

In this Maple sheet, we compute the Lax matrices using the asymptotics of the wave functions and the local diagonalization for the second element of the Painlevé 2 hierarchy.

Loading previous results on the coefficient of the spectral curve

We first use the expression of the coefficients of the spectral curve in terms of the irregular times and monodromies.

```
> restart;
CoherenceEquation1 := tinfty10+tinfty20;
tinfty20 := -tinfty10;
Pinfty01 := -tinfty11-tinfty21;
Pinfty11 := -tinfty12-tinfty22;
Pinfty21 := -tinfty13-tinfty23;
Pinfty31 := -tinfty14-tinfty24;
Pinfty62 := tinfty14*tinfty24;
Pinfty52 := tinfty13*tinfty24+tinfty14*tinfty23;
Pinfty42 := tinfty12*tinfty24+tinfty13*tinfty23+tinfty14*
tinfty22;
Pinfty32 := tinfty11*tinfty24+tinfty12*tinfty23+tinfty13*
tinfty22+tinfty14*tinfty21;
Pinfty22 := tinfty20*tinfty14+tinfty10*tinfty24+tinfty11*
tinfty23+tinfty12*tinfty22+tinfty13*tinfty21;
P1:=x-> Pinfty01+Pinfty11*x+Pinfty21*x^2+Pinfty31*x^3;
P2:=x-> Pinfty02+Pinfty12*x+Pinfty22*x^2+Pinfty32*x^3+Pinfty42*
x^4+Pinfty52*x^5+Pinfty62*x^6;
```

$$\text{CoherenceEquation1} := \text{tinfty10} + \text{tinfty20} \quad (1.1)$$

$$\text{Pinfty01} := -\text{tinfty11} - \text{tinfty21}$$

$$\text{Pinfty11} := -\text{tinfty12} - \text{tinfty22}$$

$$\text{Pinfty21} := -\text{tinfty13} - \text{tinfty23}$$

$$\text{Pinfty31} := -\text{tinfty14} - \text{tinfty24}$$

$$\text{Pinfty62} := \text{tinfty14} \text{ tinfty24}$$

$$\text{Pinfty52} := \text{tinfty13} \text{ tinfty24} + \text{tinfty14} \text{ tinfty23}$$

$$\text{Pinfty42} := \text{tinfty12} \text{ tinfty24} + \text{tinfty13} \text{ tinfty23} + \text{tinfty14} \text{ tinfty22}$$

$$\text{Pinfty32} := \text{tinfty11} \text{ tinfty24} + \text{tinfty12} \text{ tinfty23} + \text{tinfty13} \text{ tinfty22} + \text{tinfty14} \text{ tinfty21}$$

$$\text{Pinfty22} := -\text{tinfty10} \text{ tinfty14} + \text{tinfty10} \text{ tinfty24} + \text{tinfty11} \text{ tinfty23} + \text{tinfty12} \text{ tinfty22} \\ + \text{tinfty13} \text{ tinfty21}$$

Expression of the Lax matrix L

Study of the asymptotics at infinity

```
> logPsi1Infty := -tinfty14/4/h*lambda^4-tinfty13/3/h*lambda^3-
tinfty12/2/h*lambda^2-tinfty11/h*lambda-tinfty10/h*ln(lambda) +
A10-A12/(2-1)/lambda^(2-1)-A13/(3-1)/lambda^(3-1)-A14/(4-1)
/lambda^(4-1)-A15/(5-1)/lambda^(5-1)-A16/(6-1)/lambda^(6-1)-
```

```

A17/(7-1)/lambda^(7-1) ;
logPsi2Infty:=-tinfy24/4/h*lambda^4-tinfy23/3/h*lambda^3-
tinfy22/2/h*lambda^2-tinfy21/h*lambda-tinfy20/h*ln(lambda)
-1*ln(lambda)+A20-A22/(2-1)/lambda^(2-1)-A23/(3-1)/lambda^(3-1)
-A24/(4-1)/lambda^(4-1)-A25/(5-1)/lambda^(5-1)-A26/(6-1)
/lambda^(6-1)-A27/(7-1)/lambda^(7-1) ;
Llogpsi1Infty:=-Ltinfy14/4/h*lambda^4-Ltinfy13/3/h*lambda^3-
Ltinfy12/2/h*lambda^2-Ltinfy11/h*lambda-Ltinfy10/h*ln
(lambda)+LA10-LA12/(2-1)/lambda^(2-1)-LA13/(3-1)/lambda^(3-1)-
LA14/(4-1)/lambda^(4-1)-LA15/(5-1)/lambda^(5-1)-LA16/(6-1)
/lambda^(6-1)-LA17/(7-1)/lambda^(7-1) ;
Llogpsi2Infty:=-Ltinfy24/4/h*lambda^4-Ltinfy23/3/h*lambda^3-
Ltinfy22/2/h*lambda^2-Ltinfy21/h*lambda-Ltinfy20/h*ln
(lambda)+LA20-LA22/(2-1)/lambda^(2-1)-LA23/(3-1)/lambda^(3-1)-
LA24/(4-1)/lambda^(4-1)-LA25/(5-1)/lambda^(5-1)-LA26/(6-1)
/lambda^(6-1)-LA27/(7-1)/lambda^(7-1) ;
Lpsi1Infty := exp(1/h*(-tinfy14/4*lambda^4-tinfy13/3*lambda^3
-tinfy12/2*lambda^2-tinfy11*lambda-tinfy10*ln(lambda)+h*A10-
h*A12/lambda-1/2*h*A13/lambda^2-1/3*h*A14/lambda^3-1/4*h*
A15/lambda^4-1/5*h*A16/lambda^5-1/6*h*A17/lambda^6))*1/h*(-
Ltinfy14/4*lambda^4-Ltinfy13/3*lambda^3-Ltinfy12/2*lambda^2-
Ltinfy11*lambda-Ltinfy10*ln(lambda)+h*LA10-h*LA12/lambda-1/2*
h*LA13/lambda^2-1/3*h*LA14/lambda^3-1/4*h*LA15/lambda^4-1/5*h*
LA16/lambda^5-1/6*h*LA17/lambda^6) ;
Lpsi2Infty := exp(1/h*(-tinfy24/4*lambda^4-tinfy23/3*lambda^3
-tinfy22/2*lambda^2-tinfy21*lambda-tinfy20*ln(lambda)-h*ln
(lambda)+h*A20-h*A22/lambda-1/2*h*A23/lambda^2-1/3*h*
A24/lambda^3-1/4*h*A25/lambda^4-1/5*h*A26/lambda^5-1/6*h*
A27/lambda^6))*1/h*(-Ltinfy24/4*lambda^4-Ltinfy23/3*lambda^3-
Ltinfy22/2*lambda^2-Ltinfy21*lambda-Ltinfy20*ln(lambda)+h*
LA20-h*LA22/lambda-1/2*h*LA23/lambda^2-1/3*h*LA24/lambda^3-1/4*
h*LA25/lambda^4-1/5*h*LA26/lambda^5-1/6*h*LA27/lambda^6) ;
psi1Infty:=exp(logPsi1Infty) ;
psi2Infty:=exp(logPsi2Infty) ;
dpsi1dlambdaInfty:=diff(psi1Infty,lambda) :
dpsi2dlambdaInfty:=diff(psi2Infty,lambda) :
d2psi1dlambda2Infty:=diff(psi1Infty,lambda$2) :
d2psi2dlambda2Infty:=diff(psi2Infty,lambda$2) :

WronskianLambdaInfty:=h*factor(psi1Infty*dpsi2dlambdaInfty-
psi2Infty*dpsi1dlambdaInfty) :

```

WronskianLambdabisInfty:=h*simplify(factor((diff(logPsi2Infty, lambda)-diff(logPsi1Infty, lambda))*exp(logPsi1Infty+logPsi2Infty))):

WronskianTildeLambdaInfty:=h^3*factor(dpsi2dlambdaInfty*d2psi1dlambda2Infty-dpsi1dlambdaInfty*d2psi2dlambda2Infty):

$$\begin{aligned}
 \log\Psi_1\text{Infty} &:= -\frac{1}{4} \frac{\text{tiny}14 \lambda^4}{h} - \frac{1}{3} \frac{\text{tiny}13 \lambda^3}{h} - \frac{1}{2} \frac{\text{tiny}12 \lambda^2}{h} - \frac{\text{tiny}11 \lambda}{h} \\
 &\quad - \frac{\text{tiny}10 \ln(\lambda)}{h} + A_{10} - \frac{A_{12}}{\lambda} - \frac{1}{2} \frac{A_{13}}{\lambda^2} - \frac{1}{3} \frac{A_{14}}{\lambda^3} - \frac{1}{4} \frac{A_{15}}{\lambda^4} - \frac{1}{5} \frac{A_{16}}{\lambda^5} \\
 &\quad - \frac{1}{6} \frac{A_{17}}{\lambda^6} \\
 \log\Psi_2\text{Infty} &:= -\frac{1}{4} \frac{\text{tiny}24 \lambda^4}{h} - \frac{1}{3} \frac{\text{tiny}23 \lambda^3}{h} - \frac{1}{2} \frac{\text{tiny}22 \lambda^2}{h} - \frac{\text{tiny}21 \lambda}{h} \\
 &\quad + \frac{\text{tiny}10 \ln(\lambda)}{h} - \ln(\lambda) + A_{20} - \frac{A_{22}}{\lambda} - \frac{1}{2} \frac{A_{23}}{\lambda^2} - \frac{1}{3} \frac{A_{24}}{\lambda^3} - \frac{1}{4} \frac{A_{25}}{\lambda^4} \\
 &\quad - \frac{1}{5} \frac{A_{26}}{\lambda^5} - \frac{1}{6} \frac{A_{27}}{\lambda^6} \\
 L\log\psi_1\text{Infty} &:= -\frac{1}{4} \frac{L\text{tiny}14 \lambda^4}{h} - \frac{1}{3} \frac{L\text{tiny}13 \lambda^3}{h} - \frac{1}{2} \frac{L\text{tiny}12 \lambda^2}{h} - \frac{L\text{tiny}11 \lambda}{h} \\
 &\quad - \frac{L\text{tiny}10 \ln(\lambda)}{h} + LA_{10} - \frac{LA_{12}}{\lambda} - \frac{1}{2} \frac{LA_{13}}{\lambda^2} - \frac{1}{3} \frac{LA_{14}}{\lambda^3} - \frac{1}{4} \frac{LA_{15}}{\lambda^4} \\
 &\quad - \frac{1}{5} \frac{LA_{16}}{\lambda^5} - \frac{1}{6} \frac{LA_{17}}{\lambda^6} \\
 L\log\psi_2\text{Infty} &:= -\frac{1}{4} \frac{L\text{tiny}24 \lambda^4}{h} - \frac{1}{3} \frac{L\text{tiny}23 \lambda^3}{h} - \frac{1}{2} \frac{L\text{tiny}22 \lambda^2}{h} - \frac{L\text{tiny}21 \lambda}{h} \\
 &\quad - \frac{L\text{tiny}20 \ln(\lambda)}{h} + LA_{20} - \frac{LA_{22}}{\lambda} - \frac{1}{2} \frac{LA_{23}}{\lambda^2} - \frac{1}{3} \frac{LA_{24}}{\lambda^3} - \frac{1}{4} \frac{LA_{25}}{\lambda^4} \\
 &\quad - \frac{1}{5} \frac{LA_{26}}{\lambda^5} - \frac{1}{6} \frac{LA_{27}}{\lambda^6} \\
 L\psi_1\text{Infty} &:= 1 / \\
 &\quad h \left(e^{\frac{1}{h} \left(-\frac{1}{4} \text{tiny}14 \lambda^4 - \frac{1}{3} \text{tiny}13 \lambda^3 - \frac{1}{2} \text{tiny}12 \lambda^2 - \text{tiny}11 \lambda - \text{tiny}10 \ln(\lambda) + h A_{10} \right.} \right. \\
 &\quad \left. \left. - \frac{h A_{12}}{\lambda} - \frac{1}{2} \frac{h A_{13}}{\lambda^2} - \frac{1}{3} \frac{h A_{14}}{\lambda^3} - \frac{1}{4} \frac{h A_{15}}{\lambda^4} - \frac{1}{5} \frac{h A_{16}}{\lambda^5} - \frac{1}{6} \frac{h A_{17}}{\lambda^6} \right) \left(-\frac{1}{4} L\text{tiny}14 \lambda^4 \right. \right. \\
 &\quad \left. \left. - \frac{1}{3} L\text{tiny}13 \lambda^3 - \frac{1}{2} L\text{tiny}12 \lambda^2 - L\text{tiny}11 \lambda - L\text{tiny}10 \ln(\lambda) + h LA_{10} \right) \right)
 \end{aligned} \tag{2.1}$$

