In this Maple sheet, we compute the Lax matrices using the asymptotics of the wave functions and the local diagonalization for the Painlevé 5 equation.

We first use the expression of the coefficients of the spectral curve in terms of the irregular times and monodromies.

```
> restart:
P011  := t010+t020;
P111  := t110+t120;
P121  := t111+t121;
P142  := t111*t121;
P132  := t110*t121+t111*t120;
P022  := t010*t020;
CoherenceEquation1:=tinfty10+tinfty20+t010+t020+t110+t120;
CoherenceEquation2:=P012+P112;
CoherenceEquation3:=t010*t020-tinfty10*tinfty20+P112+P122;
P1:=x-> P011/x+P121/(x-1)^2+P111/(x-1);
P2:=x-> P022/x^2+P012/x+P142/(x-1)^4+P132/(x-1)^3+P122/(x-1)^2+
P112/(x-1);

P012:=solve(CoherenceEquation2,P012);
P122:=solve(CoherenceEquation3,P122);
```

$$P011 := t010 + t020 \qquad \textbf{(1)}$$
$$P111 := t110 + t120$$
$$P121 := t111 + t121$$
$$P142 := t111\, t121$$
$$P132 := t110\, t121 + t111\, t120$$
$$P022 := t010\, t020$$
$$CoherenceEquation1 := tinfty10 + tinfty20 + t010 + t020 + t110 + t120$$
$$CoherenceEquation2 := P012 + P112$$
$$CoherenceEquation3 := t010\, t020 - tinfty10\, tinfty20 + P112 + P122$$
$$P1 := x \rightarrow \frac{P011}{x} + \frac{P121}{(x-1)^2} + \frac{P111}{x-1}$$
$$P2 := x \rightarrow \frac{P022}{x^2} + \frac{P012}{x} + \frac{P142}{(x-1)^4} + \frac{P132}{(x-1)^3} + \frac{P122}{(x-1)^2} + \frac{P112}{x-1}$$
$$P012 := -P112$$
$$P122 := -t010\, t020 + tinfty10\, tinfty20 - P112$$

Expression of the Lax matrix L

Study of the asymptotics at infinity

```
> logPsi1Infty:=-tinfty10/h*ln(lambda)+A10-A12/(2-1)/lambda^(2-1)
-A13/(3-1)/lambda^(3-1)-A14/(4-1)/lambda^(4-1)-A15/(5-1)
/lambda^(5-1)-A16/(6-1)/lambda^(6-1)-A17/(7-1)/lambda^(7-1)   ;
logPsi2Infty:=-tinfty20/h*ln(lambda)-1*ln(lambda)+A20-A22/(2-1)
```

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/lambda^(2-1)-A23/(3-1)/lambda^(3-1)-A24/(4-1)/lambda^(4-1)-
A25/(5-1)/lambda^(5-1)-A26/(6-1)/lambda^(6-1)-A27/(7-1)/lambda^
(7-1)   ;
Llogpsi1Infty:=-Ltinfty10/h*ln(lambda)+LA10-LA12/(2-1)/lambda^
(2-1)-LA13/(3-1)/lambda^(3-1)-LA14/(4-1)/lambda^(4-1)-LA15/(5
-1)/lambda^(5-1)-LA16/(6-1)/lambda^(6-1)-LA17/(7-1)/lambda^(7
-1)   ;
Llogpsi2Infty:=-Ltinfty20/h*ln(lambda)+LA20-LA22/(2-1)/lambda^
(2-1)-LA23/(3-1)/lambda^(3-1)-LA24/(4-1)/lambda^(4-1)-LA25/(5
-1)/lambda^(5-1)-LA26/(6-1)/lambda^(6-1)-LA27/(7-1)/lambda^(7
-1)   ;
Lpsi1Infty := exp(1/h*(-tinfty10*ln(lambda)+h*A10-h*
A12/lambda-1/2*h*A13/lambda^2-1/3*h*A14/lambda^3-1/4*h*
A15/lambda^4-1/5*h*A16/lambda^5-1/6*h*A17/lambda^6))*1/h*(-
Ltinfty10*ln(lambda)+h*LA10-h*LA12/lambda-1/2*h*LA13/lambda^2
-1/3*h*LA14/lambda^3-1/4*h*LA15/lambda^4-1/5*h*LA16/lambda^5
-1/6*h*LA17/lambda^6);
Lpsi2Infty := exp(1/h*(-tinfty20*ln(lambda)-h*ln(lambda)+h*A20-
h*A22/lambda-1/2*h*A23/lambda^2-1/3*h*A24/lambda^3-1/4*h*
A25/lambda^4-1/5*h*A26/lambda^5-1/6*h*A27/lambda^6))*1/h*(-
Ltinfty20*ln(lambda)+h*LA20-h*LA22/lambda-1/2*h*LA23/lambda^2
-1/3*h*LA24/lambda^3-1/4*h*LA25/lambda^4-1/5*h*LA26/lambda^5
-1/6*h*LA27/lambda^6);
psi1Infty:=exp(logPsi1Infty);
psi2Infty:=exp(logPsi2Infty);
dpsi1dlambdaInfty:=diff(psi1Infty,lambda):
dpsi2dlambdaInfty:=diff(psi2Infty,lambda):
d2psi1dlambda2Infty:=diff(psi1Infty,lambda$2):
d2psi2dlambda2Infty:=diff(psi2Infty,lambda$2):
Vinfty1:=tinfty10*ln(lambda);
Vinfty2:=tinfty20*ln(lambda);


WronskianLambdaInfty:=h*factor(psi1Infty*dpsi2dlambdaInfty-
psi2Infty*dpsi1dlambdaInfty):
WronskianLambdabisInfty:=h*simplify(factor( (diff(logPsi2Infty,
lambda)-diff(logPsi1Infty,lambda))*exp(logPsi1Infty+
logPsi2Infty))):

