \\
& + 2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) (-p + t31) (p - t21) (p - t11) \left. \right) t32 + ((-q^2 t31^2 \\
& + q ((p + t11) q - t30) t31 - q^2 p t11 + t30 t11 q + t30 (t20 + t30) t12 + (t31 \\
& - t11) (((p - t11) q + t20 + t30) t31 - p (p - t11) q - p t20 - t11 t30) t22^2 + \left((-q^2 t31^2 + q ((p + t21) q - t30) t31 - q^2 p t21 + t30 t21 q - t30 t20) t12^2 + \left(2 q \left(p \right. \right. \right. \\
& - \frac{1}{2} t11 - \frac{1}{2} t21 \left. \right) t31^2 + ((-2 p^2 + 2 t11 t21) q + p t30 + t11 t20 - t21 (t20 \\
& + t30) t31 + ((t11 + t21) p - 2 t21 t11) p q + ((-t20 - t30) t11 + t21 t20) p \\
& + t21 t11 t30 \left. \right) t12 - (p - t11) (t31 - t11) (-p + t31) (p - t21) \left. \right) t22 - t12 ((((-p \\
& + t21) q + t20) t31 + p (p - t21) q + (-t20 - t30) p + t21 t30) t12 + (p - t21) (p \\
& - t11) (-p + t31) (-t21 + t31)
\end{aligned}$$

Loading the purely time-dependent terms

```

> F22:=unapply(0,tt11, tt21, tt31, tt12, tt22, tt32);
F11:=unapply(0,tt11, tt21, tt31, tt12, tt22, tt32);
F31:=unapply(h/2*tt21* (-1/(tt22-tt32)-ln(tt22-tt32)/(tt12-tt32)+
ln(tt12-tt22)/(tt12-tt32)),tt11, tt21, tt31, tt12, tt22, tt32);
F12:=unapply( h/2*tt21*tt31*(ln(tt22-tt32)/(tt12-tt32)^2-ln(tt12-
tt22)/(tt12-tt32)^2),tt11, tt21, tt31, tt12, tt22, tt32 );
F32:=unapply(h*tt21*tt31/2*((ln(tt12-tt22)-ln(tt22-tt32))/(tt12-
tt32)^2),tt11, tt21, tt31, tt12, tt22, tt32) ;
F21:=unapply(h*tt31/2*(1/(tt22-tt32) +ln(tt12-tt22)/(tt12-tt32)-
ln(tt22-tt32)/(tt12-tt32))
, tt11, tt21, tt31, tt12, tt22, tt32);

```

$$F22 := (u11, u21, u31, u12, u22, u32) \rightarrow 0$$

(3)

$$F11 := (t11, t21, t31, t12, t22, t32) \rightarrow 0$$

$$F31 := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t21 \left(-\frac{1}{t22 - t32} - \frac{\ln(t22 - t32)}{t12 - t32} + \frac{\ln(t12 - t22)}{t12 - t32} \right)$$

$$F12 := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t21 t31 \left(\frac{\ln(t22 - t32)}{(t12 - t32)^2} - \frac{\ln(t12 - t22)}{(t12 - t32)^2} \right)$$

$$F32 := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} \frac{h t21 t31 (\ln(t12 - t22) - \ln(t22 - t32))}{(t12 - t32)^2}$$

$$F21 := (t11, t21, t31, t12, t22, t32) \rightarrow \frac{1}{2} h t31 \left(\frac{1}{t22 - t32} + \frac{\ln(t12 - t22)}{t12 - t32} - \frac{\ln(t22 - t32)}{t12 - t32} \right)$$

Loading the Hamiltonians in the canonical directions

```

> Hame11 := unapply( (-p^3 + (t12+t22+t32)*q + t11+t21+t31)*p^2 + ((-t22-t32)*t12-t22*t32)*q^2 + ((-t31-t21)*t12+(-t11-t31)*t22-t32*(t11+t21))*q + t12*(-t20-t30) + t22*t20+t32*t30 + (-t31-t21)*t11-t21*t31)*p + q*(q^2*t12*t22*t32 + ((t22*t31+t32*t21)*t12+t22*t32*t11)*q + (t22*t30+t32*t20+t21*t31)*t12 + ((-t20-t30)*t32+t11*t31)*t22+t32*t11*t21) / ((t12-t32)*(t12-t22))
+ ((t20+t30)*t12+t31*t11-t20*t32-t30*t32)*t21-t11*t20*(-t32+t22) / ((-t22+t12)*(t12-t32))
+ F11(t11, t21, t31, t12, t22, t32)
, q, p);
Hame21 := unapply( (p^3 + (-t12-t22-t32)*q - t31-t11-t21)*p^2 + ((t12+t32)*t22+t12*t32)*q^2 + (t12*(t21+t31)+t22*(t11+t31)+t32*(t11+t21))*q + (h-t20)*t22 + (t30-h+t20)*t12-t32*t30+t11*(t21+t31)+t21*t31)*p - q*(q^2*t12*t22*t32 + ((t12*t31+t32*t11)*t22+t12*t32*t21)*q + (t12*t30+(-t30+h-t20)*t32+t11*t31)*t22 + ((-h+t20)*t32+t21*t31)*t12+t32*t11*t21) / ((t22-t32)*(t12-t22))
+ ((-t12+t32)*t20-t31*t11-t12*t30+t30*t32)*t21+t11*t20*(-t32+t22) / ((-t22+t12)*(-t32+t22))
+ F21(t11, t21, t31, t12, t22, t32)
, q, p);
Hame31 := unapply( (-p^3 + (t11+t21+t31+(t12+t22+t32)*q)*p^2 + ((-t12-t22)*t32-t12*t22)*q^2 + ((-t21-t11)*t32+(-t31-t21)*t12-t22*(t11+t31))*q + (-h+t30)*t32+(-t30+h-t20)*t12+t22*t20+(-t31-t21)*t11-t21*t31)*p + q*(t12*t22*t32*q^2 + ((t12*t21+t22*t11)*t32+t31*t12*t22)*q + (t12*t20+(-t30+h-t20)*t22+t11*t21)*t32 + ((-h+t30)*t22+t21*t31)*t12+t22*t11*t31) / ((t22-t32)*(t12-t32))
+ ((-t20-t30)*t32+t31*t11+t12*t20+t12*t30)*t21-t11*t20*(-t32+t22) / ((t12-t32)*(-t32+t22))
+ F31(t11, t21, t31, t12, t22, t32)
, q, p);

```



```

Hame12:=unapply( ((t32*t22*(t21+t31-2*t11)*q^3+(-t22+t32)*(t21+
t31-2*t11)*p+(-h*t32+t31*(t21+t31-2*t11))*t22+t32*t21*(t21+t31-2*
t11))*q^2+(t21+t31-2*t11)*p^2+(h*t22+h*t32-(t21+t31)*(t21+t31-2*
t11))*p+(-2*t30*t11+t21*t30+(-h+t30)*t31)*t22+(-2*t11*t20+(-h+
t20)*t21+t31*t20)*t32+t31*t21*(t21+t31-2*t11))*q-(p*h+(-2*t30-2*
t20)*t11+(t21+t31)*(t30-h+t20))*p)*t12^2+(-((-t11+t31)*t22-t32*
(t11-t21))*t32*t22*q^3+((-t11+t31)*t22^2-t32^2*(t11-t21))*p+(h*
t32-t31*(-t11+t31))*t22^2+(2*((1/2)*h*t32+(t11-t21)*(-t11+t31)))*
t32*t22+t21*t32^2*(t11-t21))*q^2+((-t11+t21)*t22+t32*(-t11+t31))
*p^2+(-h*t22^2+(-2*h*t32+2*t11*(t11-t21))*t22-h*t32^2-2*t11*(-
t11+t31)*t32)*p+(t30*t11-(-h+t30)*t31)*t22^2+((3*t20+3*t30)*t11+
(h-t20-2*t30)*t21-(-h+2*t20+t30)*t31)*t32+t31*(t11-t21)*(-2*t11+
t31))*t22+(2*((1/2)*t11*t20+(1/2)*(h-t20)*t21)*t32+(-t11+t31)*
t21*(-(1/2)*t21+t11))*t32)*q-p*((t21+t31-2*t11)*p^2+(-h*t22-h*
t32-(t11+t21+t31)*(t21+t31-2*t11))*p+(3*t20+t30)*t11+(h-t20)*t21
-(-h+2*t20+t30)*t31)*t22+(t11*(t20+3*t30)+(h-t20-2*t30)*t21-(-h+
t30)*t31)*t32+(t21+t31-2*t11)*(t11*(t21+t31)+t21*t31))*t12+t32*(
((-t11+t31)*t22-t32*(t11-t21))*p+(-h*t32-t11*(-t11+t31))*t22+t32*
t11*(t11-t21))*t22*q^2+(-((-t11+t31)*t22-t32*(t11-t21))*(t22+t32)
*p^2+((t32*h-t11^2+t31^2)*t22^2+(h*t32^2+(-2*t11^2+2*t21*t31)*
t32)*t22+(-t11^2+t21^2)*t32^2)*p+((-t20-t30)*t11+(t30-h+t20)*
t31)*t32-t11*t31*(-t11+t31))*t22^2-t32*(t11*(t20+t30)-(t30-h+
t20)*t21)*t32-(t11*(t21+t31)-2*t21*t31)*t11)*t22+t32^2*t11*t21*
(t11-t21))*q+p*(((-t11+t31)*t22-t32*(t11-t21))*p^2+((-h*t32-(-
t11+t31)*(t11+t21+t31))*t22+t32*(t11-t21)*(t11+t21+t31))*p-t20*(-
t11+t31)*t22^2+(t11*(t20+t30)+(h-t20)*t21-(-h+t30)*t31)*t32+(-
t11+t31)*(t11*(t21+t31)+t21*t31))*t22+t32*(-t31-t21)*t11+t32*t30
-t21*t31*(t11-t21)))/(2*(t12-t32)^2*(t12-t22)^2)
+((-1/2)*t12+(1/2)*t32)*t21+(t11-(1/2)*t31)*t12+(-1/2)*t22-
(1/2)*t32)*t11+(1/2)*t22*t31)*((-t20-t30)*t12-t31*t11+(t20+t30)*
t32)*t21+t11*t20*(-t32+t22))/(t12-t32)^2*(-t22+t12)^2)
+F12(t11,t21,t31,t12,t22,t32)
,q,p);