$$\left. \begin{aligned} & -\frac{h LA12}{\lambda} - \frac{1}{2} \frac{h LA13}{\lambda^2} - \frac{1}{3} \frac{h LA14}{\lambda^3} - \frac{1}{4} \frac{h LA15}{\lambda^4} - \frac{1}{5} \frac{h LA16}{\lambda^5} \\ & - \frac{1}{6} \frac{h LA17}{\lambda^6} \end{aligned} \right)$$

$Lpsi2Infty := 1 /$

$$h \left(e^{\frac{1}{h} \left(-\frac{1}{4} tinfty24 \lambda^4 - \frac{1}{3} tinfty23 \lambda^3 - \frac{1}{2} tinfty22 \lambda^2 - tinfty21 \lambda + tinfty10 \ln(\lambda) - h \ln(\lambda) \right)} + h A20 - \frac{h A22}{\lambda} - \frac{1}{2} \frac{h A23}{\lambda^2} - \frac{1}{3} \frac{h A24}{\lambda^3} - \frac{1}{4} \frac{h A25}{\lambda^4} - \frac{1}{5} \frac{h A26}{\lambda^5} - \frac{1}{6} \frac{h A27}{\lambda^6} \right) \left(-\frac{1}{4} Ltinfty24 \lambda^4 \right.$$

$$\left. - \frac{1}{3} Ltinfty23 \lambda^3 - \frac{1}{2} Ltinfty22 \lambda^2 - Ltinfty21 \lambda - Ltinfty20 \ln(\lambda) + h LA20 \right.$$

$$\left. \left. - \frac{h LA22}{\lambda} - \frac{1}{2} \frac{h LA23}{\lambda^2} - \frac{1}{3} \frac{h LA24}{\lambda^3} - \frac{1}{4} \frac{h LA25}{\lambda^4} - \frac{1}{5} \frac{h LA26}{\lambda^5} - \frac{1}{6} \frac{h LA27}{\lambda^6} \right) \right)$$

$psi1Infty :=$

$$e^{-\frac{1}{4} \frac{tinfty14 \lambda^4}{h} - \frac{1}{3} \frac{tinfty13 \lambda^3}{h} - \frac{1}{2} \frac{tinfty12 \lambda^2}{h} - \frac{tinfty11 \lambda}{h} - \frac{tinfty10 \ln(\lambda)}{h} + A10 - \frac{A12}{\lambda} - \frac{1}{2} \frac{A13}{\lambda^2} - \frac{1}{3} \frac{A14}{\lambda^3} - \frac{1}{4} \frac{A15}{\lambda^4} - \frac{1}{5} \frac{A16}{\lambda^5} - \frac{1}{6} \frac{A17}{\lambda^6}}$$

$psi2Infty :=$

$$e^{-\frac{1}{4} \frac{tinfty24 \lambda^4}{h} - \frac{1}{3} \frac{tinfty23 \lambda^3}{h} - \frac{1}{2} \frac{tinfty22 \lambda^2}{h} - \frac{tinfty21 \lambda}{h} + \frac{tinfty10 \ln(\lambda)}{h} - \ln(\lambda) + A20 - \frac{A22}{\lambda} - \frac{1}{2} \frac{A23}{\lambda^2} - \frac{1}{3} \frac{A24}{\lambda^3} - \frac{1}{4} \frac{A25}{\lambda^4} - \frac{1}{5} \frac{A26}{\lambda^5} - \frac{1}{6} \frac{A27}{\lambda^6}}$$

> L21Infty:=factor(simplify

(WronskianTildeLambdaInfty/WronskianLambdabisInfty) :

L21InftyOrderlambda7:=factor(-residue(L21Infty/lambda^8,lambda=infinity));

L21InftyOrderlambda6:=factor(-residue(L21Infty/lambda^7,lambda=infinity));

L21InftyOrderlambda5:=factor(-residue(L21Infty/lambda^6,lambda=infinity));

L21InftyOrderlambda4:=factor(-residue(L21Infty/lambda^5,lambda=infinity));

L21InftyOrderlambda3:=factor(-residue(L21Infty/lambda^4,lambda=infinity));

L21InftyOrderlambda2:=factor(-residue(L21Infty/lambda^3,lambda=

$$\begin{aligned}
& \text{infinity})); \\
L21\text{InftyOrderlambda1} &:= \text{factor}(-\text{residue}(L21\text{Infty}/\text{lambda}^2, \text{lambda}=\text{infinity})); \\
L21\text{InftyOrderlambda0} &:= \text{factor}(-\text{residue}(L21\text{Infty}/\text{lambda}^1, \text{lambda}=\text{infinity})); \\
& \text{factor}(\text{simplify}(L21\text{InftyOrderlambda6}*\text{lambda}^6+ \\
& L21\text{InftyOrderlambda5}*\text{lambda}^5+L21\text{InftyOrderlambda4}*\text{lambda}^4+ \\
& L21\text{InftyOrderlambda3}*\text{lambda}^3+L21\text{InftyOrderlambda2}*\text{lambda}^2- (- \\
& P2(\text{lambda})+P\text{infty02}+P\text{infty12}*\text{lambda}-h*\text{lambda}^2*\text{tinfty14})); \\
& L21\text{InftyOrderlambda7} := 0 \\
& L21\text{InftyOrderlambda6} := -\text{tinfty14 tinfty24} \\
& L21\text{InftyOrderlambda5} := -\text{tinfty13 tinfty24} - \text{tinfty14 tinfty23} \\
L21\text{InftyOrderlambda4} &:= -\text{tinfty12 tinfty24} - \text{tinfty13 tinfty23} - \text{tinfty14 tinfty22} \\
L21\text{InftyOrderlambda3} &:= -\text{tinfty11 tinfty24} - \text{tinfty12 tinfty23} - \text{tinfty13 tinfty22} \\
& - \text{tinfty14 tinfty21} \\
L21\text{InftyOrderlambda2} &:= -h \text{tinfty14} + \text{tinfty10 tinfty14} - \text{tinfty10 tinfty24} \\
& - \text{tinfty11 tinfty23} - \text{tinfty12 tinfty22} - \text{tinfty13 tinfty21} \\
L21\text{InftyOrderlambda1} &:= \frac{1}{-\text{tinfty24} + \text{tinfty14}} (A12 h \text{tinfty14 tinfty24} - A12 h \text{tinfty24}^2 \\
& + A22 h \text{tinfty14}^2 - A22 h \text{tinfty14 tinfty24} - h \text{tinfty13 tinfty14} + h \text{tinfty14 tinfty23} \\
& + \text{tinfty10 tinfty13 tinfty14} - \text{tinfty10 tinfty13 tinfty24} - \text{tinfty10 tinfty14 tinfty23} \\
& + \text{tinfty10 tinfty23 tinfty24} - \text{tinfty11 tinfty14 tinfty22} + \text{tinfty11 tinfty22 tinfty24} \\
& - \text{tinfty12 tinfty14 tinfty21} + \text{tinfty12 tinfty21 tinfty24}) \\
L21\text{InftyOrderlambda0} &:= \frac{1}{(-\text{tinfty24} + \text{tinfty14})^2} (A12 h \text{tinfty14}^2 \text{tinfty23} \\
& - 2 A12 h \text{tinfty14 tinfty23 tinfty24} + A12 h \text{tinfty23 tinfty24}^2 + A13 h \text{tinfty14}^2 \text{tinfty24} \\
& - 2 A13 h \text{tinfty14 tinfty24}^2 + A13 h \text{tinfty24}^3 + A22 h \text{tinfty13 tinfty14}^2 \\
& - 2 A22 h \text{tinfty13 tinfty14 tinfty24} + A22 h \text{tinfty13 tinfty24}^2 + A23 h \text{tinfty14}^3 \\
& - 2 A23 h \text{tinfty14}^2 \text{tinfty24} + A23 h \text{tinfty14 tinfty24}^2 - h \text{tinfty12 tinfty14}^2 \\
& + h \text{tinfty12 tinfty24}^2 + h \text{tinfty13}^2 \text{tinfty24} - h \text{tinfty13 tinfty14 tinfty23} \\
& - h \text{tinfty13 tinfty23 tinfty24} + 2 h \text{tinfty14}^2 \text{tinfty22} - 2 h \text{tinfty14 tinfty22 tinfty24} \\
& + h \text{tinfty14 tinfty23}^2 + \text{tinfty10 tinfty12 tinfty14}^2 - 2 \text{tinfty10 tinfty12 tinfty14 tinfty24} \\
& + \text{tinfty10 tinfty12 tinfty24}^2 - \text{tinfty10 tinfty14}^2 \text{tinfty22} \\
& + 2 \text{tinfty10 tinfty14 tinfty22 tinfty24} - \text{tinfty10 tinfty22 tinfty24}^2 \\
& - \text{tinfty11 tinfty14}^2 \text{tinfty21} + 2 \text{tinfty11 tinfty14 tinfty21 tinfty24} \\
& - \text{tinfty11 tinfty21 tinfty24}^2) \\
& 0
\end{aligned} \tag{2.2}$$