WronskianTildeLambdaInfty:=h^3*factor(dpsi2dlambdaInfty*
d2psi1dlambda2Infty-dpsi1dlambdaInfty*d2psi2dlambda2Infty):
```

$$logPsi1Infty := -\frac{tinfty10\ln(\lambda)}{h} + A10 - \frac{A12}{\lambda} - \frac{1}{2}\frac{A13}{\lambda^2} - \frac{1}{3}\frac{A14}{\lambda^3} - \frac{1}{4}\frac{A15}{\lambda^4} - \frac{1}{5}\frac{A16}{\lambda^5} - \frac{1}{6}\frac{A17}{\lambda^6} \quad \textbf{(1.1)}$$

$$logPsi2Infty := -\frac{tinfty20\ln(\lambda)}{h} - \ln(\lambda) + A20 - \frac{A22}{\lambda} - \frac{1}{2}\frac{A23}{\lambda^2} - \frac{1}{3}\frac{A24}{\lambda^3} - \frac{1}{4}\frac{A25}{\lambda^4} - \frac{1}{5}\frac{A26}{\lambda^5} - \frac{1}{6}\frac{A27}{\lambda^6}$$

$$Llogpsi1Infty := -\frac{Ltinfty10\ln(\lambda)}{h} + LA10 - \frac{LA12}{\lambda} - \frac{1}{2}\frac{LA13}{\lambda^2} - \frac{1}{3}\frac{LA14}{\lambda^3} - \frac{1}{4}\frac{LA15}{\lambda^4} - \frac{1}{5}\frac{LA16}{\lambda^5} - \frac{1}{6}\frac{LA17}{\lambda^6}$$

$$Llogpsi2Infty := -\frac{Ltinfty20\ln(\lambda)}{h} + LA20 - \frac{LA22}{\lambda} - \frac{1}{2}\frac{LA23}{\lambda^2} - \frac{1}{3}\frac{LA24}{\lambda^3} - \frac{1}{4}\frac{LA25}{\lambda^4} - \frac{1}{5}\frac{LA26}{\lambda^5} - \frac{1}{6}\frac{LA27}{\lambda^6}$$

$$Lpsi1Infty := 1 \Bigg/ \left(h \, \mathrm{e}^{\frac{-tinfty10\ln(\lambda) + hA10 - \frac{hA12}{\lambda} - \frac{1}{2}\frac{hA13}{\lambda^2} - \frac{1}{3}\frac{hA14}{\lambda^3} - \frac{1}{4}\frac{hA15}{\lambda^4} - \frac{1}{5}\frac{hA16}{\lambda^5} - \frac{1}{6}\frac{hA17}{\lambda^6}}{h}} \left(-Ltinfty10\ln(\lambda) + h\,LA10 - \frac{h\,LA12}{\lambda} - \frac{1}{2}\frac{h\,LA13}{\lambda^2} - \frac{1}{3}\frac{h\,LA14}{\lambda^3} - \frac{1}{4}\frac{h\,LA15}{\lambda^4} - \frac{1}{5}\frac{h\,LA16}{\lambda^5} - \frac{1}{6}\frac{h\,LA17}{\lambda^6} \right) \right)$$

$$Lpsi2Infty := 1 \Bigg/ \left(h \, \mathrm{e}^{\frac{-tinfty20\ln(\lambda) - h\ln(\lambda) + hA20 - \frac{hA22}{\lambda} - \frac{1}{2}\frac{hA23}{\lambda^2} - \frac{1}{3}\frac{hA24}{\lambda^3} - \frac{1}{4}\frac{hA25}{\lambda^4} - \frac{1}{5}\frac{hA26}{\lambda^5} - \frac{1}{6}\frac{hA27}{\lambda^6}}{h}} \left(-Ltinfty20\ln(\lambda) + h\,LA20 - \frac{h\,LA22}{\lambda} - \frac{1}{2}\frac{h\,LA23}{\lambda^2} - \frac{1}{3}\frac{h\,LA24}{\lambda^3} - \frac{1}{4}\frac{h\,LA25}{\lambda^4} - \frac{1}{5}\frac{h\,LA26}{\lambda^5} - \frac{1}{6}\frac{h\,LA27}{\lambda^6} \right) \right)$$

$$psi1Infty := \mathrm{e}^{-\frac{tinfty10\ln(\lambda)}{h} + A10 - \frac{A12}{\lambda} - \frac{1}{2}\frac{A13}{\lambda^2} - \frac{1}{3}\frac{A14}{\lambda^3} - \frac{1}{4}\frac{A15}{\lambda^4} - \frac{1}{5}\frac{A16}{\lambda^5} - \frac{1}{6}\frac{A17}{\lambda^6}}$$

$$psi2Infty := e^{-\frac{tinfty20\ln(\lambda)}{h} - \ln(\lambda) + A20 - \frac{A22}{\lambda} - \frac{1}{2}\frac{A23}{\lambda^2} - \frac{1}{3}\frac{A24}{\lambda^3} - \frac{1}{4}\frac{A25}{\lambda^4} - \frac{1}{5}\frac{A26}{\lambda^5} - \frac{1}{6}\frac{A27}{\lambda^6}}$$

$$Vinfty1 := tinfty10\ln(\lambda)$$

$$Vinfty2 := tinfty20\ln(\lambda)$$

```
> L21Infty:=factor(simplify
  (WronskianTildeLambdaInfty/WronskianLambdabisInfty)):
  L21InftyOrderlambda1:=factor(-residue(L21Infty/lambda^2,lambda=
  infinity));
  L21InftyOrderlambda0:=factor(-residue(L21Infty/lambda^1,lambda=
  infinity));
  L21InftyOrderlambdaMinus1:=factor(-residue(L21Infty/lambda^0,
  lambda=infinity));
  L21InftyOrderlambdaMinus2:=factor(-residue(L21Infty/lambda^
  (-1),lambda=infinity));
  L21InftyOrderlambdaMinus3:=factor(-residue(L21Infty/lambda^
  (-2),lambda=infinity));
```

$$L21InftyOrderlambda1 := 0 \tag{1.2}$$

$$L21InftyOrderlambda0 := 0$$

$$L21InftyOrderlambdaMinus1 := 0$$

$$L21InftyOrderlambdaMinus2 := -(h + tinfty20)\,tinfty10$$

$$L21InftyOrderlambdaMinus3 := -\frac{1}{h - tinfty10 + tinfty20}\big(h\,(A12\,h\,tinfty10 - A12\,h\,tinfty20 + A12\,tinfty10\,tinfty20 - A12\,tinfty20^2 - 2\,A22\,h\,tinfty10 + A22\,tinfty10^2 - A22\,tinfty10\,tinfty20)\big)$$

We get that L_{2,1} behaves at infinity like $-(h + tinfty20)\,tinfty10$/lambda^2+O(1/lambda^3)

```
> L22Infty:=factor(h*simplify(diff(WronskianLambdabisInfty,
  lambda)/WronskianLambdabisInfty)):
  L22InftyOrderlambda1:=factor(-residue(L22Infty/lambda^2,lambda=
  infinity));
  L22InftyOrderlambda0:=factor(-residue(L22Infty/lambda^1,lambda=
  infinity));
  L22InftyOrderlambdaMinus1:=factor(-residue(L22Infty/lambda^0,
  lambda=infinity));
  L22InftyOrderlambdaMinus2:=factor(-residue(L22Infty/lambda^
  (-1),lambda=infinity));
  L22InftyOrderlambdaMinus3:=factor(-residue(L22Infty/lambda^
  (-2),lambda=infinity)):
```

$$L22InftyOrderlambda1 := 0 \tag{1.3}$$

$$L22InftyOrderlambda0 := 0$$

$$L22InftyOrderlambdaMinus1 := -2\,h - tinfty10 - tinfty20$$

$$L22InftyOrderlambdaMinus2 := -\frac{h\,(A12\,tinfty10 - A12\,tinfty20 - 2\,A22\,h + A22\,tinfty10 - A22\,tinfty20)}{h - tinfty10 + tinfty20}$$

We get that L_{2,2} behaves at infinity like$(-2\,h - tinfty10 - tinfty20)$/lambda+h*O(1/lambda^2)

Study of the asymptotics at lambda= 0