```

```

Hame22:=unapply( ((t12*t32*(t11-2*t21+t31)*q^3+(-t12+t32)*(t11
-2*t21+t31)*p+(-h*t12+t11*(t11-2*t21+t31))*t32+t12*t31*(t11-2*
t21+t31))*q^2+(t11-2*t21+t31)*p^2+(h*t32+h*t12-(t11+t31)*(t11-2*
t21+t31))*p+(2*t30-2*h+2*t20)*t21+(-t30+h-t20)*t31-t11*(t20+t30)
)*t32+(-2*t21*t30+(-h+t30)*t31+t30*t11)*t12+t11*t31*(t11-2*t21+
t31))*q-(p*h+(-2*h+2*t20)*t21-t20*(t11+t31))*p)*t22^2+(-t12*t32*
(t32*(t11-t21)+t12*(-t21+t31))*q^3+((t32^2*(t11-t21)+t12^2*(-t21+
t31))*p+(h*t12-t11*(t11-t21))*t32^2-2*t12*(-(1/2)*h*t12+(t11-t21)
*(-t21+t31))*t32-t31*(-t21+t31)*t12^2)*q^2+((-t21+t31)*t32+t12*

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(t11-t21) ) *p^2+ (-h*t32^2+ (-2*h*t12-2*t21* (-t21+t31) ) *t32-t12* (h*
t12+2*t21* (t11-t21) ) ) *p+ ( (-t30+h-t20) *t21+t11* (t20+t30) ) *t32^2+ ( (
(3*h-3*t20) *t21+ (-h+2*t20+t30) *t31- (-t20+t30) *t11) *t12-t11* (-2*
t21+t11) * (-t21+t31) ) *t32-t12* ( (-t21*t30+ (-h+t30) *t31) *t12+t31*
(t11-t21) * (t31-2*t21) ) ) *q- ( (t11-2*t21+t31) *p^2+ (-h*t32-h*t12-
(t11+t21+t31) * (t11-2*t21+t31) ) *p+ ( (2*t30+h-t20) *t21+ (h-t30) *t31-
(-t20+t30) *t11) *t32+ ( (-2*t30+3*h-3*t20) *t21+ (-h+2*t20+t30) *t31+
t11* (t20+t30) ) *t12+ (t11-2*t21+t31) * ( (t11+t31) *t21+t11*t31) ) *p) *
t22+t12* ( (t32* (t11-t21) +t12* (-t21+t31) ) *p+ (-h*t12-t21* (t11-t21) ) *
t32-t12*t21* (-t21+t31) ) *t32*q^2+ ( (-t12+t32) * (t32* (t11-t21) +t12* (-
t21+t31) ) ) *p^2+ ( (t12*h+t11^2-t21^2) *t32^2+ (h*t12^2+ (2*t11*t31-2*
t21^2) *t12) *t32+t12^2* (-t21^2+t31^2) ) *p+ ( ( (-h+t20) *t21-t11*t20) *
t12-t11*t21* (t11-t21) ) *t32^2-t12* ( ( (h-t20) *t21+t31*t20) *t12+ (-t11
-t31) *t21^2+2*t11*t21*t31) *t32-t31*t12^2*t21* (-t21+t31) ) *q+ ( (t32*
(t11-t21) +t12* (-t21+t31) ) *p^2+ ( (-h*t12- (t11-t21) * (t11+t21+t31) ) *
t32-t12* (-t21+t31) * (t11+t21+t31) ) *p-t30* (t11-t21) *t32^2+ ( ( (h-t20)
*t21+ (h-t30) *t31+t11* (t20+t30) ) *t12+ (t11-t21) * ( (t11+t31) *t21+t11*
t31) ) *t32+t12* ( (t30-h+t20) *t12+ (t11+t31) *t21+t11*t31) * (-t21+t31) )
*p) / (2* (t22-t32) ^2* (t12-t22) ^2)
- ( (-t31*t11- (t20+t30) * (t12-t32) ) *t21+t11*t20* (-t32+t22) ) * ( (t12-2*
t22+t32) *t21+ (t11+t31) *t22-t11*t32-t12*t31) / (2* (-t32+t22) ^2* (-
t22+t12) ^2)
+F22 (t11 , t21 , t31 , t12 , t22 , t32)
, q, p);
Hame32:=unapply( ( (- (2* (t31- (1/2) *t11- (1/2) *t21) ) *t12*t22*q^3+ (
(2* (t31- (1/2) *t11- (1/2) *t21) ) * (t12+t22) *p+ (-h*t12- (2* (t31- (1/2) *
t11- (1/2) *t21) ) *t11) *t22- (2* (t31- (1/2) *t11- (1/2) *t21) ) *t12*t21) *
q^2+ ( (t11+t21-2*t31) *p^2+ (h*t22+h*t12+ (2* (t31- (1/2) *t11- (1/2) *
t21) ) * (t11+t21) ) *p+ ( (2*t30-2*h+2*t20) *t31+ (-t30+h-t20) *t21-t11*
(t20+t30) ) *t22+ (-2*t31*t20+ (-h+t20) *t21+t11*t20) *t12-2*t21* (t31-
(1/2) *t11- (1/2) *t21) *t11) *q-p* (p*h+ (2*t30-2*h) *t31-t30* (t11+t21) )
) *t32^2+ (t12* ( (-t11+t31) *t22+t12* (-t21+t31) ) *t22*q^3+ ( ( (t11-t31) *
t22^2-t12^2* (-t21+t31) ) *p+ (h*t12+t11* (-t11+t31) ) *t22^2-2*t12* (-
(1/2) *h*t12+ (-t21+t31) * (-t11+t31) ) *t22+t21*t12^2* (-t21+t31) ) *q^2+
( ( (t21-t31) *t22-t12* (-t11+t31) ) *p^2+ (-h*t22^2+ (-2*h*t12+2*t31* (-
t21+t31) ) *t22-h*t12^2+2*t31* (-t11+t31) *t12) *p+ ( (-t30+h-t20) *t31+
t11* (t20+t30) ) *t22^2+ ( ( (-3*t30+3*h) *t31+ (-h+t20+2*t30) *t21+ (-t20+
t30) *t11) *t12- (2* (-t21+t31) ) * (- (1/2) *t11+t31) *t11) *t22-2*t12* ( (-
(1/2) *t31*t20- (1/2) * (h-t20) *t21) *t12+ (-t11+t31) *t21* (t31- (1/2) *
t21) ) ) *q+ (2* ( (t31- (1/2) *t11- (1/2) *t21) *p^2+ ( (1/2) *h*t22+ (1/2) *h*
t12- (t31- (1/2) *t11- (1/2) *t21) * (t11+t21+t31) ) *p+ ( (- (1/2) *h-t20+
(1/2) *t30) *t31+t21* ( (1/2) *t20- (1/2) *h) - (1/2) * (-t20+t30) *t11) *t22+

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$$\begin{aligned}
& ((t_{20}+3*t_{30}*(1/2)-3*h*(1/2))*t_{31}+(-t_{30}+(1/2)*h-(1/2)*t_{20})*t_{21}- \\
& (1/2)*t_{11}*(t_{20}+t_{30}))*t_{12}+(t_{31}-(1/2)*t_{11}-(1/2)*t_{21})*((t_{11}+t_{21})* \\
& t_{31}+t_{11}*t_{21}))*p)*t_{32}-t_{12}*t_{22}*((-t_{11}+t_{31})*t_{22}+t_{12}*(-t_{21}+t_{31}))*p+ \\
& (h*t_{12}-t_{31}*(-t_{11}+t_{31}))*t_{22}-t_{12}*t_{31}*(-t_{21}+t_{31}))*q^2+((t_{12}+t_{22})*(- \\
& t_{11}+t_{31})*t_{22}+t_{12}*(-t_{21}+t_{31}))*p^2+((t_{12}*h+t_{11}^2-t_{31}^2)*t_{22}^2+(h* \\
& t_{12}^2+(2*t_{11}*t_{21}-2*t_{31}^2)*t_{12})*t_{22}+(t_{21}^2-t_{31}^2)*t_{12}^2)*p+((-h+ \\
& t_{30})*t_{31}-t_{30}*t_{11})*t_{12}+t_{11}*t_{31}*(-t_{11}+t_{31}))*t_{22}^2+((-t_{21}*t_{30}+(-h+ \\
& t_{30})*t_{31})*t_{12}+t_{31}*((t_{11}+t_{21})*t_{31}-2*t_{11}*t_{21}))*t_{12}*t_{22}+t_{31}*t_{12}^2* \\
& t_{21}*(-t_{21}+t_{31}))*q-((-t_{11}+t_{31})*t_{22}+t_{12}*(-t_{21}+t_{31}))*p^2+(h*t_{12}-(- \\
& t_{11}+t_{31})*(t_{11}+t_{21}+t_{31}))*t_{22}-t_{12}*(-t_{21}+t_{31})*(t_{11}+t_{21}+t_{31}))*p-t_{20}* \\
& (-t_{11}+t_{31})*t_{22}^2+((-h+t_{30})*t_{31}+(-h+t_{20})*t_{21}-t_{11}*(t_{20}+t_{30}))*t_{12}+ \\
& (-t_{11}+t_{31})*(t_{11}+t_{21})*t_{31}+t_{11}*t_{21}))*t_{22}+t_{12}*(t_{30}-h+t_{20})*t_{12}+ \\
& (t_{11}+t_{21})*t_{31}+t_{11}*t_{21})*(-t_{21}+t_{31}))*p)/(2*(t_{22}-t_{32})^2*(t_{12}-t_{32})^2) \\
& +((t_{20}+t_{30})*t_{32}-t_{31}*t_{11}-(t_{20}+t_{30})*t_{12})*t_{21}+t_{11}*t_{20}*(-t_{32}+t_{22}))* \\
& (t_{12}-t_{32})*t_{21}+(-t_{11}+2*t_{31})*t_{32}+(-t_{12}-t_{22})*t_{31}+t_{11}*t_{22})/(2*(-t_{32}+ \\
& t_{22})^2*(t_{12}-t_{32})^2) \\
& +F32(t_{11}, t_{21}, t_{31}, t_{12}, t_{22}, t_{32}) \\
& , q, p);
\end{aligned}$$

$$\begin{aligned}
Hame11 := (q, p) \rightarrow & \frac{1}{(-t_{22} + t_{12})(t_{12} - t_{32})} \left(-p^3 + ((t_{12} + t_{22} + t_{32})q + t_{11} + t_{21} \right. \\
& + t_{31})p^2 + (((-t_{22} - t_{32})t_{12} - t_{22}t_{32})q^2 + ((-t_{31} - t_{21})t_{12} + (-t_{11} - t_{31})t_{22} \\
& - t_{32}(t_{11} + t_{21}))q + (-t_{20} - t_{30})t_{12} + t_{20}t_{22} + t_{30}t_{32} + (-t_{31} - t_{21})t_{11} \\
& - t_{21}t_{31})p + q(q^2t_{12}t_{22}t_{32} + ((t_{21}t_{32} + t_{22}t_{31})t_{12} + t_{22}t_{32}t_{11})q + (t_{20}t_{32} \\
& + t_{21}t_{31} + t_{22}t_{30})t_{12} + ((-t_{20} - t_{30})t_{32} + t_{31}t_{11})t_{22} + t_{32}t_{11}t_{21}) \\
& \left. + \frac{((t_{20} + t_{30})t_{12} + t_{31}t_{11} - t_{32}t_{20} - t_{30}t_{32})t_{21} - t_{11}t_{20}(-t_{32} + t_{22})}{(-t_{22} + t_{12})(t_{12} - t_{32})} \right)