We get that $L_{\{2,1\}}$ behaves at infinity like

$$\begin{aligned}
L21(\text{lambda}) &= -(\text{tinfty14}*\text{tinfty24})*\text{lambda}^6 \\
& -(\text{tinfty13}*\text{tinfty24}+\text{tinfty14}*\text{tinfty23})*\text{lambda}^5 \\
& -(\text{tinfty12}*\text{tinfty24}+\text{tinfty13}*\text{tinfty23}+\text{tinfty14}*\text{tinfty22})*\text{lambda}^4 \\
& -(\text{tinfty11}*\text{tinfty24}+\text{tinfty12}*\text{tinfty23}+\text{tinfty13}*\text{tinfty22}+\text{tinfty14}*\text{tinfty21})*\text{lambda}^3 \\
& -(\text{tinfty20}*\text{tinfty14}+\text{tinfty10}*\text{tinfty24}+\text{tinfty11}*\text{tinfty23}+\text{tinfty12}*\text{tinfty22}+\text{tinfty13}*\text{tinfty21}+ \\
& h*\text{tinfty14})*\text{lambda}^2+O(\text{lambda}) \\
& \text{i.e.}
\end{aligned}$$

$$L_{\{2,1\}} = -P_2(\lambda) + P_{\infty 02} + P_{\infty 12} \lambda + C_1 \lambda + C_0 - h \lambda^2 t_{\infty 14} + o(1)$$

Computation of $L_{\{2,2\}}$

```

> L22Infty:=factor(h*simplify(diff(WronskianLambdabisInfty,
lambda)/WronskianLambdabisInfty)):
L22InftyOrderlambda7:=factor(-residue(L22Infty/lambda^8,lambda=
infinity));
L22InftyOrderlambda6:=factor(-residue(L22Infty/lambda^7,lambda=
infinity));
L22InftyOrderlambda5:=factor(-residue(L22Infty/lambda^6,lambda=
infinity));
L22InftyOrderlambda4:=factor(-residue(L22Infty/lambda^5,lambda=
infinity));
L22InftyOrderlambda3:=factor(-residue(L22Infty/lambda^4,lambda=
infinity));
L22InftyOrderlambda2:=factor(-residue(L22Infty/lambda^3,lambda=
infinity));
L22InftyOrderlambda1:=factor(-residue(L22Infty/lambda^2,lambda=
infinity));
L22InftyOrderlambda0:=factor(-residue(L22Infty/lambda^1,lambda=
infinity));
L22InftyOrderlambdaMinus1:=factor(-residue(L22Infty/lambda^0,
lambda=infinity));
L22InftyOrderlambdaMinus2:=factor(-residue(L22Infty/lambda^
(-1),lambda=infinity));
L22InftyOrderlambda7 := 0
L22InftyOrderlambda6 := 0
L22InftyOrderlambda5 := 0
L22InftyOrderlambda4 := 0
L22InftyOrderlambda3 := -t_infty14 - t_infty24
L22InftyOrderlambda2 := -t_infty13 - t_infty23
L22InftyOrderlambda1 := -t_infty12 - t_infty22
L22InftyOrderlambda0 := -t_infty11 - t_infty21
L22InftyOrderlambdaMinus1 := 2 h
L22InftyOrderlambdaMinus2 := 1/(-t_infty24 + t_infty14) (h (A12 t_infty14 - A12 t_infty24
+ A22 t_infty14 - A22 t_infty24 - t_infty13 + t_infty23))

```

(2.3)

We deduce that $L_{\{2,2\}}$ behaves at infinity like

$$L_{\{2,2\}}(\lambda) = -(t_{\infty 14} + t_{\infty 24}) \lambda^3 - (t_{\infty 13} + t_{\infty 23}) \lambda^2 - (t_{\infty 12} + t_{\infty 22}) \lambda - (t_{\infty 11} + t_{\infty 21}) + 2h/\lambda + O(1/\lambda^2)$$

$$= P_1(\lambda) + 2h/\lambda + O(1/\lambda^2)$$

Formulas for $L_{\{2,2\}}$ and $L_{\{2,1\}}$

$$L_{\{2,2\}} = P_1(\lambda) + h/(\lambda - q_1) + h/(\lambda - q_2)$$

$$L_{\{2,1\}} = -P_2(\lambda) + P_{\infty 02} + P_{\infty 12} \lambda + C_1 \lambda + C_0 - h \lambda^2 t_{\infty 14} - p_1 h/(\lambda - q_1) - p_2 h/(\lambda - q_2)$$

```
> L21Form:=-P2 (lambda)+Pinfy02+Pinfy12*lambda +C1*lambda+C0 -h*
lambda^2*tinfy14-p1*h/ (lambda-q1)-p2*h/ (lambda-q2) ;
L22Form:=P1 (lambda) +h/ (lambda-q1)+h/ (lambda-q2) ;
```

$$L21Form := -(-tinfy10 tinfy14 + tinfy10 tinfy24 + tinfy11 tinfy23 + tinfy12 tinfy22 + tinfy13 tinfy21) \lambda^2 - (tinfy11 tinfy24 + tinfy12 tinfy23 + tinfy13 tinfy22 + tinfy14 tinfy21) \lambda^3 - (tinfy12 tinfy24 + tinfy13 tinfy23 + tinfy14 tinfy22) \lambda^4 - (tinfy13 tinfy24 + tinfy14 tinfy23) \lambda^5 - tinfy14 tinfy24 \lambda^6 + C1 \lambda + C0 - h \lambda^2 tinfy14 - \frac{p1 h}{\lambda - q1} - \frac{p2 h}{\lambda - q2} \quad (2.4)$$

$$L22Form := -tinfy11 - tinfy21 + (-tinfy12 - tinfy22) \lambda + (-tinfy13 - tinfy23) \lambda^2 + (-tinfy14 - tinfy24) \lambda^3 + \frac{h}{\lambda - q1} + \frac{h}{\lambda - q2}$$

Auxiliary matrix A

The deformation operator is

$$\mathcal{L} = \hbar (\alpha_{14} \partial_{t_{\infty}^{\{1\},4}} + \alpha_{24} \partial_{t_{\infty}^{\{2\},4}} + \alpha_{13} \partial_{t_{\infty}^{\{1\},3}} + \alpha_{23} \partial_{t_{\infty}^{\{2\},3}} + \alpha_{12} \partial_{t_{\infty}^{\{1\},2}} + \alpha_{22} \partial_{t_{\infty}^{\{2\},2}} + \alpha_{11} \partial_{t_{\infty}^{\{1\},1}} + \alpha_{21} \partial_{t_{\infty}^{\{2\},1}})$$

Computation of $A_{\{1,2\}}$

```
> WronskianLInfty:=factor (psilInfty*Lpsi2Infty-psi2Infty*
LpsilInfty) :
A12Infty:=factor (simplify (WronskianLInfty/WronskianLambdaInfty)
):
Y1Infty:=h*factor (dpsildlambdaInfty/psilInfty) :
Y2Infty:=h*factor (dpsi2dlambdaInfty/psi2Infty) :
Z1Infty:=factor (LpsilInfty/psilInfty) :
Z2Infty:=factor (Lpsi2Infty/psi2Infty) :
A12bisInfty:=factor (simplify ((Z2Infty-Z1Infty) / (Y2Infty-
Y1Infty))) :
A11Infty:=factor (simplify ((Y2Infty*Z1Infty-Y1Infty*Z2Infty) /
(Y2Infty-Y1Infty))) :
factor (simplify (A12bisInfty-A12Infty)) ;
```

0

(3.1)

```
> Ltinfy14:=h*alpha14:
Ltinfy24:=h*alpha24:
Ltinfy13:=h*alpha13:
```

```

Ltiny23:=h*alpha23:
Ltiny12:=h*alpha12:
Ltiny22:=h*alpha22:
Ltiny11:=h*alpha11:
Ltiny21:=h*alpha21:
Ltiny10:=0:
Ltiny20:=0:

```

```

> A12InftyLambda4:=factor(-residue(A12Infty/lambda^5,lambda=
infinity));
A12InftyLambda3:=factor(-residue(A12Infty/lambda^4,lambda=
infinity));
A12InftyLambda2:=factor(-residue(A12Infty/lambda^3,lambda=
infinity));
A12InftyLambda1:=factor(-residue(A12Infty/lambda^2,lambda=
infinity));
A12InftyLambda0:=factor(-residue(A12Infty/lambda^1,lambda=
infinity));
A12InftyLambdaMinus1:=factor(-residue(A12Infty/lambda^0,lambda=
infinity));
A12InftyLambdaMinus2:=factor(-residue(A12Infty/lambda^(-1),
lambda=infinity));
A12InftyLambdaMinus3:=factor(-residue(A12Infty/lambda^(-2),
lambda=infinity));

```

$$A12InftyLambda4 := 0$$

$$A12InftyLambda3 := 0$$

$$A12InftyLambda2 := 0$$

(3.2)

$$A12InftyLambda1 := \frac{1}{4} \frac{\alpha 4 - \alpha 24}{-tiny24 + tiny14}$$

$$A12InftyLambda0 := \frac{1}{12} \frac{1}{(-tiny24 + tiny14)^2} (4 \alpha 3 tiny14 - 4 \alpha 3 tiny24 - 3 \alpha 4 tiny13 + 3 \alpha 4 tiny23 - 4 \alpha 23 tiny14 + 4 \alpha 23 tiny24 + 3 \alpha 24 tiny13 - 3 \alpha 24 tiny23)$$

$$A12InftyLambdaMinus1 := \frac{1}{12} \frac{1}{(-tiny24 + tiny14)^3} (6 \alpha 2 tiny14^2 - 12 \alpha 2 tiny14 tiny24 + 6 \alpha 2 tiny24^2 - 4 \alpha 3 tiny13 tiny14 + 4 \alpha 3 tiny13 tiny24 + 4 \alpha 3 tiny14 tiny23 - 4 \alpha 3 tiny23 tiny24 - 3 \alpha 4 tiny12 tiny14 + 3 \alpha 4 tiny12 tiny24 + 3 \alpha 4 tiny13^2 - 6 \alpha 4 tiny13 tiny23 + 3 \alpha 4 tiny14 tiny22 - 3 \alpha 4 tiny22 tiny24 + 3 \alpha 4 tiny23^2 - 6 \alpha 22 tiny14^2 + 12 \alpha 22 tiny14 tiny24 - 6 \alpha 22 tiny24^2 + 4 \alpha 23 tiny13 tiny14 - 4 \alpha 23 tiny13 tiny24 - 4 \alpha 23 tiny14 tiny23)$$