```
> logPsi1Zero:=t010/h*ln(lambda)+B10+B12/(2-1)*lambda^(2-1)+B13/
  (3-1)*lambda^(3-1)+B14/(4-1)*lambda^(4-1)+B15/(5-1)*lambda^(5
  -1)+B16*(6-1)*lambda^(6-1)+B17*(7-1)*lambda^(7-1)   ;
  logPsi2Zero:=t020/h*ln(lambda)+B20+B22/(2-1)*lambda^(2-1)+B23/
  (3-1)*lambda^(3-1)+B24/(4-1)*lambda^(4-1)+B25/(5-1)*lambda^(5
  -1)+B26*(6-1)*lambda^(6-1)+B27*(7-1)*lambda^(7-1)   ;
  Llogpsi1Zero:=Lt010/h*ln(lambda)+LB10+LB12/(2-1)*lambda^(2-1)+
  LB13/(3-1)*lambda^(3-1)+LB14/(4-1)*lambda^(4-1)+LB15/(5-1)*
  lambda^(5-1)+LB16*(6-1)*lambda^(6-1)+LB17*(7-1)*lambda^(7-1)   ;
  Llogpsi2Zero:=Lt020/h*ln(lambda)+LB20+LB22/(2-1)*lambda^(2-1)+
  LB23/(3-1)*lambda^(3-1)+LB24/(4-1)*lambda^(4-1)+LB25/(5-1)*
  lambda^(5-1)+LB26*(6-1)*lambda^(6-1)+LB27*(7-1)*lambda^(7-1)   ;
  Lpsi1Zero  := exp((t010/h*ln(lambda)+B10+B12/(2-1)*lambda^(2-1)+
  B13/(3-1)*lambda^(3-1)+B14/(4-1)*lambda^(4-1)+B15/(5-1)*lambda^
  (5-1)+B16*(6-1)*lambda^(6-1)+B17*(7-1)*lambda^(7-1)))
  *(Lt010/h*ln(lambda)+LB10+LB12/(2-1)*lambda^(2-1)+LB13/(3-1)*
  lambda^(3-1)+LB14/(4-1)*lambda^(4-1)+LB15/(5-1)*lambda^(5-1)+
  LB16*(6-1)*lambda^(6-1)+LB17*(7-1)*lambda^(7-1));
  Lpsi2Zero  := exp((t020/h*ln(lambda)+B20+B22/(2-1)*lambda^(2-1)+
  B23/(3-1)*lambda^(3-1)+B24/(4-1)*lambda^(4-1)+B25/(5-1)*lambda^
  (5-1)+B26*(6-1)*lambda^(6-1)+B27*(7-1)*lambda^(7-1)))
  *(Lt020/h*ln(lambda)+LB20+LB22/(2-1)*lambda^(2-1)+LB23/(3-1)*
  lambda^(3-1)+LB24/(4-1)*lambda^(4-1)+LB25/(5-1)*lambda^(5-1)+
  LB26*(6-1)*lambda^(6-1)+LB27*(7-1)*lambda^(7-1));
  psi1Zero:=exp(logPsi1Zero);
  psi2Zero:=exp(logPsi2Zero);
  dpsi1dlambdaZero:=diff(psi1Zero,lambda):
  dpsi2dlambdaZero:=diff(psi2Zero,lambda):
  d2psi1dlambda2Zero:=diff(psi1Zero,lambda$2):
  d2psi2dlambda2Zero:=diff(psi2Zero,lambda$2):
  VZero1:=t010*ln(lambda);
  VZero2:=t020*ln(lambda);


  WronskianLambdaZero:=h*factor(psi1Zero*dpsi2dlambdaZero-
  psi2Zero*dpsi1dlambdaZero):
  WronskianLambdabisZero:=h*simplify(factor( (diff(logPsi2Zero,
  lambda)-diff(logPsi1Zero,lambda))*exp(logPsi1Zero+logPsi2Zero))
  ):

  WronskianTildeLambdaZero:=h^3*factor(dpsi2dlambdaZero*
```

```
d2psi1dlambda2Zero-dpsi1dlambdaZero*d2psi2dlambda2Zero):
```

$$logPsi1Zero := \frac{t010 \ln(\lambda)}{h} + B10 + B12\lambda + \frac{1}{2} B13\lambda^2 + \frac{1}{3} B14\lambda^3 + \frac{1}{4} B15\lambda^4 + 5 B16\lambda^5 + 6 B17\lambda^6 \quad (1.4)$$

$$logPsi2Zero := \frac{t020 \ln(\lambda)}{h} + B20 + B22\lambda + \frac{1}{2} B23\lambda^2 + \frac{1}{3} B24\lambda^3 + \frac{1}{4} B25\lambda^4 + 5 B26\lambda^5 + 6 B27\lambda^6$$

$$Llogpsi1Zero := \frac{Lt010 \ln(\lambda)}{h} + LB10 + LB12\lambda + \frac{1}{2} LB13\lambda^2 + \frac{1}{3} LB14\lambda^3 + \frac{1}{4} LB15\lambda^4 + 5 LB16\lambda^5 + 6 LB17\lambda^6$$

$$Llogpsi2Zero := \frac{Lt020 \ln(\lambda)}{h} + LB20 + LB22\lambda + \frac{1}{2} LB23\lambda^2 + \frac{1}{3} LB24\lambda^3 + \frac{1}{4} LB25\lambda^4 + 5 LB26\lambda^5 + 6 LB27\lambda^6$$

$$Lpsi1Zero := e^{\frac{t010 \ln(\lambda)}{h} + B10 + B12\lambda + \frac{1}{2} B13\lambda^2 + \frac{1}{3} B14\lambda^3 + \frac{1}{4} B15\lambda^4 + 5 B16\lambda^5 + 6 B17\lambda^6} \Big(1 / h\,(Lt010 \ln(\lambda)) + LB10 + LB12\lambda + \frac{1}{2} LB13\lambda^2 + \frac{1}{3} LB14\lambda^3 + \frac{1}{4} LB15\lambda^4 + 5 LB16\lambda^5 + 6 LB17\lambda^6 \Big)$$

$$Lpsi2Zero := e^{\frac{t020 \ln(\lambda)}{h} + B20 + B22\lambda + \frac{1}{2} B23\lambda^2 + \frac{1}{3} B24\lambda^3 + \frac{1}{4} B25\lambda^4 + 5 B26\lambda^5 + 6 B27\lambda^6} \Big(1 / h\,(Lt020 \ln(\lambda)) + LB20 + LB22\lambda + \frac{1}{2} LB23\lambda^2 + \frac{1}{3} LB24\lambda^3 + \frac{1}{4} LB25\lambda^4 + 5 LB26\lambda^5 + 6 LB27\lambda^6 \Big)$$

$$psi1Zero := e^{\frac{t010 \ln(\lambda)}{h} + B10 + B12\lambda + \frac{1}{2} B13\lambda^2 + \frac{1}{3} B14\lambda^3 + \frac{1}{4} B15\lambda^4 + 5 B16\lambda^5 + 6 B17\lambda^6}$$

$$psi2Zero := e^{\frac{t020 \ln(\lambda)}{h} + B20 + B22\lambda + \frac{1}{2} B23\lambda^2 + \frac{1}{3} B24\lambda^3 + \frac{1}{4} B25\lambda^4 + 5 B26\lambda^5 + 6 B27\lambda^6}$$

$$VZero1 := t010 \ln(\lambda)$$

$$VZero2 := t020 \ln(\lambda)$$

```
> L22Zero:=factor(h*simplify(diff(WronskianLambdabisZero,lambda)
  /WronskianLambdabisZero)):
  L22ZeroOrderlambdaMinus3:=factor(residue(L22Zero*lambda^2,
  lambda=0));
  L22ZeroOrderlambdaMinus2:=factor(residue(L22Zero*lambda^1,
  lambda=0));
  L22ZeroOrderlambdaMinus1:=factor(residue(L22Zero*lambda^0,
  lambda=0));
  L22ZeroOrderlambda0:=factor(residue(L22Zero*lambda^(-1),lambda=
```

```
0));
L22ZeroOrderlambda1:=factor(residue(L22Zero*lambda^(-2),lambda=
0)):
L22ZeroOrderlambda2:=factor(residue(L22Zero*lambda^(-3),lambda=
0)):
```

$$L22ZeroOrderlambdaMinus3 := 0 \quad (1.5)$$

$$L22ZeroOrderlambdaMinus2 := 0$$

$$L22ZeroOrderlambdaMinus1 := t010 - h + t020$$

$$L22ZeroOrderlambda0 := \frac{1}{t010 - t020} \left(h \left(B12\, h + B12\, t010 - B12\, t020 - B22\, h + B22\, t010 - B22\, t020\right)\right)$$

We get that L_{2,2} behaves at lambda=0 like (t010+t020-h)/lambda +O(1)