\end{aligned} \tag{4}$$

$$\begin{aligned}
Hame21 := (q, p) \rightarrow & \frac{1}{(-t_{22} + t_{12})(-t_{32} + t_{22})} \left(p^3 + ((-t_{12} - t_{22} - t_{32})q - t_{31} - t_{11} \right. \\
& - t_{21})p^2 + (((t_{12} + t_{32})t_{22} + t_{12}t_{32})q^2 + ((t_{21} + t_{31})t_{12} + (t_{11} + t_{31})t_{22} \\
& + t_{32}(t_{11} + t_{21}))q + (h - t_{20})t_{22} + (t_{30} - h + t_{20})t_{12} - t_{30}t_{32} + t_{11}(t_{21} + t_{31}) \\
& + t_{21}t_{31})p - q(q^2t_{12}t_{22}t_{32} + ((t_{11}t_{32} + t_{12}t_{31})t_{22} + t_{12}t_{32}t_{21})q + (t_{12}t_{30} \\
& + (-t_{20} - t_{30} + h)t_{32} + t_{31}t_{11})t_{22} + ((-h + t_{20})t_{32} + t_{21}t_{31})t_{12} + t_{32}t_{11}t_{21}) \\
& \left. + \frac{((-t_{12} + t_{32})t_{20} - t_{31}t_{11} - t_{12}t_{30} + t_{30}t_{32})t_{21} + t_{11}t_{20}(-t_{32} + t_{22})}{(-t_{22} + t_{12})(-t_{32} + t_{22})} \right) \\
& + \frac{1}{2} h t_{31} \left(\frac{1}{-t_{32} + t_{22}} + \frac{\ln(-t_{22} + t_{12})}{t_{12} - t_{32}} - \frac{\ln(-t_{32} + t_{22})}{t_{12} - t_{32}} \right)
\end{aligned}$$

$$\begin{aligned}
Hame31 := (q, p) \rightarrow & \frac{1}{(-t_{32} + t_{22})(t_{12} - t_{32})} \left(-p^3 + ((t_{12} + t_{22} + t_{32})q + t_{11} + t_{21} \right. \\
& + t_{31})p^2 + (((-t_{12} - t_{22})t_{32} - t_{12}t_{22})q^2 + ((-t_{21} - t_{11})t_{32} + (-t_{31} - t_{21})t_{12} \\
& - (t_{11} + t_{31})t_{22})q + (-h + t_{30})t_{32} + (-t_{20} - t_{30} + h)t_{12} + t_{20}t_{22} + (-t_{31} \\
& - t_{21})t_{11} - t_{21}t_{31})p + q(q^2t_{12}t_{22}t_{32} + ((t_{11}t_{22} + t_{12}t_{21})t_{32} + t_{12}t_{22}t_{31})q \\
& + (t_{12}t_{20} + (-t_{20} - t_{30} + h)t_{22} + t_{21}t_{11})t_{32} + ((-h + t_{30})t_{22} + t_{21}t_{31})t_{12} \\
& \left. + t_{22}t_{11}t_{31}) \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{((-t20 - t30) t32 + t31 t11 + t12 t20 + t12 t30) t21 - t11 t20 (-t32 + t22)}{(-t32 + t22) (t12 - t32)} \\
& + \frac{1}{2} h t21 \left(-\frac{1}{-t32 + t22} - \frac{\ln(-t32 + t22)}{t12 - t32} + \frac{\ln(-t22 + t12)}{t12 - t32} \right) \\
\text{Hame12} := (q, p) & \rightarrow \frac{1}{2} \frac{1}{(t12 - t32)^2 (-t22 + t12)^2} \left((t32 t22 (t31 - 2 t11 + t21) q^3 + (\right. \\
& - (t22 + t32) (t31 - 2 t11 + t21) p + (-h t32 + t31 (t31 - 2 t11 + t21)) t22 \\
& + t32 t21 (t31 - 2 t11 + t21)) q^2 + ((t31 - 2 t11 + t21) p^2 + (h t22 + h t32 - (t21 \\
& + t31) (t31 - 2 t11 + t21)) p + (-2 t11 t30 + t21 t30 + (-h + t30) t31) t22 + (\\
& -2 t11 t20 + (-h + t20) t21 + t20 t31) t32 + t31 t21 (t31 - 2 t11 + t21)) q - (p h + (\\
& -2 t30 - 2 t20) t11 + (t21 + t31) (t30 - h + t20)) p) t12^2 + \left(-((t31 - t11) t22 \right. \\
& - t32 (t11 - t21)) t32 t22 q^3 + \left(((t31 - t11) t22^2 - t32^2 (t11 - t21)) p + (h t32 \right. \\
& - t31 (t31 - t11)) t22^2 + 2 \left(\frac{1}{2} h t32 + (t11 - t21) (t31 - t11) \right) t32 t22 + t21 t32^2 (t11 \\
& - t21) \left. \right) q^2 + \left(((-t11 + t21) t22 + t32 (t31 - t11)) p^2 + (-h t22^2 + (-2 h t32 \right. \\
& + 2 t11 (t11 - t21)) t22 - h t32^2 - 2 t11 (t31 - t11) t32) p + (t11 t30 - (-h \\
& + t30) t31) t22^2 + (((3 t20 + 3 t30) t11 + (h - t20 - 2 t30) t21 - (-h + 2 t20 \\
& + t30) t31) t32 + t31 (t11 - t21) (-2 t11 + t31)) t22 + 2 \left(\left(\frac{1}{2} t11 t20 + \frac{1}{2} (h \right. \right. \\
& - t20) t21 \left. \right) t32 + (t31 - t11) t21 \left(-\frac{1}{2} t21 + t11 \right) \left. \right) t32 \left. \right) q - p \left((t31 - 2 t11 + t21) p^2 \right. \\
& + (-h t22 - h t32 - (t11 + t21 + t31) (t31 - 2 t11 + t21)) p + ((t30 + 3 t20) t11 + (h \\
& - t20) t21 - (-h + 2 t20 + t30) t31) t22 + (t11 (t20 + 3 t30) + (h - t20 - 2 t30) t21 \\
& - (-h + t30) t31) t32 + (t31 - 2 t11 + t21) (t11 (t21 + t31) + t21 t31)) t12 \\
& + t32 ((t31 - t11) t22 - t32 (t11 - t21)) p + (-h t32 - t11 (t31 - t11)) t22 \\
& + t32 t11 (t11 - t21)) t22 q^2 + (-((t31 - t11) t22 - t32 (t11 - t21)) (t22 + t32) p^2 \\
& + ((h t32 - t11^2 + t31^2) t22^2 + (h t32^2 + (-2 t11^2 + 2 t21 t31) t32) t22 + (-t11^2 \\
& + t21^2) t32^2) p + (((-t20 - t30) t11 + (t30 - h + t20) t31) t32 - t11 t31 (t31 \\
& - t11)) t22^2 - t32 (((t20 + t30) t11 - (t30 - h + t20) t21) t32 - (t11 (t21 + t31) \\
& - 2 t21 t31) t11) t22 + t32^2 t11 t21 (t11 - t21) \left. \right) q + p \left(((t31 - t11) t22 - t32 (t11 \right. \\
& - t21)) p^2 + ((-h t32 - (t31 - t11) (t11 + t21 + t31)) t22 + t32 (t11 - t21) (t11 + t21 \\
& + t31)) p - t20 (t31 - t11) t22^2 + (((t20 + t30) t11 + (h - t20) t21 - (-h \\
& + t30) t31) t32 + (t31 - t11) (t11 (t21 + t31) + t21 t31)) t22 + t32 ((-t31 - t21) t11 \\
& + t30 t32 - t21 t31) (t11 - t21)) \left. \right) + \frac{1}{(t12 - t32)^2 (-t22 + t12)^2} \left(\left(\left(-\frac{1}{2} t12 \right. \right. \right. \\
& + \frac{1}{2} t32 \left. \right) t21 + \left(t11 - \frac{1}{2} t31 \right) t12 + \left(-\frac{1}{2} t22 - \frac{1}{2} t32 \right) t11 + \frac{1}{2} t22 t31 \left. \right) \left(((-t20 \right.
\end{aligned}$$

$$\begin{aligned}
& -t30) t12 - t31 t11 + (t20 + t30) t32) t21 + t11 t20 (-t32 + t22)) \Big) \\
& + \frac{1}{2} h t21 t31 \left(\frac{\ln(-t32 + t22)}{(t12 - t32)^2} - \frac{\ln(-t22 + t12)}{(t12 - t32)^2} \right) \\
\text{Hame22} := (q, p) \rightarrow & \frac{1}{2} \frac{1}{(-t32 + t22)^2 (-t22 + t12)^2} \left((t12 t32 (t31 + t11 - 2 t21) q^3 + (\right. \\
& - (t12 + t32) (t31 + t11 - 2 t21) p + (-h t12 + t11 (t31 + t11 - 2 t21)) t32 \\
& + t12 t31 (t31 + t11 - 2 t21)) q^2 + ((t31 + t11 - 2 t21) p^2 + (h t32 + h t12 - (t11 \\
& + t31) (t31 + t11 - 2 t21)) p + ((2 t30 - 2 h + 2 t20) t21 + (-t20 - t30 + h) t31 \\
& - (t20 + t30) t11) t32 + (-2 t21 t30 + (-h + t30) t31 + t11 t30) t12 + t11 t31 (t31 \\
& + t11 - 2 t21)) q - (p h + (-2 h + 2 t20) t21 - t20 (t11 + t31)) p) t22^2 + \left(\right. \\
& -t12 t32 (t32 (t11 - t21) + t12 (-t21 + t31)) q^3 + \left((t32^2 (t11 - t21) + t12^2 (-t21 \\
& + t31)) p + (h t12 - t11 (t11 - t21)) t32^2 - 2 t12 \left(-\frac{1}{2} h t12 + (t11 - t21) (-t21 \right. \right. \\
& \left. \left. + t31) \right) t32 - t31 (-t21 + t31) t12^2 \right) q^2 + (((-t21 + t31) t32 + t12 (t11 - t21)) p^2 + (\\
& -h t32^2 + (-2 h t12 - 2 t21 (-t21 + t31)) t32 - t12 (h t12 + 2 t21 (t11 - t21))) p + ((\\