$$\begin{aligned}
& + 4 \alpha_{23} \text{tinfty}23 \text{tinfty}24 + 3 \alpha_{24} \text{tinfty}12 \text{tinfty}14 - 3 \alpha_{24} \text{tinfty}12 \text{tinfty}24 \\
& - 3 \alpha_{24} \text{tinfty}13^2 + 6 \alpha_{24} \text{tinfty}13 \text{tinfty}23 - 3 \alpha_{24} \text{tinfty}14 \text{tinfty}22 \\
& + 3 \alpha_{24} \text{tinfty}22 \text{tinfty}24 - 3 \alpha_{24} \text{tinfty}23^2) \\
A12\text{InftyLambdaMinus2} := & \frac{1}{12} \frac{1}{(-\text{tinfty}24 + \text{tinfty}14)^4} (12 \alpha_{11} \text{tinfty}14^3 \\
& - 36 \alpha_{11} \text{tinfty}14^2 \text{tinfty}24 + 36 \alpha_{11} \text{tinfty}14 \text{tinfty}24^2 - 12 \alpha_{11} \text{tinfty}24^3 \\
& - 6 \alpha_{12} \text{tinfty}13 \text{tinfty}14^2 + 12 \alpha_{12} \text{tinfty}13 \text{tinfty}14 \text{tinfty}24 - 6 \alpha_{12} \text{tinfty}13 \text{tinfty}24^2 \\
& + 6 \alpha_{12} \text{tinfty}14^2 \text{tinfty}23 - 12 \alpha_{12} \text{tinfty}14 \text{tinfty}23 \text{tinfty}24 + 6 \alpha_{12} \text{tinfty}23 \text{tinfty}24^2 \\
& - 4 \alpha_{13} \text{tinfty}12 \text{tinfty}14^2 + 8 \alpha_{13} \text{tinfty}12 \text{tinfty}14 \text{tinfty}24 - 4 \alpha_{13} \text{tinfty}12 \text{tinfty}24^2 \\
& + 4 \alpha_{13} \text{tinfty}13^2 \text{tinfty}14 - 4 \alpha_{13} \text{tinfty}13^2 \text{tinfty}24 - 8 \alpha_{13} \text{tinfty}13 \text{tinfty}14 \text{tinfty}23 \\
& + 8 \alpha_{13} \text{tinfty}13 \text{tinfty}23 \text{tinfty}24 + 4 \alpha_{13} \text{tinfty}14^2 \text{tinfty}22 \\
& - 8 \alpha_{13} \text{tinfty}14 \text{tinfty}22 \text{tinfty}24 + 4 \alpha_{13} \text{tinfty}14 \text{tinfty}23^2 + 4 \alpha_{13} \text{tinfty}22 \text{tinfty}24^2 \\
& - 4 \alpha_{13} \text{tinfty}23^2 \text{tinfty}24 - 3 \alpha_{14} \text{tinfty}11 \text{tinfty}14^2 + 6 \alpha_{14} \text{tinfty}11 \text{tinfty}14 \text{tinfty}24 \\
& - 3 \alpha_{14} \text{tinfty}11 \text{tinfty}24^2 + 6 \alpha_{14} \text{tinfty}12 \text{tinfty}13 \text{tinfty}14 \\
& - 6 \alpha_{14} \text{tinfty}12 \text{tinfty}13 \text{tinfty}24 - 6 \alpha_{14} \text{tinfty}12 \text{tinfty}14 \text{tinfty}23 \\
& + 6 \alpha_{14} \text{tinfty}12 \text{tinfty}23 \text{tinfty}24 - 3 \alpha_{14} \text{tinfty}13^3 + 9 \alpha_{14} \text{tinfty}13^2 \text{tinfty}23 \\
& - 6 \alpha_{14} \text{tinfty}13 \text{tinfty}14 \text{tinfty}22 + 6 \alpha_{14} \text{tinfty}13 \text{tinfty}22 \text{tinfty}24 \\
& - 9 \alpha_{14} \text{tinfty}13 \text{tinfty}23^2 + 3 \alpha_{14} \text{tinfty}14^2 \text{tinfty}21 - 6 \alpha_{14} \text{tinfty}14 \text{tinfty}21 \text{tinfty}24 \\
& + 6 \alpha_{14} \text{tinfty}14 \text{tinfty}22 \text{tinfty}23 + 3 \alpha_{14} \text{tinfty}21 \text{tinfty}24^2 \\
& - 6 \alpha_{14} \text{tinfty}22 \text{tinfty}23 \text{tinfty}24 + 3 \alpha_{14} \text{tinfty}23^3 - 12 \alpha_{21} \text{tinfty}14^3 \\
& + 36 \alpha_{21} \text{tinfty}14^2 \text{tinfty}24 - 36 \alpha_{21} \text{tinfty}14 \text{tinfty}24^2 + 12 \alpha_{21} \text{tinfty}24^3 \\
& + 6 \alpha_{22} \text{tinfty}13 \text{tinfty}14^2 - 12 \alpha_{22} \text{tinfty}13 \text{tinfty}14 \text{tinfty}24 + 6 \alpha_{22} \text{tinfty}13 \text{tinfty}24^2 \\
& - 6 \alpha_{22} \text{tinfty}14^2 \text{tinfty}23 + 12 \alpha_{22} \text{tinfty}14 \text{tinfty}23 \text{tinfty}24 - 6 \alpha_{22} \text{tinfty}23 \text{tinfty}24^2 \\
& + 4 \alpha_{23} \text{tinfty}12 \text{tinfty}14^2 - 8 \alpha_{23} \text{tinfty}12 \text{tinfty}14 \text{tinfty}24 + 4 \alpha_{23} \text{tinfty}12 \text{tinfty}24^2 \\
& - 4 \alpha_{23} \text{tinfty}13^2 \text{tinfty}14 + 4 \alpha_{23} \text{tinfty}13^2 \text{tinfty}24 + 8 \alpha_{23} \text{tinfty}13 \text{tinfty}14 \text{tinfty}23 \\
& - 8 \alpha_{23} \text{tinfty}13 \text{tinfty}23 \text{tinfty}24 - 4 \alpha_{23} \text{tinfty}14^2 \text{tinfty}22 \\
& + 8 \alpha_{23} \text{tinfty}14 \text{tinfty}22 \text{tinfty}24 - 4 \alpha_{23} \text{tinfty}14 \text{tinfty}23^2 - 4 \alpha_{23} \text{tinfty}22 \text{tinfty}24^2 \\
& + 4 \alpha_{23} \text{tinfty}23^2 \text{tinfty}24 + 3 \alpha_{24} \text{tinfty}11 \text{tinfty}14^2 - 6 \alpha_{24} \text{tinfty}11 \text{tinfty}14 \text{tinfty}24 \\
& + 3 \alpha_{24} \text{tinfty}11 \text{tinfty}24^2 - 6 \alpha_{24} \text{tinfty}12 \text{tinfty}13 \text{tinfty}14 \\
& + 6 \alpha_{24} \text{tinfty}12 \text{tinfty}13 \text{tinfty}24 + 6 \alpha_{24} \text{tinfty}12 \text{tinfty}14 \text{tinfty}23 \\
& - 6 \alpha_{24} \text{tinfty}12 \text{tinfty}23 \text{tinfty}24 + 3 \alpha_{24} \text{tinfty}13^3 - 9 \alpha_{24} \text{tinfty}13^2 \text{tinfty}23 \\
& + 6 \alpha_{24} \text{tinfty}13 \text{tinfty}14 \text{tinfty}22 - 6 \alpha_{24} \text{tinfty}13 \text{tinfty}22 \text{tinfty}24 \\
& + 9 \alpha_{24} \text{tinfty}13 \text{tinfty}23^2 - 3 \alpha_{24} \text{tinfty}14^2 \text{tinfty}21 + 6 \alpha_{24} \text{tinfty}14 \text{tinfty}21 \text{tinfty}24 \\
& - 6 \alpha_{24} \text{tinfty}14 \text{tinfty}22 \text{tinfty}23 - 3 \alpha_{24} \text{tinfty}21 \text{tinfty}24^2 \\
& + 6 \alpha_{24} \text{tinfty}22 \text{tinfty}23 \text{tinfty}24 - 3 \alpha_{24} \text{tinfty}23^3)
\end{aligned}$$

We get that $A_{\{1,2\}} = (\alpha_{14} - \alpha_{24}) / 4 / (-\text{tinfty}14 - \text{tinfty}24) * \text{lambda} + \text{nu} + \text{mu}_1 / (\text{lambda} - \text{q}_1) + \text{mu}_2 / (\text{lambda} - \text{q}_2)$

> $A12\text{Form} := (\alpha_{14} - \alpha_{24}) / 4 / (-\text{tinfty}24 + \text{tinfty}14) * \text{lambda} + \text{nu} + \text{mu}_1 / (\text{lambda} - \text{q}_1) + \text{mu}_2 / (\text{lambda} - \text{q}_2);$
 $A12\text{Formbis} := (\alpha_{14} - \alpha_{24}) / 4 / (-\text{tinfty}24 + \text{tinfty}14) * \text{lambda} + \text{nu} +$

```

(nu1*(lambda-(q1+q2))+nu2)/(lambda-q1)/(lambda-q2);
Equationmu1:=-residue(A12Form,lambda=infinity)-
A12InftyLambdaMinus1:
Equationmu2:=-residue(A12Form*lambda,lambda=infinity)-
A12InftyLambdaMinus2:
mu1:= -(nu1*q2-nu2)/(q1-q2);
mu2:= (nu1*q1-nu2)/(q1-q2);
simplify(-residue(A12Form/lambda^2,lambda=infinity)-
A12InftyLambda1);
factor(simplify(A12Form-A12Formbis));
solve({factor(-residue(A12Form/lambda,lambda=infinity))=
A12InftyLambda0},{nu});
nu:=(3*alpha14*tinfty13-3*alpha14*tinfty23-3*alpha24*
tinfty13+3*alpha24*tinfty23-4*alpha13*tinfty14+4*alpha13*
tinfty24+4*alpha23*tinfty14-4*alpha23*tinfty24)/(12*(tinfty14^2
-2*tinfty14*tinfty24+tinfty24^2));
nubis:=(1/4)*(tinfty13-tinfty23)/(tinfty14-tinfty24)^2*
(alpha14-alpha24)+1/3*(alpha13-alpha23)/(tinfty14-tinfty24);
factor(nu-nubis);

```

$$\begin{aligned}
A12Form &:= \frac{1}{4} \frac{(\alpha14 - \alpha24) \lambda}{-tinfty24 + tinfty14} + v + \frac{\mu1}{\lambda - q1} + \frac{\mu2}{\lambda - q2} \\
A12Formbis &:= \frac{1}{4} \frac{(\alpha14 - \alpha24) \lambda}{-tinfty24 + tinfty14} + v + \frac{v1(\lambda - q1 - q2) + v2}{(\lambda - q1)(\lambda - q2)} \\
\mu1 &:= -\frac{v1 q2 - v2}{q1 - q2} \\
\mu2 &:= \frac{v1 q1 - v2}{q1 - q2} \\
&\quad 0 \\
&\quad 0
\end{aligned} \tag{3.3}$$

$$\left\{ v = \frac{1}{12} \frac{1}{(-tinfty24 + tinfty14)^2} (4 \alpha13 tinfty14 - 4 \alpha13 tinfty24 - 3 \alpha14 tinfty13 + 3 \alpha14 tinfty23 - 4 \alpha23 tinfty14 + 4 \alpha23 tinfty24 + 3 \alpha24 tinfty13 - 3 \alpha24 tinfty23) \right\}$$

$$nubis := -\frac{1}{4} \frac{(-tinfty23 + tinfty13)(\alpha14 - \alpha24)}{(-tinfty24 + tinfty14)^2} + \frac{1}{3} \frac{\alpha13 - \alpha23}{-tinfty24 + tinfty14}$$

```

> nu1:=A12InftyLambdaMinus1:
nu2:=A12InftyLambdaMinus2:
simplify(Equationmu1);
simplify(Equationmu2);