```
> L21Zero:=factor(simplify
(WronskianTildeLambdaZero/WronskianLambdabisZero)):
L21ZeroOrderlambdaMinus5:=factor(residue(L21Zero*lambda^4,
lambda=0));
L21ZeroOrderlambdaMinus4:=factor(residue(L21Zero*lambda^3,
lambda=0));
L21ZeroOrderlambdaMinus3:=factor(residue(L21Zero*lambda^2,
lambda=0));
L21ZeroOrderlambdaMinus2:=factor(residue(L21Zero*lambda^1,
lambda=0));
L21ZeroOrderlambdaMinus1:=factor(residue(L21Zero*lambda^0,
lambda=0));
L21ZeroOrderlambda0:=factor(residue(L21Zero*lambda^(-1),lambda=
0));
L21ZeroOrderlambda1:=factor(residue(L21Zero*lambda^(-2),lambda=
0)):
L21ZeroOrderlambda2:=factor(residue(L21Zero*lambda^(-3),lambda=
0)):
```

$$L21ZeroOrderlambdaMinus5 := 0 \quad (1.6)$$

$$L21ZeroOrderlambdaMinus4 := 0$$

$$L21ZeroOrderlambdaMinus3 := 0$$

$$L21ZeroOrderlambdaMinus2 := -t010\, t020$$

$$L21ZeroOrderlambdaMinus1 := -\frac{1}{t010 - t020} \left(h \left(B12\, h\, t020 + B12\, t010\, t020 - B12\, t020^2 - B22\, h\, t010 + B22\, t010^2 - B22\, t010\, t020\right)\right)$$

$$L21ZeroOrderlambda0 := \frac{1}{(t010 - t020)^2} \left(h \left(B12^2 h^2 t020 - B12\, B22\, h^2 t010 - B12\, B22\, h^2 t020 - B12\, B22\, h\, t010^2 + 2\, B12\, B22\, h\, t010\, t020 - B12\, B22\, h\, t020^2 + B22^2 h^2 t010 - 2\, B13\, h\, t010\, t020 + 2\, B13\, h\, t020^2 - B13\, t010^2 t020 + 2\, B13\, t010\, t020^2 - B13\, t020^3 + 2\, B23\, h\, t010^2 - 2\, B23\, h\, t010\, t020 - B23\, t010^3 + 2\, B23\, t010^2 t020 - B23\, t010\, t020^2\right)\right)$$

We get that L_{2,1} behaves at lambda=0 like -t010*t020/lambda^2+ O(1/lambda)

Study of the asymptotics at lambda=1

```
> logPsi1Un:=-t111/h/(lambda-1)+t110/h*ln(lambda-1)+C10+C12/(2-1)
*(lambda-1)^(2-1)+C13/(3-1)*(lambda-1)^(3-1)+C14/(4-1)*
(lambda-1)^(4-1)+C15/(5-1)*(lambda-1)^(5-1)+C16*(6-1)*
(lambda-1)^(6-1)+C17*(7-1)*(lambda-1)^(7-1)   ;
logPsi2Un:=-t121/h/(lambda-1)+t120/h*ln(lambda-1)+C20+C22/(2-1)
*(lambda-1)^(2-1)+C23/(3-1)*(lambda-1)^(3-1)+C24/(4-1)*
(lambda-1)^(4-1)+C25/(5-1)*(lambda-1)^(5-1)+C26*(6-1)*
(lambda-1)^(6-1)+C27*(7-1)*(lambda-1)^(7-1)   ;
Llogpsi1Un:=-Lt111/h/(lambda-1)+Lt110/h*ln(lambda-1)+LC10+LC12/
(2-1)*(lambda-1)^(2-1)+LC13/(3-1)*(lambda-1)^(3-1)+LC14/(4-1)*
(lambda-1)^(4-1)+LC15/(5-1)*(lambda-1)^(5-1)+LC16*(6-1)*
(lambda-1)^(6-1)+LC17*(7-1)*(lambda-1)^(7-1)   ;
Llogpsi2Un:=-Lt121/h/(lambda-1)+Lt120/h*ln(lambda-1)+LC20+LC22/
(2-1)*(lambda-1)^(2-1)+LC23/(3-1)*(lambda-1)^(3-1)+LC24/(4-1)*
(lambda-1)^(4-1)+LC25/(5-1)*(lambda-1)^(5-1)+LC26*(6-1)*
(lambda-1)^(6-1)+LC27*(7-1)*(lambda-1)^(7-1)   ;
Lpsi1Un := exp((-t111/h/(lambda-1)+t110/h*ln(lambda-1)+C10+C12/
(2-1)*(lambda-1)^(2-1)+C13/(3-1)*(lambda-1)^(3-1)+C14/(4-1)*
(lambda-1)^(4-1)+C15/(5-1)*(lambda-1)^(5-1)+C16*(6-1)*
(lambda-1)^(6-1)+C17*(7-1)*(lambda-1)^(7-1)))
*(-Lt111/h/(lambda-1)+Lt110/h*ln(lambda-1)+LC10+LC12/(2-1)*
(lambda-1)^(2-1)+LC13/(3-1)*(lambda-1)^(3-1)+LC14/(4-1)*
(lambda-1)^(4-1)+LC15/(5-1)*(lambda-1)^(5-1)+LC16*(6-1)*
(lambda-1)^(6-1)+LC17*(7-1)*(lambda-1)^(7-1));
Lpsi2Un := exp((-t121/h/(lambda-1)+t120/h*ln(lambda-1)+C20+C22/
(2-1)*(lambda-1)^(2-1)+C23/(3-1)*(lambda-1)^(3-1)+C24/(4-1)*
(lambda-1)^(4-1)+C25/(5-1)*(lambda-1)^(5-1)+C26*(6-1)*
(lambda-1)^(6-1)+C27*(7-1)*(lambda-1)^(7-1)))
*(-Lt121/h/(lambda-1)+Lt120/h*ln(lambda-1)+LC20+LC22/(2-1)*
(lambda-1)^(2-1)+LC23/(3-1)*(lambda-1)^(3-1)+LC24/(4-1)*
(lambda-1)^(4-1)+LC25/(5-1)*(lambda-1)^(5-1)+LC26*(6-1)*
(lambda-1)^(6-1)+LC27*(7-1)*(lambda-1)^(7-1));
psi1Un:=exp(logPsi1Un);
psi2Un:=exp(logPsi2Un);
dpsi1dlambdaUn:=diff(psi1Un,lambda):
dpsi2dlambdaUn:=diff(psi2Un,lambda):
d2psi1dlambda2Un:=diff(psi1Un,lambda$2):
d2psi2dlambda2Un:=diff(psi2Un,lambda$2):


WronskianLambdaUn:=h*factor(psi1Un*dpsi2dlambdaUn-psi2Un*
```

```
dpsi1dlambdaUn):
WronskianLambdabisUn:=h*simplify(factor( (diff(logPsi2Un,
lambda)-diff(logPsi1Un,lambda))*exp(logPsi1Un+logPsi2Un))):