& -t20 - t30 + h) t21 + (t20 + t30) t11) t32^2 + ((3 h - 3 t20) t21 + (-h + 2 t20 \\
& + t30) t31 - t11 (-t20 + t30)) t12 - t11 (-2 t21 + t11) (-t21 + t31)) t32 - t12 ((\\
& -t21 t30 + (-h + t30) t31) t12 + t31 (t11 - t21) (t31 - 2 t21)) q - ((t31 + t11 \\
& - 2 t21) p^2 + (-h t32 - h t12 - (t11 + t21 + t31) (t31 + t11 - 2 t21)) p + ((2 t30 + h \\
& - t20) t21 + (h - t30) t31 - t11 (-t20 + t30)) t32 + ((-2 t30 + 3 h - 3 t20) t21 + (-h \\
& + 2 t20 + t30) t31 + (t20 + t30) t11) t12 + (t31 + t11 - 2 t21) ((t11 + t31) t21 \\
& + t31 t11)) p) t22 + t12 ((t32 (t11 - t21) + t12 (-t21 + t31)) p + (-h t12 - t21 (t11 \\
& - t21)) t32 - t12 t21 (-t21 + t31)) t32 q^2 + (- (t12 + t32) (t32 (t11 - t21) + t12 (-t21 \\
& + t31)) p^2 + ((h t12 + t11^2 - t21^2) t32^2 + (h t12^2 + (2 t11 t31 - 2 t21^2) t12) t32 \\
& + t12^2 (-t21^2 + t31^2)) p + (((-h + t20) t21 - t11 t20) t12 - t11 t21 (t11 - t21)) t32^2 \\
& - t12 ((h - t20) t21 + t20 t31) t12 + (-t11 - t31) t21^2 + 2 t11 t21 t31) t32 \\
& - t31 t12^2 t21 (-t21 + t31)) q + ((t32 (t11 - t21) + t12 (-t21 + t31)) p^2 + ((-h t12 \\
& - (t11 - t21) (t11 + t21 + t31)) t32 - t12 (-t21 + t31) (t11 + t21 + t31)) p - t30 (t11 \\
& - t21) t32^2 + ((h - t20) t21 + (h - t30) t31 + (t20 + t30) t11) t12 + (t11 \\
& - t21) ((t11 + t31) t21 + t31 t11)) t32 + t12 ((t30 - h + t20) t12 + (t11 + t31) t21 \\
& + t31 t11) (-t21 + t31)) p) - \frac{1}{2} \frac{1}{(-t32 + t22)^2 (-t22 + t12)^2} (((-t31 t11 - (t20 \\
& + t30) (t12 - t32)) t21 + t11 t20 (-t32 + t22)) ((t12 - 2 t22 + t32) t21 + (t11 \\
& + t31) t22 - t32 t11 - t12 t31)) \\
\text{Hame32} := (q, p) \rightarrow & \frac{1}{2} \frac{1}{(-t32 + t22)^2 (t12 - t32)^2} \left(\left(-2 \left(t31 - \frac{1}{2} t11 \right. \right. \right. \\
& \left. \left. - \frac{1}{2} t21 \right) t12 t22 q^3 + \left(2 \left(t31 - \frac{1}{2} t11 - \frac{1}{2} t21 \right) (t12 + t22) p + \left(-h t12 - 2 \left(t31 \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} t_{11} - \frac{1}{2} t_{21}) t_{11}) t_{22} - 2 \left(t_{31} - \frac{1}{2} t_{11} - \frac{1}{2} t_{21} \right) t_{12} t_{21}) q^2 + \left((t_{11} + t_{21} \right. \\
& - 2 t_{31}) p^2 + \left(h t_{22} + h t_{12} + 2 \left(t_{31} - \frac{1}{2} t_{11} - \frac{1}{2} t_{21} \right) (t_{11} + t_{21}) \right) p + \left((2 t_{30} \right. \\
& - 2 h + 2 t_{20}) t_{31} + (-t_{20} - t_{30} + h) t_{21} - (t_{20} + t_{30}) t_{11}) t_{22} + (-2 t_{20} t_{31} + (-h \\
& + t_{20}) t_{21} + t_{11} t_{20}) t_{12} - 2 t_{21} \left(t_{31} - \frac{1}{2} t_{11} - \frac{1}{2} t_{21} \right) t_{11}) q - p (p h + (2 t_{30} \\
& - 2 h) t_{31} - t_{30} (t_{11} + t_{21})) t_{32}^2 + \left(t_{12} ((t_{31} - t_{11}) t_{22} + t_{12} (-t_{21} + t_{31})) t_{22} q^3 \right. \\
& + \left((t_{11} - t_{31}) t_{22}^2 - t_{12}^2 (-t_{21} + t_{31}) \right) p + (h t_{12} + t_{11} (t_{31} - t_{11})) t_{22}^2 - 2 t_{12} \left(\right. \\
& - \frac{1}{2} h t_{12} + (-t_{21} + t_{31}) (t_{31} - t_{11})) t_{22} + t_{21} t_{12}^2 (-t_{21} + t_{31}) \left. \right) q^2 + \left((t_{21} \right. \\
& - t_{31}) t_{22} - t_{12} (t_{31} - t_{11})) p^2 + (-h t_{22}^2 + (-2 h t_{12} + 2 t_{31} (-t_{21} + t_{31})) t_{22} \\
& - h t_{12}^2 + 2 t_{31} (t_{31} - t_{11}) t_{12}) p + ((-t_{20} - t_{30} + h) t_{31} + (t_{20} + t_{30}) t_{11}) t_{22}^2 \\
& + \left(((-3 t_{30} + 3 h) t_{31} + (-h + t_{20} + 2 t_{30}) t_{21} + t_{11} (-t_{20} + t_{30})) t_{12} - 2 (-t_{21} \right. \\
& + t_{31}) \left(-\frac{1}{2} t_{11} + t_{31} \right) t_{11}) t_{22} - 2 t_{12} \left(\left(-\frac{1}{2} t_{20} t_{31} - \frac{1}{2} (h - t_{20}) t_{21} \right) t_{12} + (t_{31} \right. \\
& - t_{11}) t_{21} \left(t_{31} - \frac{1}{2} t_{21} \right) \left. \right) q + 2 \left(\left(t_{31} - \frac{1}{2} t_{11} - \frac{1}{2} t_{21} \right) p^2 + \left(\frac{1}{2} h t_{22} \right. \right. \\
& + \frac{1}{2} h t_{12} - \left(t_{31} - \frac{1}{2} t_{11} - \frac{1}{2} t_{21} \right) (t_{11} + t_{21} + t_{31}) \left. \right) p + \left(\left(-\frac{1}{2} h - t_{20} \right. \right. \\
& + \frac{1}{2} t_{30}) t_{31} + t_{21} \left(\frac{1}{2} t_{20} - \frac{1}{2} h \right) - \frac{1}{2} t_{11} (-t_{20} + t_{30}) \left. \right) t_{22} + \left(\left(t_{20} + \frac{3}{2} t_{30} \right. \right. \\
& - \frac{3}{2} h) t_{31} + \left(-t_{30} + \frac{1}{2} h - \frac{1}{2} t_{20} \right) t_{21} - \frac{1}{2} (t_{20} + t_{30}) t_{11}) t_{12} + \left(t_{31} - \frac{1}{2} t_{11} \right. \\
& - \frac{1}{2} t_{21}) (t_{31} (t_{11} + t_{21}) + t_{21} t_{11}) \left. \right) p \left. \right) t_{32} - t_{12} t_{22} ((t_{31} - t_{11}) t_{22} + t_{12} (-t_{21} \\
& + t_{31})) p + (h t_{12} - t_{31} (t_{31} - t_{11})) t_{22} - t_{12} t_{31} (-t_{21} + t_{31})) q^2 + ((t_{12} \\
& + t_{22}) ((t_{31} - t_{11}) t_{22} + t_{12} (-t_{21} + t_{31})) p^2 + ((h t_{12} + t_{11}^2 - t_{31}^2) t_{22}^2 + (h t_{12}^2 \\
& + (2 t_{11} t_{21} - 2 t_{31}^2) t_{12}) t_{22} + (t_{21}^2 - t_{31}^2) t_{12}^2) p + (((-h + t_{30}) t_{31} \\
& - t_{11} t_{30}) t_{12} + t_{11} t_{31} (t_{31} - t_{11})) t_{22}^2 + ((-t_{21} t_{30} + (-h + t_{30}) t_{31}) t_{12} \\
& + t_{31} (t_{31} (t_{11} + t_{21}) - 2 t_{21} t_{11})) t_{12} t_{22} + t_{31} t_{12}^2 t_{21} (-t_{21} + t_{31})) q - ((t_{31} \\
& - t_{11}) t_{22} + t_{12} (-t_{21} + t_{31})) p^2 + ((h t_{12} - (t_{31} - t_{11}) (t_{11} + t_{21} + t_{31})) t_{22} \\
& - t_{12} (-t_{21} + t_{31}) (t_{11} + t_{21} + t_{31})) p - t_{20} (t_{31} - t_{11}) t_{22}^2 + (((-h + t_{30}) t_{31} + (-h \\
& + t_{20}) t_{21} - (t_{20} + t_{30}) t_{11}) t_{12} + (t_{31} - t_{11}) (t_{31} (t_{11} + t_{21}) + t_{21} t_{11})) t_{22} \\
& + t_{12} ((t_{30} - h + t_{20}) t_{12} + t_{31} (t_{11} + t_{21}) + t_{21} t_{11}) (-t_{21} + t_{31})) p \\
& + \frac{1}{2} \frac{1}{(-t_{32} + t_{22})^2 (t_{12} - t_{32})^2} (((t_{20} + t_{30}) t_{32} - t_{31} t_{11} - (t_{20} + t_{30}) t_{12}) t_{21} \\
& + t_{11} t_{20} (-t_{32} + t_{22})) ((t_{12} - t_{32}) t_{21} + (-t_{11} + 2 t_{31}) t_{32} + (-t_{12} - t_{22}) t_{31} \\
& + t_{22} t_{11})) + \frac{1}{2} \frac{h t_{21} t_{31} (\ln(-t_{22} + t_{12}) - \ln(-t_{32} + t_{22}))}{(t_{12} - t_{32})^2}
\end{aligned}$$

Matching the JMU tau-function with the Hamiltonians evaluated at $\hbar=0$