```

0

(3.4)

0

```
> A12InftyLambdaMinus1bis := -(1/4) * ((tinfy12-tinfy22) * (tinfy14-
tinfy24) - (tinfy13-tinfy23)^2) / (tinfy14-tinfy24)^3 *
(alpha14-alpha24)
- (1/3) * (tinfy13-tinfy23) / (tinfy14-tinfy24)^2 * (alpha13-
alpha23)
+ (alpha12-alpha22) / (2 * (tinfy14-tinfy24));
factor(series(A12InftyLambdaMinus1-A12InftyLambdaMinus1bis,
alpha13=0));
```

$A12InftyLambdaMinus1bis :=$ (3.5)

$$-\frac{1}{4} \frac{1}{(-tinfy24 + tinfy14)^3} \left((-tinfy22 + tinfy12) (-tinfy24 + tinfy14) - (-tinfy23 + tinfy13)^2 \right) (\alpha14 - \alpha24) - \frac{1}{3} \frac{(-tinfy23 + tinfy13) (\alpha13 - \alpha23)}{(-tinfy24 + tinfy14)^2} + \frac{\alpha12 - \alpha22}{-2 tinfy24 + 2 tinfy14}$$

0

```
> A12InftyLambdaMinus2bis := -(1/4) * ((tinfy11-tinfy21) * (tinfy14-
tinfy24)^2 - 2 * (tinfy12-tinfy22) * (tinfy14-tinfy24) * (tinfy13-
tinfy23) + (tinfy13-tinfy23)^3) / (tinfy14-tinfy24)^4 *
(alpha14-alpha24)
- (1/3) * ((tinfy12-tinfy22) * (tinfy14-tinfy24) - (tinfy13-
tinfy23)^2) / (tinfy14-tinfy24)^3 * (alpha13-alpha23)
- (1/2) * (-tinfy23+tinfy13) / (tinfy14-tinfy24)^2 * (alpha12-
alpha22)
+ (alpha11-alpha21) / (tinfy14-tinfy24);
factor(series(A12InftyLambdaMinus2-A12InftyLambdaMinus2bis, g=0)
);
```

$A12InftyLambdaMinus2bis :=$ (3.6)

$$-\frac{1}{4} \frac{1}{(-tinfy24 + tinfy14)^4} \left((-tinfy21 + tinfy11) (-tinfy24 + tinfy14)^2 - 2 (-tinfy22 + tinfy12) (-tinfy24 + tinfy14) (-tinfy23 + tinfy13) + (-tinfy23 + tinfy13)^3 \right) (\alpha14 - \alpha24) - \frac{1}{3} \frac{1}{(-tinfy24 + tinfy14)^3} \left((-tinfy22 + tinfy12) (-tinfy24 + tinfy14) - (-tinfy23 + tinfy13)^2 \right) (\alpha13 - \alpha23) - \frac{1}{2} \frac{(-tinfy23 + tinfy13) (\alpha12 - \alpha22)}{(-tinfy24 + tinfy14)^2} + \frac{\alpha11 - \alpha21}{-tinfy24 + tinfy14}$$

0

Computation of $A_{\{1,1\}}$

```
> A11InftyLambda6 := factor(-residue(A11Infty/lambda^7, lambda=
infinity));
```

```

AllInftyLambda5:=factor(-residue (AllInfty/lambda^6 ,lambda=
infinity) );
AllInftyLambda4:=factor(-residue (AllInfty/lambda^5 ,lambda=
infinity) );
AllInftyLambda3:=factor(-residue (AllInfty/lambda^4 ,lambda=
infinity) );
AllInftyLambda2:=factor(-residue (AllInfty/lambda^3 ,lambda=
infinity) );
AllInftyLambda1:=factor(-residue (AllInfty/lambda^2 ,lambda=
infinity) );
AllInftyLambda0:=factor(-residue (AllInfty/lambda^1 ,lambda=
infinity) ):

```

$$AllInftyLambda6 := 0$$

$$AllInftyLambda5 := 0$$

(3.7)

$$AllInftyLambda4 := \frac{1}{4} \frac{\alpha_4 t_{24} - \alpha_4 t_{14}}{-t_{24} + t_{14}}$$

$$AllInftyLambda3 := \frac{1}{12} \frac{1}{(-t_{24} + t_{14})^2} (4 \alpha_3 t_{14} t_{24} - 4 \alpha_3 t_{24}^2 - 3 \alpha_4 t_{13} t_{24} + 3 \alpha_4 t_{14} t_{23} - 4 \alpha_2 t_{14}^2 + 4 \alpha_2 t_{14} t_{24} + 3 \alpha_2 t_{13} t_{24} - 3 \alpha_2 t_{14} t_{23})$$

$$AllInftyLambda2 := \frac{1}{12} \frac{1}{(-t_{24} + t_{14})^3} (6 \alpha_2 t_{14}^2 t_{24} - 12 \alpha_2 t_{14} t_{24}^2 + 6 \alpha_2 t_{24}^3 - 4 \alpha_3 t_{13} t_{14} t_{24} + 4 \alpha_3 t_{13} t_{24}^2 + 4 \alpha_3 t_{14}^2 t_{23} - 4 \alpha_3 t_{14} t_{23} t_{24} - 3 \alpha_4 t_{12} t_{14} t_{24} + 3 \alpha_4 t_{12} t_{24}^2 + 3 \alpha_4 t_{13}^2 t_{24} - 3 \alpha_4 t_{13} t_{14} t_{23} - 3 \alpha_4 t_{13} t_{23} t_{24} + 3 \alpha_4 t_{14}^2 t_{22} - 3 \alpha_4 t_{14} t_{22} t_{24} + 3 \alpha_4 t_{14} t_{23}^2 - 6 \alpha_2 t_{14}^3 + 12 \alpha_2 t_{14}^2 t_{24} - 6 \alpha_2 t_{14} t_{24}^2 + 4 \alpha_2 t_{13} t_{14} t_{24} - 4 \alpha_2 t_{13} t_{24}^2 - 4 \alpha_2 t_{14}^2 t_{23} + 4 \alpha_2 t_{14} t_{23} t_{24} + 3 \alpha_2 t_{12} t_{14} t_{24} - 3 \alpha_2 t_{12} t_{24}^2 - 3 \alpha_2 t_{13}^2 t_{24} + 3 \alpha_2 t_{13} t_{14} t_{23} + 3 \alpha_2 t_{13} t_{23} t_{24} - 3 \alpha_2 t_{14}^2 t_{22} + 3 \alpha_2 t_{14} t_{22} t_{24} - 3 \alpha_2 t_{14} t_{23}^2)$$

$$AllInftyLambda1 := \frac{1}{12} \frac{1}{(-t_{24} + t_{14})^4} (12 \alpha_1 t_{14}^3 t_{24} - 36 \alpha_1 t_{14}^2 t_{24}^2 + 36 \alpha_1 t_{14} t_{24}^3 - 12 \alpha_1 t_{24}^4 - 6 \alpha_2 t_{13} t_{14}^2 t_{24} + 12 \alpha_2 t_{13} t_{14} t_{24}^2 - 6 \alpha_2 t_{13} t_{24}^3 + 6 \alpha_2 t_{14}^3 t_{23} - 12 \alpha_2 t_{14}^2 t_{23} t_{24} + 6 \alpha_2 t_{14} t_{23} t_{24}^2 - 4 \alpha_3 t_{12} t_{14}^2 t_{24} + 8 \alpha_3 t_{12} t_{14} t_{24}^2 - 4 \alpha_3 t_{12} t_{24}^3 + 4 \alpha_3 t_{13}^2 t_{14} t_{24} - 4 \alpha_3 t_{13}^2 t_{24}^2)$$