WronskianTildeLambdaUn:=h^3*factor(dpsi2dlambdaUn*
d2psi1dlambda2Un-dpsi1dlambdaUn*d2psi2dlambda2Un):
```

$$logPsi1Un := -\frac{t111}{h\,(\lambda-1)} + \frac{t110\ln(\lambda-1)}{h} + C10 + C12\,(\lambda-1) + \frac{1}{2}\,C13\,(\lambda-1)^2 + \frac{1}{3}\,C14\,(\lambda-1)^3 + \frac{1}{4}\,C15\,(\lambda-1)^4 + 5\,C16\,(\lambda-1)^5 + 6\,C17\,(\lambda-1)^6 \quad (1.7)$$

$$logPsi2Un := -\frac{t121}{h\,(\lambda-1)} + \frac{t120\ln(\lambda-1)}{h} + C20 + C22\,(\lambda-1) + \frac{1}{2}\,C23\,(\lambda-1)^2 + \frac{1}{3}\,C24\,(\lambda-1)^3 + \frac{1}{4}\,C25\,(\lambda-1)^4 + 5\,C26\,(\lambda-1)^5 + 6\,C27\,(\lambda-1)^6$$

$$Llogpsi1Un := -\frac{Lt111}{h\,(\lambda-1)} + \frac{Lt110\ln(\lambda-1)}{h} + LC10 + LC12\,(\lambda-1) + \frac{1}{2}\,LC13\,(\lambda-1)^2 + \frac{1}{3}\,LC14\,(\lambda-1)^3 + \frac{1}{4}\,LC15\,(\lambda-1)^4 + 5\,LC16\,(\lambda-1)^5 + 6\,LC17\,(\lambda-1)^6$$

$$Llogpsi2Un := -\frac{Lt121}{h\,(\lambda-1)} + \frac{Lt120\ln(\lambda-1)}{h} + LC20 + LC22\,(\lambda-1) + \frac{1}{2}\,LC23\,(\lambda-1)^2 + \frac{1}{3}\,LC24\,(\lambda-1)^3 + \frac{1}{4}\,LC25\,(\lambda-1)^4 + 5\,LC26\,(\lambda-1)^5 + 6\,LC27\,(\lambda-1)^6$$

$$Lpsi1Un := e^{-\frac{t111}{h(\lambda-1)} + \frac{t110\ln(\lambda-1)}{h} + C10 + C12(\lambda-1) + \frac{1}{2}C13(\lambda-1)^2 + \frac{1}{3}C14(\lambda-1)^3 + \frac{1}{4}C15(\lambda-1)^4 + 5C16(\lambda-1)^5 + 6C17(\lambda-1)^6} \left(-\frac{Lt111}{h\,(\lambda-1)} + \frac{Lt110\ln(\lambda-1)}{h} + LC10 + LC12\,(\lambda-1) + \frac{1}{2}\,LC13\,(\lambda-1)^2 + \frac{1}{3}\,LC14\,(\lambda-1)^3 + \frac{1}{4}\,LC15\,(\lambda-1)^4 + 5\,LC16\,(\lambda-1)^5 + 6\,LC17\,(\lambda-1)^6\right)$$

$$Lpsi2Un := e^{-\frac{t121}{h(\lambda-1)} + \frac{t120\ln(\lambda-1)}{h} + C20 + C22(\lambda-1) + \frac{1}{2}C23(\lambda-1)^2 + \frac{1}{3}C24(\lambda-1)^3 + \frac{1}{4}C25(\lambda-1)^4 + 5C26(\lambda-1)^5 + 6C27(\lambda-1)^6} \left(-\frac{Lt121}{h\,(\lambda-1)} + \frac{Lt120\ln(\lambda-1)}{h} + LC20 + LC22\,(\lambda-1) + \frac{1}{2}\,LC23\,(\lambda-1)^2 + \frac{1}{3}\,LC24\,(\lambda-1)^3 + \frac{1}{4}\,LC25\,(\lambda-1)^4\right.$$

$$+5\,LC26\,(\lambda-1)^5+6\,LC27\,(\lambda-1)^6\Big)$$

$psi1Un :=$

$$e^{-\frac{t111}{h(\lambda-1)}+\frac{t110\ln(\lambda-1)}{h}+C10+C12(\lambda-1)+\frac{1}{2}C13(\lambda-1)^2+\frac{1}{3}C14(\lambda-1)^3+\frac{1}{4}C15(\lambda-1)^4+5\,C16(\lambda-1)^5+6\,C17(\lambda-1)^6}$$

$psi2Un :=$

$$e^{-\frac{t121}{h(\lambda-1)}+\frac{t120\ln(\lambda-1)}{h}+C20+C22(\lambda-1)+\frac{1}{2}C23(\lambda-1)^2+\frac{1}{3}C24(\lambda-1)^3+\frac{1}{4}C25(\lambda-1)^4+5\,C26(\lambda-1)^5+6\,C27(\lambda-1)^6}$$

```
> L22Un:=factor(h*simplify(diff(WronskianLambdabisUn,lambda)
  /WronskianLambdabisUn)):
  L22UnOrderlambdaMinus3:=factor(residue(L22Un*(lambda-1)^2,
  lambda=1));
  L22UnOrderlambdaMinus2:=factor(residue(L22Un*(lambda-1)^1,
  lambda=1));
  L22UnOrderlambdaMinus1:=factor(residue(L22Un*(lambda-1)^0,
  lambda=1));
  L22UnOrderlambda0:=factor(residue(L22Un*(lambda-1)^(-1),lambda=
  1));
  L22UnOrderlambda1:=factor(residue(L22Un*(lambda-1)^(-2),lambda=
  1)):
  L22UnOrderlambda2:=factor(residue(L22Un*(lambda-1)^(-3),lambda=
  1)):
```

$$L22UnOrderlambdaMinus3 := 0 \qquad \textbf{(1.8)}$$
$$L22UnOrderlambdaMinus2 := t111+t121$$
$$L22UnOrderlambdaMinus1 := -2\,h+t110+t120$$
$$L22UnOrderlambda0 := \frac{h\,(C12\,t111-C12\,t121+C22\,t111-C22\,t121+t110-t120)}{t111-t121}$$

We get that L_{2,2} behave at lambda=1 like $(t111+t121)$/(lambda-1)^2+ (t110+t120-2h)/(lambda-1) +O(1)

```
> L21Un:=factor(simplify
  (WronskianTildeLambdaUn/WronskianLambdabisUn)):
  L21UnOrderlambdaMinus5:=factor(residue(L21Un*(lambda-1)^4,
  lambda=1));
  L21UnOrderlambdaMinus4:=factor(residue(L21Un*(lambda-1)^3,
  lambda=1));
  L21UnOrderlambdaMinus3:=factor(residue(L21Un*(lambda-1)^2,
  lambda=1));
  L21UnOrderlambdaMinus2:=factor(residue(L21Un*(lambda-1)^1,
```

```
lambda=1));
```

$$L21UnOrderlambdaMinus5 := 0 \quad (1.9)$$
$$L21UnOrderlambdaMinus4 := -t111\,t121$$
$$L21UnOrderlambdaMinus3 := -t110\,t121 - t111\,t120$$
$$L21UnOrderlambdaMinus2 := -\frac{1}{t111 - t121}\left(C12\,h\,t111\,t121 - C12\,h\,t121^2 + C22\,h\,t111^2 - C22\,h\,t111\,t121 + h\,t110\,t121 - h\,t111\,t120 + t110\,t111\,t120 - t110\,t120\,t121\right)$$

We get that L_{2,1} behave at lambda=1 like $-t111\,t121$/(lambda-1)^4- ($t121\,t110 + t120\,t111$)/(lambda-1)^3+ O((lambda-1)^{-2})