```

> Ct11:=simplify(omegaJMUdt11 - (-residue(Hame11(q,p)/h,h=0) +F11
(t11,t21,t31,t12,t22,t32) ));
Ct21:=simplify(omegaJMUdt21 - (-residue(Hame21(q,p)/h,h=0) +F21
(t11,t21,t31,t12,t22,t32) ));
Ct31:=simplify(omegaJMUdt31 - (-residue(Hame31(q,p)/h,h=0) +F31
(t11,t21,t31,t12,t22,t32)));

Ct12:=simplify(omegaJMUdt12- (-residue(Hame12(q,p)/h,h=0) +F12
(t11,t21,t31,t12,t22,t32) ));

Ct22:=simplify(simplify(omegaJMUdt22- (-residue(Hame22(q,p)/h,h=0)
+F22(t11,t21,t31,t12,t22,t32) )));

Ct32:=simplify(simplify(omegaJMUdt32- (-residue(Hame32(q,p)/h,h=0)
+F32(t11,t21,t31,t12,t22,t32) )));

```

$$Ct11 := \frac{(t20 + t30)(t11 - t31)}{t12 - t32} \quad (5)$$

$$Ct21 := \frac{1}{2} \frac{1}{(-t32 + t22)(t12 - t32)} (-h t31 (-t32 + t22) \ln(-t22 + t12) + h t31 (-t32 + t22) \ln(-t32 + t22) - (t12 - t32) ((h - 2 t20) t31 + 2 t21 t20))$$

$$Ct31 := \frac{1}{2} \frac{1}{(-t32 + t22)(t12 - t32)} (-h t21 (-t32 + t22) \ln(-t22 + t12) + h t21 (-t32 + t22) \ln(-t32 + t22) + ((-h - 2 t20) t21 + 2 t11 t20 + 2 t30 (t11 - t31)) t32 + t12 (h + 2 t20) t21 - 2 (t20 + t30) (t11 - t31) t22 - 2 t12 t20 t31)$$

$$Ct12 := \frac{1}{2} \frac{1}{(t12 - t32)^2 (-t22 + t12)} (h t21 t31 (-t22 + t12) \ln(-t22 + t12) - h t21 t31 (-t22 + t12) \ln(-t32 + t22) - (t20 + t30) ((t20 + t30) t12^2 + ((-t22 - t32) t30 - 2 t32 t20 + (t11 - t31)^2) t12 + t22 t30 t32 + t20 t32^2 - t22 (t11 - t31)^2))$$

$$Ct22 := \frac{1}{2} \frac{1}{(-t32 + t22)^2 (-t22 + t12)} (t20 (t20 t22^2 + ((-2 t20 - t30) t32 + t12 t30 - (t21 - t31)^2) t22 + (-t30 t32 + (t21 - t31)^2) t12 + (t20 + t30) t32^2))$$

$$Ct32 := \frac{1}{2} \frac{1}{(-t32 + t22)^2 (t12 - t32)^2} (-h t21 t31 (-t32 + t22)^2 \ln(-t22 + t12) + h t21 t31 (-t32 + t22)^2 \ln(-t32 + t22) - t30^2 t32^3 + (t30 (t20 + 2 t30) t22 + ((-2 t11 + 2 t21) t31 + t11^2 - t12 t30 - t21^2) t20 + t30 (t11^2 - 2 t11 t31 + t12 t30 + t31^2)) t32^2 + (-t30 (t20 + t30) t22^2 + (-2 (t11 - t31)^2 t20 - 2 t30 (t11^2 - 2 t11 t31 + t12 t30 + t31^2)) t22 + t12 t20 (t12 t30 + 2 t21^2 - 4 t21 t31 + 2 t31^2)) t32 + (t20 + t30) (t11^2 - 2 t11 t31 + t12 t30 + t31^2) t22^2 - t12^2 t20 t22 t30 - t12^2 t20 (t21 - t31)^2)$$

```

> GSolOldCoordinatesbis := -t20*t21*(t21-2*t31)/(2*t22-2*t32)+(t20+
t30)*t11*(-2*t31+t11)/(2*t12-2*t32)-t31^2*(t30*t32+(-t20-t30)*
t22+t12*t20)/((2*(-t32+t22))*(t12-t32))+(1/2)*ln(-t32+t22)*t20*
t30-(1/2)*ln(t22-t12)*t20^2-(1/2)*ln(t22-t12)*t20*t30-(1/2)*ln
(t12-t32)*t20*t30-(1/2)*t30^2*ln(t12-t32);

```

(6)

$$\begin{aligned}
GSolOldCoordinatesbis := & -\frac{t20 t21 (t21 - 2 t31)}{2 t22 - 2 t32} + \frac{(t20 + t30) t11 (-2 t31 + t11)}{2 t12 - 2 t32} \\
& - \frac{1}{2} \frac{t31^2 (t30 t32 + (-t20 - t30) t22 + t12 t20)}{(-t32 + t22) (t12 - t32)} + \frac{1}{2} \ln(-t32 + t22) t20 t30 \\
& - \frac{1}{2} \ln(t22 - t12) t20^2 - \frac{1}{2} \ln(t22 - t12) t20 t30 - \frac{1}{2} \ln(t12 - t32) t20 t30 \\
& - \frac{1}{2} t30^2 \ln(t12 - t32)
\end{aligned}$$

(6)

We can express the JMU differential using the times (T_1,...,T_5,tau)