$$\begin{aligned}
& -4 \alpha_3 \text{tinfty}13 \text{tinfty}14^2 \text{tinfty}23 + 4 \alpha_3 \text{tinfty}13 \text{tinfty}23 \text{tinfty}24^2 \\
& + 4 \alpha_3 \text{tinfty}14^3 \text{tinfty}22 - 8 \alpha_3 \text{tinfty}14^2 \text{tinfty}22 \text{tinfty}24 + 4 \alpha_3 \text{tinfty}14^2 \text{tinfty}23^2 \\
& + 4 \alpha_3 \text{tinfty}14 \text{tinfty}22 \text{tinfty}24^2 - 4 \alpha_3 \text{tinfty}14 \text{tinfty}23^2 \text{tinfty}24 \\
& - 3 \alpha_4 \text{tinfty}11 \text{tinfty}14^2 \text{tinfty}24 + 6 \alpha_4 \text{tinfty}11 \text{tinfty}14 \text{tinfty}24^2 \\
& - 3 \alpha_4 \text{tinfty}11 \text{tinfty}24^3 + 6 \alpha_4 \text{tinfty}12 \text{tinfty}13 \text{tinfty}14 \text{tinfty}24 \\
& - 6 \alpha_4 \text{tinfty}12 \text{tinfty}13 \text{tinfty}24^2 - 3 \alpha_4 \text{tinfty}12 \text{tinfty}14^2 \text{tinfty}23 \\
& + 3 \alpha_4 \text{tinfty}12 \text{tinfty}23 \text{tinfty}24^2 - 3 \alpha_4 \text{tinfty}13^3 \text{tinfty}24 \\
& + 3 \alpha_4 \text{tinfty}13^2 \text{tinfty}14 \text{tinfty}23 + 6 \alpha_4 \text{tinfty}13^2 \text{tinfty}23 \text{tinfty}24 \\
& - 3 \alpha_4 \text{tinfty}13 \text{tinfty}14^2 \text{tinfty}22 - 6 \alpha_4 \text{tinfty}13 \text{tinfty}14 \text{tinfty}23^2 \\
& + 3 \alpha_4 \text{tinfty}13 \text{tinfty}22 \text{tinfty}24^2 - 3 \alpha_4 \text{tinfty}13 \text{tinfty}23^2 \text{tinfty}24 \\
& + 3 \alpha_4 \text{tinfty}14^3 \text{tinfty}21 - 6 \alpha_4 \text{tinfty}14^2 \text{tinfty}21 \text{tinfty}24 \\
& + 6 \alpha_4 \text{tinfty}14^2 \text{tinfty}22 \text{tinfty}23 + 3 \alpha_4 \text{tinfty}14 \text{tinfty}21 \text{tinfty}24^2 \\
& - 6 \alpha_4 \text{tinfty}14 \text{tinfty}22 \text{tinfty}23 \text{tinfty}24 + 3 \alpha_4 \text{tinfty}14 \text{tinfty}23^3 - 12 \alpha_21 \text{tinfty}14^4 \\
& + 36 \alpha_21 \text{tinfty}14^3 \text{tinfty}24 - 36 \alpha_21 \text{tinfty}14^2 \text{tinfty}24^2 + 12 \alpha_21 \text{tinfty}14 \text{tinfty}24^3 \\
& + 6 \alpha_22 \text{tinfty}13 \text{tinfty}14^2 \text{tinfty}24 - 12 \alpha_22 \text{tinfty}13 \text{tinfty}14 \text{tinfty}24^2 \\
& + 6 \alpha_22 \text{tinfty}13 \text{tinfty}24^3 - 6 \alpha_22 \text{tinfty}14^3 \text{tinfty}23 + 12 \alpha_22 \text{tinfty}14^2 \text{tinfty}23 \text{tinfty}24 \\
& - 6 \alpha_22 \text{tinfty}14 \text{tinfty}23 \text{tinfty}24^2 + 4 \alpha_23 \text{tinfty}12 \text{tinfty}14^2 \text{tinfty}24 \\
& - 8 \alpha_23 \text{tinfty}12 \text{tinfty}14 \text{tinfty}24^2 + 4 \alpha_23 \text{tinfty}12 \text{tinfty}24^3 \\
& - 4 \alpha_23 \text{tinfty}13^2 \text{tinfty}14 \text{tinfty}24 + 4 \alpha_23 \text{tinfty}13^2 \text{tinfty}24^2 \\
& + 4 \alpha_23 \text{tinfty}13 \text{tinfty}14^2 \text{tinfty}23 - 4 \alpha_23 \text{tinfty}13 \text{tinfty}23 \text{tinfty}24^2 \\
& - 4 \alpha_23 \text{tinfty}14^3 \text{tinfty}22 + 8 \alpha_23 \text{tinfty}14^2 \text{tinfty}22 \text{tinfty}24 - 4 \alpha_23 \text{tinfty}14^2 \text{tinfty}23^2 \\
& - 4 \alpha_23 \text{tinfty}14 \text{tinfty}22 \text{tinfty}24^2 + 4 \alpha_23 \text{tinfty}14 \text{tinfty}23^2 \text{tinfty}24 \\
& + 3 \alpha_24 \text{tinfty}11 \text{tinfty}14^2 \text{tinfty}24 - 6 \alpha_24 \text{tinfty}11 \text{tinfty}14 \text{tinfty}24^2 \\
& + 3 \alpha_24 \text{tinfty}11 \text{tinfty}24^3 - 6 \alpha_24 \text{tinfty}12 \text{tinfty}13 \text{tinfty}14 \text{tinfty}24 \\
& + 6 \alpha_24 \text{tinfty}12 \text{tinfty}13 \text{tinfty}24^2 + 3 \alpha_24 \text{tinfty}12 \text{tinfty}14^2 \text{tinfty}23 \\
& - 3 \alpha_24 \text{tinfty}12 \text{tinfty}23 \text{tinfty}24^2 + 3 \alpha_24 \text{tinfty}13^3 \text{tinfty}24 \\
& - 3 \alpha_24 \text{tinfty}13^2 \text{tinfty}14 \text{tinfty}23 - 6 \alpha_24 \text{tinfty}13^2 \text{tinfty}23 \text{tinfty}24 \\
& + 3 \alpha_24 \text{tinfty}13 \text{tinfty}14^2 \text{tinfty}22 + 6 \alpha_24 \text{tinfty}13 \text{tinfty}14 \text{tinfty}23^2 \\
& - 3 \alpha_24 \text{tinfty}13 \text{tinfty}22 \text{tinfty}24^2 + 3 \alpha_24 \text{tinfty}13 \text{tinfty}23^2 \text{tinfty}24 \\
& - 3 \alpha_24 \text{tinfty}14^3 \text{tinfty}21 + 6 \alpha_24 \text{tinfty}14^2 \text{tinfty}21 \text{tinfty}24 \\
& - 6 \alpha_24 \text{tinfty}14^2 \text{tinfty}22 \text{tinfty}23 - 3 \alpha_24 \text{tinfty}14 \text{tinfty}21 \text{tinfty}24^2 \\
& + 6 \alpha_24 \text{tinfty}14 \text{tinfty}22 \text{tinfty}23 \text{tinfty}24 - 3 \alpha_24 \text{tinfty}14 \text{tinfty}23^3)
\end{aligned}$$

We get that $A_{\{1,1\}} = 1/4 * (\alpha_14 * \text{tinfty}24 - \alpha_24 * \text{tinfty}14) / (\text{tinfty}14 - \text{tinfty}24) * \text{lambda}^4 + c_3 * \text{lambda}^3 + c_2 * \text{lambda}^2 + c_1 * \text{lambda} + c_0 + \text{rho}_1 / (\text{lambda} - q_1) + \text{rho}_2 / (\text{lambda} - q_2)$

> **A11Form:=1/4*(alpha14*tinfty24-alpha24*tinfty14)/(tinfty14-tinfty24)*lambda^4+c3*lambda^3+c2*lambda^2+c1*lambda+c0+rho1/(lambda-q1)+rho2/(lambda-q2);**

simplify(-residue(A11Form/lambda^5,lambda=infinity)-A11InftyLambda4);

```
solve({factor(-residue(A11Form/lambda^4,lambda=infinity))=
A11InftyLambda3,factor(-residue(A11Form/lambda^3,lambda=
infinity))=A11InftyLambda2,factor(-residue(A11Form/lambda^2,
lambda=infinity))=A11InftyLambda1},{c3,c2,c1}):
```

$$A11Form := \frac{1}{4} \frac{(\alpha_4 \text{infty}^{24} - \alpha_{24} \text{infty}^{14}) \lambda^4}{-\text{infty}^{24} + \text{infty}^{14}} + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 + \frac{\rho_1}{\lambda - q_1} \quad (3.8)$$

$$+ \frac{\rho_2}{\lambda - q_2}$$

0

```
> c1:= -(3*alpha14*infty11*infty14^2*infty24-6*alpha14*
infty11*infty14*infty24^2+3*alpha14*infty11*infty24^3-6*
alpha14*infty12*infty13*infty14*infty24+6*alpha14*infty12*
infty13*infty24^2+3*alpha14*infty12*infty14^2*infty23-3*
alpha14*infty12*infty23*infty24^2+3*alpha14*infty13^3*
infty24-3*alpha14*infty13^2*infty14*infty23-6*alpha14*
infty13^2*infty23*infty24+3*alpha14*infty13*infty14^2*
infty22+6*alpha14*infty13*infty14*infty23^2-3*alpha14*
infty13*infty22*infty24^2+3*alpha14*infty13*infty23^2*
infty24-3*alpha14*infty14^3*infty21+6*alpha14*infty14^2*
infty21*infty24-6*alpha14*infty14^2*infty22*infty23-3*
alpha14*infty14*infty21*infty24^2+6*alpha14*infty14*
infty22*infty23*infty24-3*alpha14*infty14*infty23^3-3*
alpha24*infty11*infty14^2*infty24+6*alpha24*infty11*
infty14*infty24^2-3*alpha24*infty11*infty24^3+6*alpha24*
infty12*infty13*infty14*infty24-6*alpha24*infty12*
infty13*infty24^2-3*alpha24*infty12*infty14^2*infty23+3*
alpha24*infty12*infty23*infty24^2-3*alpha24*infty13^3*
infty24+3*alpha24*infty13^2*infty14*infty23+6*alpha24*
infty13^2*infty23*infty24-3*alpha24*infty13*infty14^2*
infty22-6*alpha24*infty13*infty14*infty23^2+3*alpha24*
infty13*infty22*infty24^2-3*alpha24*infty13*infty23^2*
infty24+3*alpha24*infty14^3*infty21-6*alpha24*infty14^2*
infty21*infty24+6*alpha24*infty14^2*infty22*infty23+3*
alpha24*infty14*infty21*infty24^2-6*alpha24*infty14*
infty22*infty23*infty24+3*alpha24*infty14*infty23^3+4*
alpha13*infty12*infty14^2*infty24-8*alpha13*infty12*
infty14*infty24^2+4*alpha13*infty12*infty24^3-4*alpha13*
infty13^2*infty14*infty24+4*alpha13*infty13^2*infty24^2+4*
alpha13*infty13*infty14^2*infty23-4*alpha13*infty13*
infty23*infty24^2-4*alpha13*infty14^3*infty22+8*alpha13*
infty14^2*infty22*infty24-4*alpha13*infty14^2*infty23^2-4*
alpha13*infty14*infty22*infty24^2+4*alpha13*infty14*
```

$$\begin{aligned} & \text{tinfy23}^2 * \text{tinfy24} - 4 * \alpha_{23} * \text{tinfy12} * \text{tinfy14}^2 * \text{tinfy24} + 8 * \\ & \alpha_{23} * \text{tinfy12} * \text{tinfy14} * \text{tinfy24}^2 - 4 * \alpha_{23} * \text{tinfy12} * \\ & \text{tinfy24}^3 + 4 * \alpha_{23} * \text{tinfy13}^2 * \text{tinfy14} * \text{tinfy24} - 4 * \alpha_{23} * \\ & \text{tinfy13}^2 * \text{tinfy24}^2 - 4 * \alpha_{23} * \text{tinfy13} * \text{tinfy14}^2 * \text{tinfy23} + 4 * \\ & \alpha_{23} * \text{tinfy13} * \text{tinfy23} * \text{tinfy24}^2 + 4 * \alpha_{23} * \text{tinfy14}^3 * \\ & \text{tinfy22} - 8 * \alpha_{23} * \text{tinfy14}^2 * \text{tinfy22} * \text{tinfy24} + 4 * \alpha_{23} * \\ & \text{tinfy14}^2 * \text{tinfy23}^2 + 4 * \alpha_{23} * \text{tinfy14} * \text{tinfy22} * \text{tinfy24}^2 - 4 * \\ & \alpha_{23} * \text{tinfy14} * \text{tinfy23}^2 * \text{tinfy24} + 6 * \alpha_{12} * \text{tinfy13} * \\ & \text{tinfy14}^2 * \text{tinfy24} - 12 * \alpha_{12} * \text{tinfy13} * \text{tinfy14} * \text{tinfy24}^2 + 6 * \\ & \alpha_{12} * \text{tinfy13} * \text{tinfy24}^3 - 6 * \alpha_{12} * \text{tinfy14}^3 * \text{tinfy23} + 12 * \\ & \alpha_{12} * \text{tinfy14}^2 * \text{tinfy23} * \text{tinfy24} - 6 * \alpha_{12} * \text{tinfy14} * \\ & \text{tinfy23} * \text{tinfy24}^2 - 6 * \alpha_{22} * \text{tinfy13} * \text{tinfy14}^2 * \text{tinfy24} + 12 * \\ & \alpha_{22} * \text{tinfy13} * \text{tinfy14} * \text{tinfy24}^2 - 6 * \alpha_{22} * \text{tinfy13} * \\ & \text{tinfy24}^3 + 6 * \alpha_{22} * \text{tinfy14}^3 * \text{tinfy23} - 12 * \alpha_{22} * \text{tinfy14}^2 * \\ & \text{tinfy23} * \text{tinfy24} + 6 * \alpha_{22} * \text{tinfy14} * \text{tinfy23} * \text{tinfy24}^2 - 12 * \\ & \alpha_{11} * \text{tinfy14}^3 * \text{tinfy24} + 36 * \alpha_{11} * \text{tinfy14}^2 * \text{tinfy24}^2 \\ & - 36 * \alpha_{11} * \text{tinfy14} * \text{tinfy24}^3 + 12 * \alpha_{11} * \text{tinfy24}^4 + 12 * \\ & \alpha_{21} * \text{tinfy14}^4 - 36 * \alpha_{21} * \text{tinfy14}^3 * \text{tinfy24} + 36 * \alpha_{21} * \\ & \text{tinfy14}^2 * \text{tinfy24}^2 - 12 * \alpha_{21} * \text{tinfy14} * \text{tinfy24}^3) / (12 * \\ & (\text{tinfy14}^4 - 4 * \text{tinfy14}^3 * \text{tinfy24} + 6 * \text{tinfy14}^2 * \text{tinfy24}^2 - 4 * \\ & \text{tinfy14} * \text{tinfy24}^3 + \text{tinfy24}^4)) : \end{aligned}$$