```
> -residue( (t111+t121)/(lambda-1)^2+  (t110+t120-2*h)/
  (lambda-1)+ (t010+t020-h)/lambda  +h/(lambda-q),lambda=infinity)-CoherenceEquation1;
```

$$-2\,h - tinfty10 - tinfty20 \quad (1.10)$$

Formulas for L_{2,2} and L_{2,1}

We have L_{2,2} behaves at lambda=1 like (t111+t121)/(lambda-1)^2+ (t110+t120-2h)/(lambda-1) +O(1)

L_{2,2} behaves at lambda=0 like (t010+t020-h)/lambda +O(1)

L_{2,2} behaves at lambda=infinity like-(tinfty10+tinfty20+2h)/lambda+ O(1/lambda^2)

Thus,

L_{2,2}= (t111+t121)/(lambda-1)^2+ (t110+t120-2h)/(lambda-1)+ (t010+t020-h)/lambda +h/(lambda-q)

=P1(lambda)+h/(lambda-q)-h/lambda-2*h/(lambda-1)

with the additional condition that t110+t120+t010+t020+tinfty10+tinfty20=0 to get the correct asymptotic behavior at infinity.

We have L_{2,1} behaves at lambda=1 like -t111*t121/(lambda-1)^4-(t121*t110+t120*t111)/(lambda-1)^3+ O((lambda-1)^{-2})

L_{2,1} behaves at lambda=0 like -t010*t020/lambda^2+ O(1/lambda)

L_{2,1} behaves at lambda=infinity like -tinfty10*(tinfty20+h)/lambda^2+ O(1/lambda^3)

Thus,

L_{2,1}=-t111*t121/(lambda-1)^4-(t121*t110+t120*t111)/(lambda-1)^3 -a_2/(lambda-1) ^2 -a_1/(lambda-1) -t010*t020/lambda^2 -a_0/lambda - p*h/(lambda-q)

=-P2(lambda)+P012/lambda+P122/(lambda-1)^2+P112/(lambda-1)-a0/lambda -a2/(lambda-1)^2 -a1/(lambda-1) -h*p/(lambda-q)

with the additional relation **a_0+a_1+h*p=0 to ensure the correct behavior at infinity**

```
> P1(lambda)+h/(lambda-q)-h/lambda-2*h/(lambda-1);
```

$$\frac{t010 + t020}{\lambda} + \frac{t111 + t121}{(\lambda - 1)^2} + \frac{t110 + t120}{\lambda - 1} + \frac{h}{\lambda - q} - \frac{h}{\lambda} - \frac{2\,h}{\lambda - 1} \quad (1.11)$$

```
> -P2(lambda)+P012/lambda+P122/(lambda-1)^2+P112/(lambda-1)-
  a0/lambda -a2/(lambda-1)^2 -a1/(lambda-1) -h*p/(lambda-q);
```

$$-\frac{t010\,t020}{\lambda^2} - \frac{t111\,t121}{(\lambda - 1)^4} - \frac{t110\,t121 + t111\,t120}{(\lambda - 1)^3} - \frac{a0}{\lambda} - \frac{a2}{(\lambda - 1)^2} - \frac{a1}{\lambda - 1} - \frac{h\,p}{\lambda - q} \quad (1.12)$$

```
> EqCoeffLambdaMinus1Infinity:= residue(-P2(lambda)+P012/lambda+
P122/(lambda-1)^2+P112/(lambda-1)-a0/lambda -a2/(lambda-1)^2 -
a1/(lambda-1) -h*p/(lambda-q),lambda=infinity);
EqCoeffLambdaMinus1Infinity:=-residue((-P2(lambda)+P012/lambda+
P122/(lambda-1)^2+P112/(lambda-1)-a0/lambda -a2/(lambda-1)^2 -
a1/(lambda-1) -h*p/(lambda-q))*lambda,lambda=infinity) -(-
tinfty10*(tinfty20+h));
```

$$EqCoeffLambdaMinus1Infinity := h\,p + a0 + a1 \tag{1.13}$$

$$EqCoeffLambdaMinus1Infinity := -h\,p\,q - t010\,t020 - a1 - a2 + (h + tinfty20)\,tinfty10$$

```
> L22:=P1(lambda)+h/(lambda-q)-h/lambda-2*h/(lambda-1);
L21:=-P2(lambda)+P012/lambda+P122/(lambda-1)^2+P112/(lambda-1)-
a0/lambda -a2/(lambda-1)^2 -a1/(lambda-1) -h*p/(lambda-q);
```

$$L22 := \frac{t010 + t020}{\lambda} + \frac{t111 + t121}{(\lambda-1)^2} + \frac{t110 + t120}{\lambda - 1} + \frac{h}{\lambda - q} - \frac{h}{\lambda} - \frac{2\,h}{\lambda - 1} \tag{1.14}$$

$$L21 := -\frac{t010\,t020}{\lambda^2} - \frac{t111\,t121}{(\lambda-1)^4} - \frac{t110\,t121 + t111\,t120}{(\lambda-1)^3} - \frac{a0}{\lambda} - \frac{a2}{(\lambda-1)^2} - \frac{a1}{\lambda - 1} - \frac{h\,p}{\lambda - q}$$

Auxiliary Matrix A.

The deformation operator is
\mathcal{L}=\hbar*(alpha111*\partial_{t_{1^{(1)},1}}+ alpha121* \partial_{t_{1^{(2)},1}})

```
> WronskianLInfty:=factor(psi1Infty*Lpsi2Infty-psi2Infty*
Lpsi1Infty):
WronskianLZero:=factor(psi1Zero*Lpsi2Zero-psi2Zero*Lpsi1Zero):
WronskianLUn:=factor(psi1Un*Lpsi2Un-psi2Un*Lpsi1Un):
A12Infty:=factor(simplify(WronskianLInfty/WronskianLambdaInfty)
):
A12Zero:=factor(simplify(WronskianLZero/WronskianLambdaZero)):
A12Un:=factor(simplify(WronskianLUn/WronskianLambdaUn)):
Y1Infty:=h*factor(dpsi1dlambdaInfty/psi1Infty):
Y2Infty:=h*factor(dpsi2dlambdaInfty/psi2Infty):
Y1Zero:=h*factor(dpsi1dlambdaZero/psi1Zero):
Y2Zero:=h*factor(dpsi2dlambdaZero/psi2Zero):
Y1Un:=h*factor(dpsi1dlambdaUn/psi1Un):
Y2Un:=h*factor(dpsi2dlambdaUn/psi2Un):
Z1Infty:=factor(Lpsi1Infty/psi1Infty):
Z2Infty:=factor(Lpsi2Infty/psi2Infty):
Z1Zero:=factor(Lpsi1Zero/psi1Zero):
Z2Zero:=factor(Lpsi2Zero/psi2Zero):
Z1Un:=factor(Lpsi1Un/psi1Un):
Z2Un:=factor(Lpsi2Un/psi2Un):
```

```
A12bisInfty:=factor(simplify((Z2Infty-Z1Infty)/(Y2Infty-
Y1Infty))):
A12bisZero:=factor(simplify((Z2Zero-Z1Zero)/(Y2Zero-Y1Zero))):
A12bisUn:=factor(simplify((Z2Un-Z1Un)/(Y2Un-Y1Un))):
A11Infty:=factor(simplify( (Y2Infty*Z1Infty-Y1Infty*Z2Infty)/
(Y2Infty-Y1Infty) )):
A11Zero:=factor(simplify( (Y2Zero*Z1Zero-Y1Zero*Z2Zero)/
(Y2Zero-Y1Zero) )):
A11Un:=factor(simplify( (Y2Un*Z1Un-Y1Un*Z2Un)/(Y2Un-Y1Un) )):
factor(simplify(A12bisInfty-A12Infty));
factor(simplify(A12bisZero-A12Zero));
factor(simplify(A12bisZero-A12Zero));
```

$$0 \quad (2.1)$$
$$0$$
$$0$$

```
> Lt020:=0:
Lt120:=0:
Lt010:=0:
Lt110:=0:
Ltinfty20:=0:
Ltinfty10:=0:
Lt111:=h*alpha111:
Lt121:=h*alpha121:
```