```

> TT1:=unapply (t12+t22+t32, t11, t21, t31, t12, t22, t32) ;
TT2:=unapply (t11+t21+t31, t11, t21, t31, t12, t22, t32) ;
TT3:=unapply (t22, t11, t21, t31, t12, t22, t32) ;
TT4:=unapply (t11, t11, t21, t31, t12, t22, t32) ;
TT5:=unapply (t32, t11, t21, t31, t12, t22, t32) ;

solve ({TT1 (t11, t21, t31, t12, t22, t32)=T1, TT2 (t11, t21, t31, t12, t22,
t32)=T2, TT3 (t11, t21, t31, t12, t22, t32)=T3, TT4 (t11, t21, t31, t12, t22,
t32)=T4, TT5 (t11, t21, t31, t12, t22, t32)=T5, taufunction (t11, t21, t31,
t12, t22, t32)=tau }, {t11, t21, t31, t12, t22, t32}) ;

t11function := T4;
t12function := -T3-T5+T1;
t21function := (tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))+T1*
T2-T1*T4-2*T3*T2-T5*T2+3*T4*T3)/(2*T1-3*T3-3*T5);
t22function := T3;
t31function := -(tau*sqrt((-2*T3-T5+T1)*(-T3-2*T5+T1)*(T3-T5))-
T1*T2+T1*T4+T3*T2+2*T5*T2-3*T4*T5)/(2*T1-3*T3-3*T5);
t32function := T5;
simplify (TT1 (t11function, t21function, t31function, t12function,
t22function, t32function)) ;
simplify (TT2 (t11function, t21function, t31function, t12function,
t22function, t32function)) ;
simplify (TT3 (t11function, t21function, t31function, t12function,
t22function, t32function)) ;
simplify (TT4 (t11function, t21function, t31function, t12function,
t22function, t32function)) ;
simplify (TT5 (t11function, t21function, t31function, t12function,
t22function, t32function)) ;
simplify (taufunction (t11function, t21function, t31function,
t12function, t22function, t32function)) ;

partialtaufunction:=unapply ( diff (t11function, tau)*partialt11 +
diff (t21function, tau)*partialt21+diff (t31function, tau)*

```



```

partialt31+diff(t12function,tau)*partialt12+diff(t22function,tau)
*partialt22+diff(t32function,tau)*partialt32,T1,T2,T3,T4,T5,tau):
partialtau:=partialtaufunction(TT1(t11,t21,t31,t12,t22,t32),TT2
(t11,t21,t31,t12,t22,t32),TT3(t11,t21,t31,t12,t22,t32),TT4(t11,
t21,t31,t12,t22,t32),TT5(t11,t21,t31,t12,t22,t32),taufunction
(t11,t21,t31,t12,t22,t32)):

```

```

Coefft21:=residue(partialtau/partialt21^2,partialt21=0):

```

```

Coefft31:=residue(partialtau/partialt31^2,partialt31=0):

```

```

Hamttau:=unapply(simplify(Coefft21*Hame21(q,p)+Coefft31*Hame31
(q,p)),q,p):

```

```

Ltauq:=diff(Hamttau(q,p),p):

```

```

Ltauq:=-diff(Hamttau(q,p),q):

```

```

SOLL:=solve({checkqfunction(q,p)=checkq,checkpfunction(q,p)=
checkp},{q,p}):

```

```

Solp:=simplify(rhs(SOLL[1])):

```

```

Solq:=simplify(rhs(SOLL[2])):

```

```

simplify(checkqfunction(Solq,Solp)-checkq);

```

```

simplify(checkpfunction(Solq,Solp)-checkp);

```

```

Ltauchekqfunction:=unapply(simplify(diff(checkqfunction(q,p),
q)*Ltauq+diff(checkqfunction(q,p),p)*Ltauq+Coefft21*h*diff
(checkqfunction(q,p),t21)+Coefft31*h*diff(checkqfunction(q,p),
t31)),q,p):

```

```

Ltauchekpfunction:=unapply(simplify(diff(checkpfunction(q,p),q)
*Ltauq+diff(checkpfunction(q,p),p)*Ltauq+Coefft21*h*diff
(checkpfunction(q,p),t21)+Coefft31*h*diff(checkpfunction(q,p),
t31)),q,p):

```

```

Ltauchekqfunction2:=unapply(simplify(Ltauchekqfunction(Solq,
Solp)),t11,t21,t31,t12,t22,t32):

```

```

Ltauchekpfunction2:=unapply(simplify(Ltauchekpfunction(Solq,
Solp)),t11,t21,t31,t12,t22,t32):

```

```

Ltauchekq:=simplify(Ltauchekqfunction2(t11function,t21function
,t31function,t12function,t22function,t32function)):

```

```

Ltauchekp:=simplify(Ltauchekpfunction2(t11function,t21function
,t31function,t12function,t22function,t32function)):

```

```

HamttauchekCoordinates:=unapply(simplify(int(Ltauchekq,checkp)-
int(simplify(diff(int(Ltauchekq,checkp),checkq)+Ltauchekp),
checkq),checkq,checkp)):

```

```

simplify(Ltauchekq-diff(HamttauchekCoordinates(checkq,checkp),

```

```

checkp) );
simplify(Ltauchekp+diff (HamtauchekCoordinates (checkq, checkp) ,
checkq) );

```

$$\begin{aligned}
TT1 &:= (t11, t21, t31, t12, t22, t32) \rightarrow t12 + t22 + t32 \\
TT2 &:= (t11, t21, t31, t12, t22, t32) \rightarrow t11 + t21 + t31 \\
TT3 &:= (t11, t21, t31, t12, t22, t32) \rightarrow t22 \\
TT4 &:= (t11, t21, t31, t12, t22, t32) \rightarrow t11 \\
TT5 &:= (t11, t21, t31, t12, t22, t32) \rightarrow t32
\end{aligned}$$

(7)

$$\left\{ t11 = T4, t12 = -T3 - T5 + T1, t21 \right.$$

$$= \frac{1}{2 T1 - 3 T3 - 3 T5} \left(\tau \sqrt{(-2 T3 - T5 + T1) (-T3 - 2 T5 + T1) (-T5 + T3)} + T1 T2 - T1 T4 - 2 T3 T2 - T5 T2 + 3 T3 T4 \right), t22 = T3, t31 =$$

$$- \frac{1}{2 T1 - 3 T3 - 3 T5} \left(\tau \sqrt{(-2 T3 - T5 + T1) (-T3 - 2 T5 + T1) (-T5 + T3)} - T1 T2 + T1 T4 + T3 T2 + 2 T5 T2 - 3 T5 T4 \right), t32 = T5 \left. \right\}$$

$$t11function := T4$$

$$t12function := -T3 - T5 + T1$$

$$t21function := \frac{1}{2 T1 - 3 T3 - 3 T5} \left(\tau \sqrt{(-2 T3 - T5 + T1) (-T3 - 2 T5 + T1) (-T5 + T3)} + T1 T2 - T1 T4 - 2 T3 T2 - T5 T2 + 3 T3 T4 \right)$$

$$t22function := T3$$

$$t31function :=$$

$$- \frac{1}{2 T1 - 3 T3 - 3 T5} \left(\tau \sqrt{(-2 T3 - T5 + T1) (-T3 - 2 T5 + T1) (-T5 + T3)} - T1 T2 + T1 T4 + T3 T2 + 2 T5 T2 - 3 T5 T4 \right)$$

$$t32function := T5$$

T1

T2

T3

T4

T5

 τ

0

0

0

0

```

> omegaJMU:=omegaJMUdt11*dt11+omegaJMUdt21*dt21+omegaJMUdt31*
dt31+omegaJMUdt12*dt12+omegaJMUdt22*dt22+omegaJMUdt32*dt32:
omegaJMUfunction1:=unapply (omegaJMU, q, p) :
omegaJMUfunction2:=unapply (simplify (omegaJMUfunction1 (Solq, Solp)
), t11, t21, t31, t12, t22, t32) :
omegaJMUfunction3:=unapply (simplify (omegaJMUfunction2
(t11function, t21function, t31function, t12function, t22function,
t32function) ), dt11, dt21, dt31, dt12, dt22, dt32) :

```

```

> dt11function:=simplify(diff(t11function,T1)*dT1+ diff
(t11function,T2)*dT2+diff(t11function,T3)*dT3+diff(t11function,
T4)*dT4+diff(t11function,T5)*dT5+diff(t11function,tau)*dtau);
dt21function:=simplify(diff(t21function,T1)*dT1+ diff
(t21function,T2)*dT2+diff(t21function,T3)*dT3+diff(t21function,
T4)*dT4+diff(t21function,T5)*dT5+diff(t21function,tau)*dtau):
dt31function:=simplify(diff(t31function,T1)*dT1+ diff
(t31function,T2)*dT2+diff(t31function,T3)*dT3+diff(t31function,
T4)*dT4+diff(t31function,T5)*dT5+diff(t31function,tau)*dtau):
dt12function:=simplify(diff(t12function,T1)*dT1+ diff
(t12function,T2)*dT2+diff(t12function,T3)*dT3+diff(t12function,
T4)*dT4+diff(t12function,T5)*dT5+diff(t12function,tau)*dtau);
dt22function:=simplify(diff(t22function,T1)*dT1+ diff
(t22function,T2)*dT2+diff(t22function,T3)*dT3+diff(t22function,
T4)*dT4+diff(t22function,T5)*dT5+diff(t22function,tau)*dtau);
dt32function:=simplify(diff(t32function,T1)*dT1+ diff
(t32function,T2)*dT2+diff(t32function,T3)*dT3+diff(t32function,
T4)*dT4+diff(t32function,T5)*dT5+diff(t32function,tau)*dtau);

```

$$\begin{aligned}
dt11function &:= dT4 \\
dt12function &:= dT1 - dT3 - dT5 \\
dt22function &:= dT3 \\
dt32function &:= dT5
\end{aligned}$$

(8)