$$\begin{aligned} c2 := & - (3 * \alpha_{14} * \text{tinfy12} * \text{tinfy14} * \text{tinfy24} - 3 * \alpha_{14} * \text{tinfy12} * \\ & \text{tinfy24}^2 - 3 * \alpha_{14} * \text{tinfy13}^2 * \text{tinfy24} + 3 * \alpha_{14} * \text{tinfy13} * \\ & \text{tinfy14} * \text{tinfy23} + 3 * \alpha_{14} * \text{tinfy13} * \text{tinfy23} * \text{tinfy24} - 3 * \\ & \alpha_{14} * \text{tinfy14}^2 * \text{tinfy22} + 3 * \alpha_{14} * \text{tinfy14} * \text{tinfy22} * \\ & \text{tinfy24} - 3 * \alpha_{14} * \text{tinfy14} * \text{tinfy23}^2 - 3 * \alpha_{24} * \text{tinfy12} * \\ & \text{tinfy14} * \text{tinfy24} + 3 * \alpha_{24} * \text{tinfy12} * \text{tinfy24}^2 + 3 * \alpha_{24} * \\ & \text{tinfy13}^2 * \text{tinfy24} - 3 * \alpha_{24} * \text{tinfy13} * \text{tinfy14} * \text{tinfy23} - 3 * \\ & \alpha_{24} * \text{tinfy13} * \text{tinfy23} * \text{tinfy24} + 3 * \alpha_{24} * \text{tinfy14}^2 * \\ & \text{tinfy22} - 3 * \alpha_{24} * \text{tinfy14} * \text{tinfy22} * \text{tinfy24} + 3 * \alpha_{24} * \\ & \text{tinfy14} * \text{tinfy23}^2 + 4 * \alpha_{13} * \text{tinfy13} * \text{tinfy14} * \text{tinfy24} - 4 * \\ & \alpha_{13} * \text{tinfy13} * \text{tinfy24}^2 - 4 * \alpha_{13} * \text{tinfy14}^2 * \text{tinfy23} + 4 * \\ & \alpha_{13} * \text{tinfy14} * \text{tinfy23} * \text{tinfy24} - 4 * \alpha_{23} * \text{tinfy13} * \text{tinfy14} * \\ & \text{tinfy24} + 4 * \alpha_{23} * \text{tinfy13} * \text{tinfy24}^2 + 4 * \alpha_{23} * \text{tinfy14}^2 * \\ & \text{tinfy23} - 4 * \alpha_{23} * \text{tinfy14} * \text{tinfy23} * \text{tinfy24} - 6 * \alpha_{12} * \\ & \text{tinfy14}^2 * \text{tinfy24} + 12 * \alpha_{12} * \text{tinfy14} * \text{tinfy24}^2 - 6 * \alpha_{12} * \\ & \text{tinfy24}^3 + 6 * \alpha_{22} * \text{tinfy14}^3 - 12 * \alpha_{22} * \text{tinfy14}^2 * \\ & \text{tinfy24} + 6 * \alpha_{22} * \text{tinfy14} * \text{tinfy24}^2) / (12 * (\text{tinfy14}^3 - 3 * \\ & \text{tinfy14}^2 * \text{tinfy24} + 3 * \text{tinfy14} * \text{tinfy24}^2 - \text{tinfy24}^3)) : \end{aligned}$$

$$c3 := - (3 * \alpha_{14} * \text{tinfy13} * \text{tinfy24} - 3 * \alpha_{14} * \text{tinfy14} * \text{tinfy23}$$

```
-3*alpha24*tinfty13*tinfty24+3*alpha24*tinfty14*tinfty23-4*
alpha13*tinfty14*tinfty24+4*alpha13*tinfty24^2+4*alpha23*
tinfty14^2-4*alpha23*tinfty14*tinfty24) / (12*(tinfty14^2-2*
tinfty14*tinfty24+tinfty24^2)) :
```

```
c4:=(1/4)*(alpha14*tinfty24-alpha24*tinfty14)/(tinfty14-
tinfty24) :
```

```
simplify(factor(-residue(AllForm/lambda^4,lambda=infinity))-
AllInftyLambda3) ;
simplify(factor(-residue(AllForm/lambda^3,lambda=infinity))-
AllInftyLambda2) ;
simplify(factor(-residue(AllForm/lambda^2,lambda=infinity))-
AllInftyLambda1) ;
```

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \quad (3.9)$$

```
> c3bis:=- (1/4)*(tinfty13*tinfty24-tinfty14*tinfty23)/(tinfty14-
tinfty24)^2*(alpha14-alpha24)
+1/3*(alpha13*tinfty24-alpha23*tinfty14)/(tinfty14-tinfty24) ;
factor(series(c3-c3bis,alpha14=0)) ;
```

$$c3bis := -\frac{1}{4} \frac{(\alpha_{13} \text{tinfty}24 - \alpha_{14} \text{tinfty}23) (\alpha_{14} - \alpha_{24})}{(-\text{tinfty}24 + \text{tinfty}14)^2} + \frac{1}{3} \frac{\alpha_{13} \text{tinfty}24 - \alpha_{23} \text{tinfty}14}{-\text{tinfty}24 + \text{tinfty}14} \quad (3.10)$$

```
> c2bis:=- (1/4)*((tinfty14-tinfty24)*(tinfty24*tinfty12-tinfty14*
tinfty22)-(-tinfty23+tinfty13)*(tinfty13*tinfty24-tinfty14*
tinfty23))/(tinfty14-tinfty24)^3*(alpha14-alpha24)
- (1/3)*(tinfty13*tinfty24-tinfty14*tinfty23)/(tinfty14-
tinfty24)^2*(alpha13-alpha23)
+ (alpha12*tinfty24-alpha22*tinfty14)/(2*(tinfty14-tinfty24)) ;
factor(series(c2-c2bis,alpha14=0)) ;
```

$$c2bis := -\frac{1}{4} \frac{1}{(-\text{tinfty}24 + \text{tinfty}14)^3} \left(((-\text{tinfty}24 + \text{tinfty}14) (\text{tinfty}12 \text{tinfty}24 - \text{tinfty}14 \text{tinfty}22) - (-\text{tinfty}23 + \text{tinfty}13) (\text{tinfty}13 \text{tinfty}24 - \text{tinfty}14 \text{tinfty}23)) (\alpha_{14} - \alpha_{24}) \right) - \frac{1}{3} \frac{(\text{tinfty}13 \text{tinfty}24 - \text{tinfty}14 \text{tinfty}23) (\alpha_{13} - \alpha_{23})}{(-\text{tinfty}24 + \text{tinfty}14)^2} + \frac{\alpha_{12} \text{tinfty}24 - \alpha_{22} \text{tinfty}14}{-2 \text{tinfty}24 + 2 \text{tinfty}14} \quad (3.11)$$

0


```

> c1bis := (tinfty24*alpha11-tinfty14*alpha21) / (tinfty14-tinfty24)
- (1/2) * (tinfty13*tinfty24-tinfty14*tinfty23) / (tinfty14-
tinfty24) ^2 * (alpha12-alpha22)
- (1/3) * ((tinfty14-tinfty24) * (tinfty24*tinfty12-tinfty14*
tinfty22) - (-tinfty23+tinfty13) * (tinfty13*tinfty24-tinfty14*
tinfty23)) / (tinfty14-tinfty24) ^3 * (alpha13-alpha23)
- 1/4 * (tinfty24*tinfty13^3-tinfty14*tinfty23^3+ (2*tinfty14+
tinfty24) *tinfty13*tinfty23^2- (2*tinfty24+tinfty14) *tinfty13^2*
tinfty23
- (tinfty14-tinfty24) * (2*tinfty12*tinfty24-tinfty14*tinfty22-
tinfty22*tinfty24) *tinfty13
- (tinfty14-tinfty24) * (2*tinfty22*tinfty14-tinfty14*tinfty12-
tinfty12*tinfty24) *tinfty23
+ (tinfty14-tinfty24) ^2 * (tinfty11*tinfty24-tinfty14*tinfty21)) /
(tinfty14-tinfty24) ^4 * (alpha14-alpha24)
;
factor(series(c1-c1bis,e=0));