```
> A12InftyLambda3:=factor(-residue(A12Infty/lambda^4,lambda=
infinity));
A12InftyLambda2:=factor(-residue(A12Infty/lambda^3,lambda=
infinity));
A12InftyLambda1:=factor(-residue(A12Infty/lambda^2,lambda=
infinity));
A12InftyLambda0:=factor(-residue(A12Infty/lambda^1,lambda=
infinity));
A12InftyLambdaMinus1:=factor(-residue(A12Infty/lambda^0,lambda=
infinity)):
```

$$A12InftyLambda3 := 0 \quad (2.2)$$
$$A12InftyLambda2 := 0$$
$$A12InftyLambda1 := \frac{LA10 - LA20}{h - tinfty10 + tinfty20}$$
$$A12InftyLambda0 := -\frac{1}{(h - tinfty10 + tinfty20)^2} (A12\ LA10\ h - A12\ LA20\ h - A22\ LA10\ h + A22\ LA20\ h + LA12\ h - LA12\ tinfty10 + LA12\ tinfty20 - LA22\ h + LA22\ tinfty10 - LA22\ tinfty20)$$

```
> A12ZeroLambdaMinus3:=factor(residue(A12Zero*lambda^2,lambda=0))
;
```

```
A12ZeroLambdaMinus2:=factor(residue(A12Zero*lambda^1,lambda=0))
;
A12ZeroLambdaMinus1:=factor(residue(A12Zero*lambda^0,lambda=0))
;
A12ZeroLambda0:=factor(residue(A12Zero*lambda^(-1),lambda=0));
A12ZeroLambda1:=factor(residue(A12Zero*lambda^(-2),lambda=0));
```

$$A12ZeroLambdaMinus3 := 0 \tag{2.3}$$
$$A12ZeroLambdaMinus2 := 0$$
$$A12ZeroLambdaMinus1 := 0$$
$$A12ZeroLambda0 := 0$$
$$A12ZeroLambda1 := \frac{LB10 - LB20}{t010 - t020}$$

```
> A12UnLambdaMinus3:=factor(residue(A12Un*(lambda-1)^2,lambda=1))
;
A12UnLambdaMinus2:=factor(residue(A12Un*(lambda-1)^1,lambda=1))
;
A12UnLambdaMinus1:=factor(residue(A12Un*(lambda-1)^0,lambda=1))
;
A12UnLambda0:=factor(residue(A12Un*(lambda-1)^(-1),lambda=1));
A12UnLambda1:=factor(residue(A12Un*(lambda-1)^(-2),lambda=1));
A12UnLambda2:=factor(residue(A12Un*(lambda-1)^(-3),lambda=1));
```

$$A12UnLambdaMinus3 := 0 \tag{2.4}$$
$$A12UnLambdaMinus2 := 0$$
$$A12UnLambdaMinus1 := 0$$
$$A12UnLambda0 := 0$$
$$A12UnLambda1 := -\frac{\alpha l11 - \alpha l21}{t111 - t121}$$
$$A12UnLambda2 := \frac{1}{(t111 - t121)^2}\left(LC10\, t111 - LC10\, t121 - LC20\, t111 + LC20\, t121 + t110\, \alpha l11 - t110\, \alpha l21 - t120\, \alpha l11 + t120\, \alpha l21\right)$$

```
> A11InftyLambda3:=factor(-residue(A11Infty/lambda^4,lambda=
infinity));
A11InftyLambda2:=factor(-residue(A11Infty/lambda^3,lambda=
infinity));
A11InftyLambda1:=factor(-residue(A11Infty/lambda^2,lambda=
infinity));
A11InftyLambda0:=factor(-residue(A11Infty/lambda^1,lambda=
infinity));
A11InftyLambdaMinus1:=factor(-residue(A11Infty/lambda^0,lambda=
infinity)):

A11ZeroLambdaMinus3:=factor(residue(A11Zero*lambda^2,lambda=0))
;
A11ZeroLambdaMinus2:=factor(residue(A11Zero*lambda^1,lambda=0))
```

```
;
A11ZeroLambdaMinus1:=factor(residue(A11Zero*lambda^0,lambda=0))
;
A11ZeroLambda0:=factor(residue(A11Zero*lambda^(-1),lambda=0));
A11ZeroLambda1:=factor(residue(A11Zero*lambda^(-2),lambda=0)):

A11UnLambdaMinus3:=factor(residue(A11Un*(lambda-1)^2,lambda=1))
;
A11UnLambdaMinus2:=factor(residue(A11Un*(lambda-1)^1,lambda=1))
;
A11UnLambdaMinus1:=factor(residue(A11Un*(lambda-1)^0,lambda=1))
;
A11UnLambda0:=factor(residue(A11Un*(lambda-1)^(-1),lambda=1));
A11UnLambda1:=factor(residue(A11Un*(lambda-1)^(-2),lambda=1)):
```

$$A11InftyLambda3 := 0 \tag{2.5}$$

$$A11InftyLambda2 := 0$$

$$A11InftyLambda1 := 0$$

$$A11InftyLambda0 := \frac{LA10\, h + LA10\, tinfty20 - LA20\, tinfty10}{h - tinfty10 + tinfty20}$$

$$A11ZeroLambdaMinus3 := 0$$

$$A11ZeroLambdaMinus2 := 0$$

$$A11ZeroLambdaMinus1 := 0$$

$$A11ZeroLambda0 := -\frac{LB10\, t020 - LB20\, t010}{t010 - t020}$$

$$A11UnLambdaMinus3 := 0$$

$$A11UnLambdaMinus2 := 0$$

$$A11UnLambdaMinus1 := -\frac{t111\, \alpha l21 - t121\, \alpha l11}{t111 - t121}$$

$$A11UnLambda0 := -\frac{1}{(t111 - t121)^2} \big(LC10\, t111\, t121 - LC10\, t121^2 - LC20\, t111^2 + LC20\, t111\, t121 + t110\, t121\, \alpha l11 - t110\, t121\, \alpha l21 - t111\, t120\, \alpha l11 + t111\, t120\, \alpha l21 \big)$$

```
> A12Form:=nu0+nuMinus1*lambda+ mu/(lambda-q);
EQ1:=residue(A12Form/(lambda-1),lambda=1);
EQ2:=residue(A12Form/(lambda-1)^2 ,lambda=1) -(-(alpha111-
alpha121)/(t111-t121));
EQ3:=residue(A12Form/lambda,lambda=0);
solve({EQ1,EQ2,EQ3},{nu0,nuMinus1,mu});
```

$$A12Form := \nu 0 + nuMinus1\, \lambda + \frac{\mu}{\lambda - q} \tag{2.6}$$

$$EQ1 := \frac{-\nu 0\, q - q\, nuMinus1 + \mu + \nu 0 + nuMinus1}{-q + 1}$$

$$EQ2 := \frac{-q\, nuMinus1 + \nu 0 + 2\, nuMinus1 + \frac{-\nu 0\, q - q\, nuMinus1 + \mu + \nu 0 + nuMinus1}{q-1}}{-q+1} + \frac{\alpha 11 - \alpha 21}{t111 - t121}$$

$$EQ3 := -\frac{-\nu 0\, q + \mu}{q}$$

$$\left\{\mu = \frac{(q-1)\,(q\,\alpha 11 - q\,\alpha 21 - \alpha 11 + \alpha 21)\, q}{t111 - t121},\ \nu 0 = \frac{(q-1)\,(q\,\alpha 11 - q\,\alpha 21 - \alpha 11 + \alpha 21)}{t111 - t121},\ nuMinus1 = \frac{q\,\alpha 11 - q\,\alpha 21 - \alpha 11 + \alpha 21}{t111 - t121}\right\}$$

```
> factor((alpha111*q-alpha121*q-alpha111+alpha121)/(t111-t121));
```

$$\frac{(\alpha 11 - \alpha 21)\,(q-1)}{t111 - t121} \tag{2.7}$$