```

> omegaJMUfunctioncheckCoordinates:=simplify(omegaJMUfunction3
(dt11function,dt21function,dt31function,dt12function,dt22function,
dt32function)):

```

```

> omegaJMUfunctioncheckCoordinatesdT1:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dT1^2,dT1=0)):
omegaJMUfunctioncheckCoordinatesdT2:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dT2^2,dT2=0)):
omegaJMUfunctioncheckCoordinatesdT3:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dT3^2,dT3=0)):
omegaJMUfunctioncheckCoordinatesdT4:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dT4^2,dT4=0)):
omegaJMUfunctioncheckCoordinatesdT5:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dT5^2,dT5=0)):
omegaJMUfunctioncheckCoordinatesdtau:=simplify(residue
(omegaJMUfunctioncheckCoordinates/dtau^2,dtau=0)):

```

```

> omegaJMUfunctioncheckCoordinatesdT1function:=unapply
(omegaJMUfunctioncheckCoordinatesdT1,T1,T2,T3,T4,T5,tau):
omegaJMUfunctioncheckCoordinatesdT2function:=unapply
(omegaJMUfunctioncheckCoordinatesdT2,T1,T2,T3,T4,T5,tau):
omegaJMUfunctioncheckCoordinatesdT3function:=unapply

```

```

(omegaJMUfunctioncheckCoordinatesdT3,T1,T2,T3,T4,T5,tau):
omegaJMUfunctioncheckCoordinatesdT4function:=unapply
(omegaJMUfunctioncheckCoordinatesdT4,T1,T2,T3,T4,T5,tau):
omegaJMUfunctioncheckCoordinatesdT5function:=unapply
(omegaJMUfunctioncheckCoordinatesdT5,T1,T2,T3,T4,T5,tau):
omegaJMUfunctioncheckCoordinatesdtaufunction:=unapply
(omegaJMUfunctioncheckCoordinatesdtau,T1,T2,T3,T4,T5,tau):
> t12function-t22function;
t22function-t32function;
t12function-t32function;
assume(-2*TTT3-TTT5+TTT1>0 and TTT3-TTT5>0 and -TTT3-2*TTT5+
TTT1>0);
TermdT1function:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdT1function(TTT1,TTT2,TTT3,TTT4,
TTT5,tau)),TTT1,TTT2,TTT3,TTT4,TTT5):
TermdT2function:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdT2function(TTT1,TTT2,TTT3,TTT4,
TTT5,tau)),TTT1,TTT2,TTT3,TTT4,TTT5):
TermdT3function:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdT3function(TTT1,TTT2,TTT3,TTT4,
TTT5,tau)),TTT1,TTT2,TTT3,TTT4,TTT5):
TermdT4function:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdT4function(TTT1,TTT2,TTT3,TTT4,
TTT5,tau)),TTT1,TTT2,TTT3,TTT4,TTT5):
TermdT5function:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdT5function(TTT1,TTT2,TTT3,TTT4,
TTT5,tau)),TTT1,TTT2,TTT3,TTT4,TTT5):

```

$$\begin{aligned}
& -2 T_3 - T_5 + T_1 \\
& -T_5 + T_3 \\
& -T_3 - 2 T_5 + T_1
\end{aligned}
\tag{9}$$

```

> omegaJMUfunctioncheckCoordinatesdtaubis:=- (checkp^2*
checkq+checkq^2*checkp -checkq*checkp*tau-t20*checkp +t10*checkq)
;
Termdtaufunction:=unapply(simplify
(omegaJMUfunctioncheckCoordinatesdtaufunction(TTT1,TTT2,TTT3,
TTT4,TTT5,tau)-omegaJMUfunctioncheckCoordinatesdtaubis),TTT1,
TTT2,TTT3,TTT4,TTT5):
Termdtaufunction(T1,T2,T3,T4,T5);
simplify(omegaJMUfunctioncheckCoordinatesdtaubis-
(HamtaucheckCoordinatesTheo(checkq,checkp) +h*checkq));

```

$$\begin{aligned}
\omega_{JMU}functioncheckCoordinatesdtaubis := & -checkp^2 checkq - checkq^2 checkp \\
& + checkq checkp \tau + t_{20} checkp - checkq (-t_{20} - t_{30})
\end{aligned}
\tag{10}$$

$$- \left(4 \sqrt{-2 T3 - T5 + T1} \left(\sqrt{-T3 - 2 T5 + T1} \tau \sqrt{-T5 + T3} \left(\left(-\frac{5}{4} t20 - \frac{1}{4} t30 \right) T3 + \left(-\frac{7}{4} t20 + \frac{1}{4} t30 \right) T5 + T1 t20 \right) \sqrt{-2 T3 - T5 + T1} - \frac{1}{4} (t20 - t30) (-T5 + T3) (T2 - 3 T4) (-T3 - 2 T5 + T1) \right) \right) / \left(\sqrt{-T3 - 2 T5 + T1} \sqrt{-T5 + T3} (2 T1 - 3 T3 - 3 T5)^2 \right)$$

0

```
> pdsolve({
diff(G(T1,T2,T3,T4,T5,tau),tau) = Termdtaufunction(T1,T2,T3,T4,
T5),
diff(G(T1,T2,T3,T4,T5,tau),T1) = TermdT1function(T1,T2,T3,T4,T5),
diff(G(T1,T2,T3,T4,T5,tau),T2) = TermdT2function(T1,T2,T3,T4,T5),
diff(G(T1,T2,T3,T4,T5,tau),T3) = TermdT3function(T1,T2,T3,T4,T5),
diff(G(T1,T2,T3,T4,T5,tau),T4) = TermdT4function(T1,T2,T3,T4,T5),
diff(G(T1,T2,T3,T4,T5,tau),T5) = TermdT5function(T1,T2,T3,T4,T5)}
,{G(T1,T2,T3,T4,T5,tau)});
```

$$\left\{ G(T1, T2, T3, T4, T5, \tau) = \frac{1}{\sqrt{-T5 + T3} (2 T1 - 3 T3 - 3 T5)^2} \left(\tau \sqrt{-T3 - 2 T5 + T1} (t20 - t30) (-T5 + T3) (T2 - 3 T4) \sqrt{-2 T3 - T5 + T1} + 2 \sqrt{-T5 + T3} \left(-t30 \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 (t20 + t30) \ln(T3 + 2 T5 - T1) - \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 (t20 + t30) t20 \ln(2 T3 + T5 - T1) + t30 \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 t20 \ln(-T5 + T3) - (-2 T3 - T5 + T1) \left(-\frac{5}{4} \tau^2 T3 - \frac{7}{4} \tau^2 T5 + \tau^2 T1 - \frac{1}{4} (T2 - 3 T4)^2 \right) t20 + \left(-\frac{1}{2} T3^2 \tau^2 + \left(\frac{1}{4} \tau^2 T5 + \frac{1}{4} \tau^2 T1 - \frac{1}{4} (T2 - 3 T4)^2 \right) T3 + \frac{1}{4} \tau^2 T5^2 + \left(-\frac{1}{4} \tau^2 T1 - \frac{1}{2} (T2 - 3 T4)^2 \right) T5 + \frac{1}{4} T1 (T2 - 3 T4)^2 \right) t30 + 2_C1 \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 \right) \right\} \quad (11)$$

```
> GSol :=unapply( (tau*sqrt(-T3-2*T5+T1)*(t20-t30)*(-T5+T3)*(T2-3*
T4)*sqrt(-2*T3-T5+T1)+(2*(-(T1-3*T3*(1/2)-3*T5*(1/2))^2*t30*(t20+
t30)*ln(T3+2*T5-T1)-(T1-3*T3*(1/2)-3*T5*(1/2))^2*t20*(t20+t30)*ln
(2*T3+T5-T1)+(T1-3*T3*(1/2)-3*T5*(1/2))^2*t30*t20*ln(-T5+T3)-(-2*
T3-T5+T1)*(-5*tau^2*T3*(1/4)-7*tau^2*T5*(1/4)+tau^2*T1-(1/4)*(T2
-3*T4)^2)*t20+(-(1/2)*T3^2*tau^2+(1/4)*tau^2*T5+(1/4)*tau^2*T1-
(1/4)*(T2-3*T4)^2)*T3+(1/4)*T5^2*tau^2+(-(1/4)*tau^2*T1-(1/2)*(T2
-3*T4)^2)*T5+(1/4)*T1*(T2-3*T4)^2)*t30)*sqrt(-T5+T3))/(sqrt(-T5+
T3)*(2*T1-3*T3-3*T5)^2) ,T1,T2,T3,T4,T5,tau);
```

```

simplify(diff(GSol(T1,T2,T3,T4,T5,tau),tau) -Termdtaufunction(T1,
T2,T3,T4,T5));
simplify(diff(GSol(T1,T2,T3,T4,T5,tau),T1) -TermdT1function(T1,
T2,T3,T4,T5));
simplify(diff(GSol(T1,T2,T3,T4,T5,tau),T2) -TermdT2function(T1,
T2,T3,T4,T5));
simplify(diff(GSol(T1,T2,T3,T4,T5,tau),T3) -TermdT3function(T1,
T2,T3,T4,T5));
simplify(diff(GSol(T1,T2,T3,T4,T5,tau),T4) -TermdT4function(T1,
T2,T3,T4,T5));
simplify(diff(GSol(T1,T2,T3,T4,T5,tau),T5) -TermdT5function(T1,
T2,T3,T4,T5));

```