```

$$\begin{aligned}
c1bis := & \frac{\alpha 1 \text{ tinfty}24 - \alpha 21 \text{ tinfty}14}{-\text{tinfty}24 + \text{tinfty}14} \\
& - \frac{1}{2} \frac{(\text{tinfty}13 \text{ tinfty}24 - \text{tinfty}14 \text{ tinfty}23) (\alpha 2 - \alpha 22)}{(-\text{tinfty}24 + \text{tinfty}14)^2} \\
& - \frac{1}{3} \frac{1}{(-\text{tinfty}24 + \text{tinfty}14)^3} \left(((-\text{tinfty}24 + \text{tinfty}14) (\text{tinfty}12 \text{ tinfty}24 \right. \\
& \left. - \text{tinfty}14 \text{ tinfty}22) - (-\text{tinfty}23 + \text{tinfty}13) (\text{tinfty}13 \text{ tinfty}24 - \text{tinfty}14 \text{ tinfty}23) \right) \\
& (\alpha 3 - \alpha 23) - \frac{1}{4} \frac{1}{(-\text{tinfty}24 + \text{tinfty}14)^4} \left((\text{tinfty}24 \text{ tinfty}13^3 \right. \\
& \left. - \text{tinfty}14 \text{ tinfty}23^3 + (2 \text{ tinfty}14 + \text{tinfty}24) \text{ tinfty}13 \text{ tinfty}23^2 - (2 \text{ tinfty}24 \right. \\
& \left. + \text{tinfty}14) \text{ tinfty}13^2 \text{ tinfty}23 - (-\text{tinfty}24 + \text{tinfty}14) (2 \text{ tinfty}12 \text{ tinfty}24 \right. \\
& \left. - \text{tinfty}14 \text{ tinfty}22 - \text{tinfty}22 \text{ tinfty}24) \text{ tinfty}13 - (-\text{tinfty}24 + \text{tinfty}14) (\right. \\
& \left. -\text{tinfty}12 \text{ tinfty}14 - \text{tinfty}12 \text{ tinfty}24 + 2 \text{ tinfty}14 \text{ tinfty}22) \text{ tinfty}23 + (-\text{tinfty}24 \right. \\
& \left. + \text{tinfty}14)^2 (\text{tinfty}11 \text{ tinfty}24 - \text{tinfty}14 \text{ tinfty}21) \right) (\alpha 4 - \alpha 24) \\
& 0
\end{aligned} \tag{3.12}$$

```

> c1function:=unapply(c1,alpha14,
alpha24,alpha13,alpha23,alpha12,alpha22,alpha11,alpha21):
c2function:=unapply(c2,alpha14,alpha24,alpha13,alpha23,alpha12,
alpha22,alpha11,alpha21):
c3function:=unapply(c3,alpha14,alpha24,alpha13,alpha23,alpha12,
alpha22,alpha11,alpha21):
nufunction:=unapply(nu,alpha14,alpha24,alpha13,alpha23,alpha12,
alpha22,alpha11,alpha21):
nulfunction:=unapply(-(1/4)*((tinfty12-tinfty22)*(tinfty14-
tinfty24) - (tinfty13-tinfty23)^2 ) / (tinfty14-tinfty24) ^3*

```

$$\begin{aligned}
& (\alpha_{14}-\alpha_{24}) \\
& - (1/3) * (\text{tiny13}-\text{tiny23}) / (\text{tiny14}-\text{tiny24})^2 * (\alpha_{13}-\alpha_{23}) \\
& + (\alpha_{12}-\alpha_{22}) / (2 * (\text{tiny14}-\text{tiny24})), \alpha_{14}, \alpha_{24}, \alpha_{13}, \alpha_{23}, \alpha_{12}, \alpha_{22}, \alpha_{11}, \alpha_{21} ; \\
& \text{nu2function} := \text{unapply}(- (1/4) * ((\text{tiny11}-\text{tiny21}) * (\text{tiny14}-\text{tiny24})^2 - 2 * (\text{tiny12}-\text{tiny22}) * (\text{tiny14}-\text{tiny24}) * (\text{tiny13}-\text{tiny23}) + (\text{tiny13}-\text{tiny23})^3) / (\text{tiny14}-\text{tiny24})^4 * \\
& (\alpha_{14}-\alpha_{24}) \\
& - (1/3) * ((\text{tiny12}-\text{tiny22}) * (\text{tiny14}-\text{tiny24}) - (\text{tiny13}-\text{tiny23})^2) / (\text{tiny14}-\text{tiny24})^3 * (\alpha_{13}-\alpha_{23}) \\
& - (1/2) * (-\text{tiny23}+\text{tiny13}) / (\text{tiny14}-\text{tiny24})^2 * (\alpha_{12}-\alpha_{22}) \\
& + (\alpha_{11}-\alpha_{21}) / (\text{tiny14}-\text{tiny24}), \alpha_{14}, \alpha_{24}, \alpha_{13}, \alpha_{23}, \alpha_{12}, \alpha_{22}, \alpha_{11}, \alpha_{21} ; \\
& \text{mulfunction} := (\alpha_4, \alpha_{24}, \alpha_3, \alpha_{23}, \alpha_2, \alpha_{22}, \alpha_1, \alpha_{21}) \rightarrow \tag{3.13} \\
& - \frac{1}{4} \frac{1}{(-\text{tiny24} + \text{tiny14})^3} \left(((-\text{tiny22} + \text{tiny12}) (-\text{tiny24} + \text{tiny14}) \right. \\
& \left. - (-\text{tiny23} + \text{tiny13})^3 \right) (\alpha_4 - \alpha_{24}) - \frac{1}{3} \frac{(-\text{tiny23} + \text{tiny13}) (\alpha_3 - \alpha_{23})}{(-\text{tiny24} + \text{tiny14})^2} \\
& + \frac{\alpha_2 - \alpha_{22}}{-2 \text{tiny24} + 2 \text{tiny14}} \\
& \text{nu2function} := (\alpha_4, \alpha_{24}, \alpha_3, \alpha_{23}, \alpha_2, \alpha_{22}, \alpha_1, \alpha_{21}) \rightarrow \\
& - \frac{1}{4} \frac{1}{(-\text{tiny24} + \text{tiny14})^4} \left(((-\text{tiny21} + \text{tiny11}) (-\text{tiny24} + \text{tiny14})^2 - 2 (\right. \\
& \left. -\text{tiny22} + \text{tiny12}) (-\text{tiny24} + \text{tiny14}) (-\text{tiny23} + \text{tiny13}) + (-\text{tiny23} \right. \\
& \left. + \text{tiny13})^3 \right) (\alpha_4 - \alpha_{24}) \\
& - \frac{1}{3} \frac{1}{(-\text{tiny24} + \text{tiny14})^3} \left(((-\text{tiny22} + \text{tiny12}) (-\text{tiny24} + \text{tiny14}) \right. \\
& \left. - (-\text{tiny23} + \text{tiny13})^3 \right) (\alpha_3 - \alpha_{23}) - \frac{1}{2} \frac{(-\text{tiny23} + \text{tiny13}) (\alpha_2 - \alpha_{22})}{(-\text{tiny24} + \text{tiny14})^2} \\
& + \frac{\alpha_1 - \alpha_{21}}{-\text{tiny24} + \text{tiny14}}
\end{aligned}$$

Verification of the theoretical formulas for c1, c2, c3, c4

```

> with(LinearAlgebra) :
m:=4:
M0:=Matrix(m,m,0) :
for i from 1 to m do M0[i,i]:=(tiny14-tiny24): od:
M0[2,1]:=tiny13-tiny23:
M0[3,1]:=tiny12-tiny22:
M0[3,2]:=tiny13-tiny23:

```

```

M0[4,1]:=tinfty11-tinfty21:
M0[4,2]:=tinfty12-tinfty22:
M0[4,3]:=tinfty13-tinfty23:
M0;

```

$$\begin{bmatrix}
-tinfty24 + tinfty14 & 0 & 0 & 0 \\
-tinfty23 + tinfty13 & -tinfty24 + tinfty14 & 0 & 0 \\
-tinfty22 + tinfty12 & -tinfty23 + tinfty13 & -tinfty24 + tinfty14 & 0 \\
-tinfty21 + tinfty11 & -tinfty22 + tinfty12 & -tinfty23 + tinfty13 & -tinfty24 + tinfty14
\end{bmatrix} \quad (4.1)$$

```

> c3bis:=- (1/4) * (tinfty13*tinfty24-tinfty14*tinfty23) / (tinfty14-
tinfty24) ^2 * (alpha14-alpha24)
+1/3 * (alpha13*tinfty24-alpha23*tinfty14) / (tinfty14-tinfty24) :
c2bis:=- (1/4) * ( (tinfty14-tinfty24) * (tinfty24*tinfty12-tinfty14*
tinfty22) - (-tinfty23+tinfty13) * (tinfty13*tinfty24-tinfty14*
tinfty23) ) / (tinfty14-tinfty24) ^3 * (alpha14-alpha24)
- (1/3) * (tinfty13*tinfty24-tinfty14*tinfty23) / (tinfty14-
tinfty24) ^2 * (alpha13-alpha23)
+ (alpha12*tinfty24-alpha22*tinfty14) / (2 * (tinfty14-tinfty24)) :
c1bis:=(tinfty24*alpha11-tinfty14*alpha21) / (tinfty14-tinfty24)
- (1/2) * (tinfty13*tinfty24-tinfty14*tinfty23) / (tinfty14-
tinfty24) ^2 * (alpha12-alpha22)
- (1/3) * ( (tinfty14-tinfty24) * (tinfty24*tinfty12-tinfty14*
tinfty22) - (-tinfty23+tinfty13) * (tinfty13*tinfty24-tinfty14*
tinfty23) ) / (tinfty14-tinfty24) ^3 * (alpha13-alpha23)
-1/4 * (tinfty24*tinfty13^3-tinfty14*tinfty23^3+ (2*tinfty14+
tinfty24) * tinfty13*tinfty23^2 - (2*tinfty24+tinfty14) * tinfty13^2 *
tinfty23
- (tinfty14-tinfty24) * (2*tinfty12*tinfty24-tinfty14*tinfty22-
tinfty22*tinfty24) * tinfty13
- (tinfty14-tinfty24) * (2*tinfty22*tinfty14-tinfty14*tinfty12-
tinfty12*tinfty24) * tinfty23
+ (tinfty14-tinfty24) ^2 * (tinfty11*tinfty24-tinfty14*tinfty21) ) /
(tinfty14-tinfty24) ^4 * (alpha14-alpha24) :

c4bis:=1/4 * (alpha14*tinfty24-alpha24*tinfty14) / (tinfty14-
tinfty24) :
cvector:=Matrix(4,1,0) :
cvector[1,1]:=c4bis:
cvector[2,1]:=c3bis:
cvector[3,1]:=c2bis:
cvector[4,1]:=c1bis:

> RHSTheo:=Multiply(M0,cvector) :

```

```

RHSTheo2:=Matrix(4,1,0):
RHSTheo2[1,1]:=(tinfy24*alpha14-tinfy14*alpha24)/4:
RHSTheo2[2,1]:=(tinfy24*alpha13-tinfy14*alpha23)/3+(tinfy23*
alpha14-tinfy13*alpha24)/4:
RHSTheo2[3,1]:=(tinfy24*alpha12-tinfy14*alpha22)/2+(tinfy23*
alpha13-tinfy13*alpha23)/3+(tinfy22*alpha14-tinfy12*alpha24)
/4:
RHSTheo2[4,1]:=(tinfy24*alpha11-tinfy14*alpha21)/1+(tinfy23*
alpha12-tinfy13*alpha22)/2+(tinfy22*alpha13-tinfy12*alpha23)
/3+(tinfy21*alpha14-tinfy11*alpha24)/4:
simplify(RHSTheo-RHSTheo2);

```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(4.2)