```
> mu:=(alpha111-alpha121)*q*(q-1)^2/(t111-t121):
  nu0:=(alpha111-alpha121)*(q-1)^2/(t111-t121);
  nuMinus1:=(alpha111-alpha121)*(q-1)/(t111-t121);
  A12Form:=nu0+nuMinus1*lambda+ mu/(lambda-q);
  simplify(series(A12Form,lambda=1));
  simplify(series(A12Form,lambda=0));
  series(A12Form,lambda=infinity);
  factor(A12Form);
```

$$\nu 0 := \frac{(\alpha 11 - \alpha 21)\,(q-1)^2}{t111 - t121} \tag{2.8}$$

$$nuMinus1 := \frac{(\alpha 11 - \alpha 21)\,(q-1)}{t111 - t121}$$

$$A12Form := \frac{(\alpha 11 - \alpha 21)\,(q-1)^2}{t111 - t121} + \frac{(\alpha 11 - \alpha 21)\,(q-1)\,\lambda}{t111 - t121} + \frac{(\alpha 11 - \alpha 21)\, q\,(q-1)^2}{(t111 - t121)\,(\lambda - q)}$$

$$\frac{-\alpha 11 + \alpha 21}{t111 - t121}(\lambda - 1) - \frac{(\alpha 11 - \alpha 21)\, q}{(t111 - t121)\,(q-1)}(\lambda-1)^2 - \frac{(\alpha 11 - \alpha 21)\, q}{(q-1)^2\,(t111 - t121)}(\lambda-1)^3 - \frac{(\alpha 11 - \alpha 21)\, q}{(q-1)^3\,(t111 - t121)}(\lambda-1)^4 - \frac{(\alpha 11 - \alpha 21)\, q}{(q-1)^4\,(t111 - t121)}(\lambda-1)^5 + \mathrm{O}\left((\lambda-1)^6\right)$$

$$\frac{(\alpha 11 - \alpha 21)\,(q-1)}{(t111 - t121)\, q}\lambda - \frac{(\alpha 11 - \alpha 21)\,(q-1)^2}{(t111 - t121)\, q^2}\lambda^2 - \frac{(\alpha 11 - \alpha 21)\,(q-1)^2}{(t111 - t121)\, q^3}\lambda^3 - \frac{(\alpha 11 - \alpha 21)\,(q-1)^2}{(t111 - t121)\, q^4}\lambda^4$$

$$-\frac{(\alpha 11-\alpha 21)(q-1)^2}{(t111-t121)q^5}\lambda^5+\mathrm{O}(\lambda^6)$$

$$\frac{(\alpha 11-\alpha 21)(q-1)\lambda}{t111-t121}+\frac{(\alpha 11-\alpha 21)(q-1)^2}{t111-t121}+\frac{(\alpha 11-\alpha 21)q(q-1)^2}{(t111-t121)\lambda}+\frac{(\alpha 11-\alpha 21)q^2(q-1)^2}{(t111-t121)\lambda^2}+\frac{(\alpha 11-\alpha 21)q^3(q-1)^2}{(t111-t121)\lambda^3}+\frac{(\alpha 11-\alpha 21)q^4(q-1)^2}{(t111-t121)\lambda^4}+\frac{(\alpha 11-\alpha 21)q^5(q-1)^2}{(t111-t121)\lambda^5}+\mathrm{O}\left(\frac{1}{\lambda^6}\right)$$

$$\frac{(\alpha 11-\alpha 21)(q-1)\lambda(\lambda-1)}{(t111-t121)(\lambda-q)}$$

A_{1,2} is O(\lambda) at lambda=0 and at lambda=infinity. It is of the form -(alpha111-alpha121)/(t111-t121)(lambda-1) at lambda=1.
Thus we get that
A_{1,2}= (alpha111-alpha121)*(q-1)*lambda*(lambda-1)/((t111-t121)*(lambda-q))
=nu0+nuMinus1*lambda+ mu/(lambda-q)
with
mu:=(alpha111-alpha121)*q*(q-1)^2/(t111-t121):
nu0:=(alpha111-alpha121)*(q-1)^2/(t111-t121);
nuMinus1:=(alpha111-alpha121)*(q-1)/(t111-t121);

```
> A12:=(alpha111-alpha121)*(q-1)*lambda*(lambda-1)/((t111-t121)*
  (lambda-q));
  simplify(A12- ( nu0+nuMinus1*lambda+ mu/(lambda-q)));
```

$$A12:=\frac{(\alpha 11-\alpha 21)(q-1)\lambda(\lambda-1)}{(t111-t121)(\lambda-q)} \tag{2.9}$$

$$0$$

> A_{1,1} is O(1) at lambda=0 an at \lambda=infinity. It behaves like $\frac{\alpha 11\, t121-\alpha 21\, t111}{t111-t121}$ / (lambda-1) at \lambda=1
Thus we get that
A_{1,1}=(alpha111*t121-alpha121*t111)/(t111-t121)/(lambda-1) +c_0+ rho/(lambda-q)
A_{1,1}=c1/(lambda-1) +c0+rho/(lambda-q)
with c_1=(alpha111*t121-alpha121*t111)/(t111-t121)

```
> A11:=c1/(lambda-1) +c0+rho/(lambda-q);
  c1:=(alpha111*t121-alpha121*t111)/(t111-t121);
  series(A11,lambda=1,2);
  series(A11,lambda=0,2);
  series(A11,lambda=infinity,2);
```

$$A11:=\frac{c1}{\lambda-1}+c0+\frac{\rho}{\lambda-q} \tag{2.10}$$

$$c1:=\frac{-t111\,\alpha 21+t121\,\alpha 11}{t111-t121}$$

$$\frac{\frac{-t111\ \alpha l21 + t121\ \alpha l11}{t111 - t121}}{\lambda - 1} + c0 + \frac{\rho}{-q+1} + \frac{\rho}{(q-1)(-q+1)}(\lambda - 1) + O((\lambda - 1)^2)$$

$$-\frac{-t111\ \alpha l21 + t121\ \alpha l11}{t111 - t121} + c0 - \frac{\rho}{q} + \left(\frac{t111\ \alpha l21 - t121\ \alpha l11}{t111 - t121} - \frac{\rho}{q^2}\right)\lambda + O(\lambda^2)$$

$$c0 + \frac{\frac{-t111\ \alpha l21 + t121\ \alpha l11}{t111 - t121} + \rho}{\lambda} + O\left(\frac{1}{\lambda^2}\right)$$

Summary of the results (c_0 does not play any role)

```
> mu:=(alpha111-alpha121)*q*(q-1)^2/(t111-t121):
nu0:=(alpha111-alpha121)*(q-1)^2/(t111-t121);
nuMinus1:=(alpha111-alpha121)*(q-1)/(t111-t121);
A12:=(alpha111-alpha121)*(q-1)*lambda*(lambda-1)/((t111-t121)*
(lambda-q));
simplify(A12- ( nu0+nuMinus1*lambda+ mu/(lambda-q)));
A11:=c1/(lambda-1) +c0+rho/(lambda-q);
c1:=(alpha111*t121-alpha121*t111)/(t111-t121);
a0+a1+h*p=0;
h*p*q+t010*t020+a1+a2-(h+tinfty20)*tinfty10=0;
```

$$\nu 0 := \frac{(\alpha l11 - \alpha l21)(q-1)^2}{t111 - t121} \qquad (2.11)$$

$$nuMinus1 := \frac{(\alpha l11 - \alpha l21)(q-1)}{t111 - t121}$$

$$A12 := \frac{(\alpha l11 - \alpha l21)(q-1)\lambda(\lambda - 1)}{(t111 - t121)(\lambda - q)}$$

$$0$$

$$A11 := \frac{-t111\ \alpha l21 + t121\ \alpha l11}{(t111 - t121)(\lambda - 1)} + c0 + \frac{\rho}{\lambda - q}$$

$$c1 := \frac{-t111\ \alpha l21 + t121\ \alpha l11}{t111 - t121}$$