```

GSolbis:=1/2*(-(t20+t30)*(t20*ln(2*T3+T5-T1)+t30*ln(T3+2*T5-T1))+
t20*t30*ln(T3-T5)
+(T2-3*T4)^2*((T1-2*T3-T5)*t20+t30*(T1-T3-2*T5))/(2*(2*T1-3*T3-3*
T5)^2)
+sqrt(T1-T3-2*T5)*sqrt(T1-2*T3-T5)*sqrt(T3-T5)*(T2-3*T4)*(t20-
t30)*tau/(2*T1-3*T3-3*T5)^2
-(((4*T1-5*T3-7*T5)*t20-t30*(T3-T5))*(T1-2*T3-T5)*tau^2/2/(2*T1
-3*T3-3*T5)^2);
simplify(series(simplify(GSol(T1,T2,T3,T4,T5,tau)-GSolbis),
tau=0));

```

$GSol := (T1, T2, T3, T4, T5, \tau)$

(12)

$$\begin{aligned}
&\rightarrow \frac{1}{\sqrt{-T5+T3} (2 T1 - 3 T3 - 3 T5)^2} \left(\tau \sqrt{-T3 - 2 T5 + T1} (t20 - t30) (-T5 \right. \\
&+ T3) (T2 - 3 T4) \sqrt{-2 T3 - T5 + T1} + 2 \left(-t30 \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 (t20 \right. \\
&+ t30) \ln(T3 + 2 T5 - T1) - \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 (t20 + t30) t20 \ln(2 T3 + T5 \\
&- T1) + t30 \left(T1 - \frac{3}{2} T3 - \frac{3}{2} T5 \right)^2 t20 \ln(-T5 + T3) - (-2 T3 - T5 + T1) \left(-\frac{5}{4} \tau^2 T3 \right. \\
&- \frac{7}{4} \tau^2 T5 + \tau^2 T1 - \frac{1}{4} (T2 - 3 T4)^2 \left. \right) t20 + \left(-\frac{1}{2} T3^2 \tau^2 + \left(\frac{1}{4} \tau^2 T5 + \frac{1}{4} \tau^2 T1 \right. \right. \\
&- \frac{1}{4} (T2 - 3 T4)^2 \left. \right) T3 + \frac{1}{4} \tau^2 T5^2 + \left(-\frac{1}{4} \tau^2 T1 - \frac{1}{2} (T2 - 3 T4)^2 \right) T5 \\
&+ \left. \frac{1}{4} T1 (T2 - 3 T4)^2 \right) t30 \sqrt{-T5 + T3} \\
&0 \\
&0 \\
&0 \\
&0 \\
&0 \\
&0
\end{aligned}$$

$$\begin{aligned}
GSolbis := & -\frac{1}{2} (t20 + t30) (t20 \ln(2 T3 + T5 - T1) + t30 \ln(T3 + 2 T5 - T1)) \\
& + \frac{1}{2} t30 t20 \ln(-T5 + T3) \\
& + \frac{1}{2} \frac{(T2 - 3 T4)^2 ((-2 T3 - T5 + T1) t20 + (-T3 - 2 T5 + T1) t30)}{(2 T1 - 3 T3 - 3 T5)^2} \\
& + \frac{\sqrt{-T3 - 2 T5 + T1} \sqrt{-2 T3 - T5 + T1} \sqrt{-T5 + T3} (T2 - 3 T4) (t20 - t30) \tau}{(2 T1 - 3 T3 - 3 T5)^2} \\
& - \frac{1}{2} \frac{((4 T1 - 5 T3 - 7 T5) t20 - t30 (-T5 + T3)) (-2 T3 - T5 + T1) \tau^2}{(2 T1 - 3 T3 - 3 T5)^2} \\
& \quad 0
\end{aligned}$$

```

> GSolOldCoordinatesfunction:=unapply(simplify(GSol(TT1(t11, t21,
t31, t12, t22, t32),TT2(t11, t21, t31, t12, t22, t32),TT3(t11,
t21, t31, t12, t22, t32),TT4(t11, t21, t31, t12, t22, t32),TT5
(t11, t21, t31, t12, t22, t32),taufunction(t11, t21, t31, t12,
t22, t32))),t11, t21, t31, t12, t22, t32):
assume(tt12>tt22 and tt22>tt32 and tt12>tt32):
GSolOldCoordinatesfunction2:=unapply(simplify
(GSolOldCoordinatesfunction(tt11, tt21, tt31, tt12, tt22, tt32)
),tt11, tt21, tt31, tt12, tt22, tt32):
GSolOldCoordinates:=GSolOldCoordinatesfunction2(t11,t21,t31,t12,
t22,t32);

```

$$\begin{aligned}
GSolOldCoordinates := & \frac{1}{2} \frac{1}{(t12 - t32) (-t32 + t22)} (-t30 (t12 - t32) (-t32 + t22) (t20 \\
& + t30) \ln(-t12 + t32) - t20 (t12 - t32) (-t32 + t22) (t20 + t30) \ln(t22 - t12) \\
& + t20 t30 (t12 - t32) (-t32 + t22) \ln(-t32 + t22) + (- (t11 - t21) (t11 + t21 \\
& - 2 t31) t32 + t22 (t11 - t31)^2 - t12 (t21 - t31)^2) t20 + t30 (t11 - t31)^2 (-t32 + t22)
\end{aligned} \tag{13}$$

```

> GSolOldCoordinatesbis:= 1/2*(t30*t10*ln(-t12+t32) +t10*t20*ln
(t22-t12)+ln(-t32+t22)*t20*t30)
-t10*t11^2/2/(t12-t32)-t20*t21^2/2/(t22-t32)
+t30*t31^2/2/(t12-t32)
-(1/2)*(t12-t22)*t31^2*t20/((t22-t32)*(t12-t32))
+t31*t11*t10/(t12-t32)+t20*t31*t21/(t22-t32);
simplify(series(simplify(GSolOldCoordinates-
GSolOldCoordinatesbis),t11));

```

$$\begin{aligned}
GSolOldCoordinatesbis := & \frac{1}{2} t30 (-t20 - t30) \ln(-t12 + t32) + \frac{1}{2} (-t20 - t30) t20 \ln(t22 \\
& - t12) + \frac{1}{2} \ln(-t32 + t22) t20 t30 - \frac{1}{2} \frac{(-t20 - t30) t11^2}{t12 - t32} - \frac{1}{2} \frac{t20 t21^2}{-t32 + t22} \\
& + \frac{1}{2} \frac{t30 t31^2}{t12 - t32} - \frac{1}{2} \frac{(-t22 + t12) t31^2 t20}{(-t32 + t22) (t12 - t32)} + \frac{t31 t11 (-t20 - t30)}{t12 - t32} \\
& + \frac{t20 t31 t21}{-t32 + t22}
\end{aligned} \tag{14}$$

0

```
> simplify(omegaJMUdt11 - (1/((t12-t32)*(t22-t12))*(-p^3+ P1(q)*p^2
-P2(q)*p+P3(q))-t11*t21*t31/(t12-t32)/(t12-t22)+t10*t21/(t12-t22)
+t11*t20*(t22-t32)/((t12-t22)*(t12-t32)) +diff
(GSolOldCoordinatesbis,t11) ) );
```

```
simplify(omegaJMUdt21 - (1/((t22-t32)*(t12-t22))*(-p^3+ P1(q)*p^2
-P2(q)*p+P3(q))+t11*t21*t31/(t12-t22)/(t22-t32)-t20*t11/(t12-t22)
-t21*t10*(t12-t32)/((t12-t22)*(t22-t32)) +diff
(GSolOldCoordinatesbis,t21) ) );
```

```
simplify(omegaJMUdt31 - (-1/((t22-t32)*(t12-t32))*(-p^3+ P1(q)*
p^2-P2(q)*p+P3(q))-t11*t21*t31/(t12-t32)/(t22-t32)-t30*t21/(t22-
t32)+t20*(t22*t11+(t21-t11)*t32-t12*t21)/((t22-t32)*(t12-t32))
+diff(GSolOldCoordinatesbis,t31) ) );
```

0
0
0

(15)

```
> simplify(omegaJMUdt12- simplify(-residue(Hame12(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t12) ));
simplify(omegaJMUdt22- simplify(-residue(Hame22(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t22)));
simplify(omegaJMUdt32- simplify(-residue(Hame32(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t32)));
```

```
simplify(omegaJMUdt11- simplify(-residue(Hame11(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t11) ));
simplify(omegaJMUdt21- simplify(-residue(Hame21(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t21)));
simplify(omegaJMUdt31- simplify(-residue(Hame31(q,p)/h,h=0)+diff
(GSolOldCoordinatesbis,t31)));
```

0
0
0
0
0
0

(16)

```
> GSolOldCoordinatesbisfunction:=unapply(GSolOldCoordinatesbis,t11,
t21,t31,t12,t22,t32,t20,t30);
```

$GSolOldCoordinatesbisfunction := (t11, t21, t31, t12, t22, t32, t20, t30) \rightarrow \frac{1}{2} t30 (-t20$ (17)

$-t30) \ln(-t12 + t32) + \frac{1}{2} (-t20 - t30) t20 \ln(t22 - t12) + \frac{1}{2} \ln(t22 - t32) t20 t30$

$$\left[\begin{aligned}
 & -\frac{1}{2} \frac{(-t_{20} - t_{30}) t_1^2}{t_{12} - t_{32}} - \frac{1}{2} \frac{t_{20} t_2^2}{t_{22} - t_{32}} + \frac{1}{2} \frac{t_{30} t_3^2}{t_{12} - t_{32}} \\
 & -\frac{1}{2} \frac{(t_{12} - t_{22}) t_3^2 t_{20}}{(t_{22} - t_{32})(t_{12} - t_{32})} + \frac{t_{31} t_1 (-t_{20} - t_{30})}{t_{12} - t_{32}} + \frac{t_{20} t_{31} t_2}{t_{22} - t_{32}}
 \end{aligned} \right]$$