Image and video colorization by variational approaches

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Overview

Overview

Luminance-chrominance space

Luminance :

$$Y = 0.299R + 0.587G + 0.114B$$

(U, V) are defined such that:

$$[0, 255]^3 \to [a1, a2] \times [b1, b2] \times [c1, c2]$$
$$(R, B, G) \mapsto (Y, U, V)$$

is a bijection.

- The human eye is sensitive to Y;
- If Y is a gray-scale image to colorize, the computation of U and V gives the color image.

Color image.

Gray-scale version (Y).

Manual colorization.

- State-of-the-art: Levin *et al.* 2004, Sapiro 2005, Horiuchi *et al.* 2004 ;
- Need some tedious work for the user;
- Regular results;
- Most of methods work in luminance-chrominance spaces.

Figure : Manual colorization of Levin *et al.*

The problem of exemplar-based colorization.

Source.

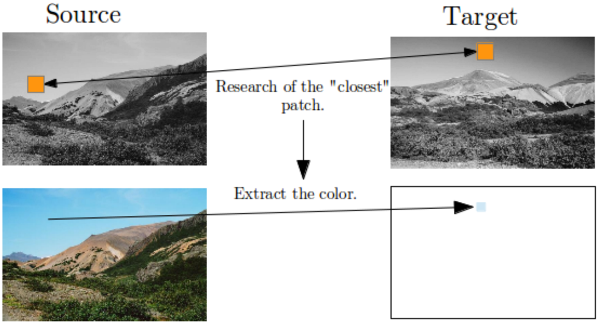
Target.

Result.

- Correspondence between textures and colors;
- Regularization of the result for a realistic image;
- A new difficulty: find an appropriate source image.

Add of a prior with textures.

- State-of-the-art: Welsh *et al.* 2002;
- No regularization;
- Problem of the choice of metric between patches;
- Method of Welsh *et al.* works in the $l\alpha\beta$ color-space.



Speed up the search.

- Sub-sampling of 200 pixels of the image on a grid.
- Use of a fast algorithm such as patchmatch (Barnes et al 2009) to find a close patch.

Add of a prior with textures.

- State-of-the-art: Charpiat *et al.* (*Lab*) 2008, Bugeau *et al.* 2013 (*YUV*), Gupta *et al.* 2013 (*YUV*).
- Two steps:
  - Search of information: candidates extraction from textures criteria;
  - Spatial regularization of colors.

Metrics.

Used criteria:

- standard-deviation: $\rho_1(p,q,P) := |\sigma^2(P_p) - \sigma^2(P_q)|$, where $\sigma(P_p)$ stands for the standard-deviation of the patch around the pixel p;
- the DFT: $\rho_2(p,q,P) := \sum_\xi \left| ||\hat{P}_p(\xi)||_2 - ||\hat{P}_q(\xi)||_2 \right|$, where $\hat{P}_p$ is the DFT of the patch around the pixel p;
- cumulative histogram: $\rho_3(p,q,P) := \sum_i |H_{P_p}(i) - H_{P_q}(i)|$, where H is the cumulative histogram of the patch around the pixel p.

Overview

Luminance-chrominance coupled model (Pierre et al 2014)

$$F(u,W) := \quad TV_{\mathcal{C}}(u) + \frac{\lambda}{2} \int_\Omega \sum_{i=1}^N w_i ||u - c_i||^2$$

$$+\chi_{u\in R} + \chi_{W\in\Delta}.$$

u: chrominance vector, to be computed.
R is a rectangle (where the chromaticity lives).
$\Delta = \left\{ w \in \mathbb{R}^N \text{ s.t. } 0 \leq w_i \leq 1 \text{ , } \forall i \in [1..N] \text{ and } \sum_{i=1}^N w_i = 1 \right\}.$
c_i are chrominance candidates.

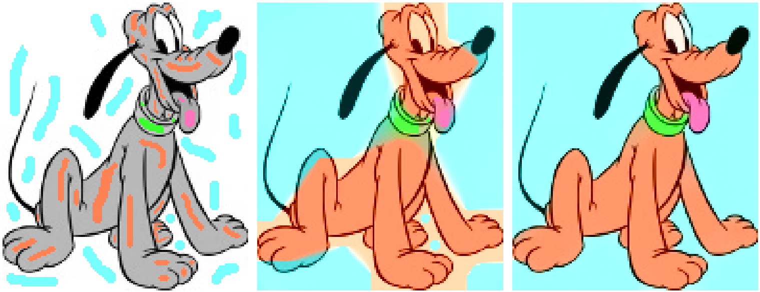
$$TV_{\mathcal{C}}(u) = \int_\Omega \sqrt{\color{red}{\gamma \partial_x Y^2 + \gamma \partial_y Y^2} + \partial_x U^2 + \partial_y U^2 + \partial_x V^2 + \partial_y V^2}$$

Intuitions about coupling.

Consider the following model:

$$F(u, W) := TV_{\mathcal{C}}(u) + \frac{\lambda}{2} \int_{\Omega} \|M(u - c)\|^2$$

With M a mask and c seeds of color put by the user.



Scribbles Without coupling With coupling

Different regularizations with different coupling:

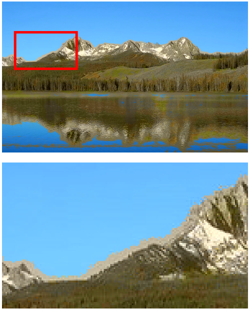
Target | Scribbles | $\gamma = 0$ | $\gamma = 1$ | $\gamma = 10$

Small γ: contours of low perimeter for chrominance channels.

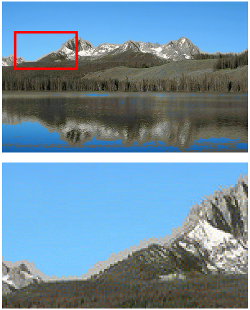
Comparison

Target image

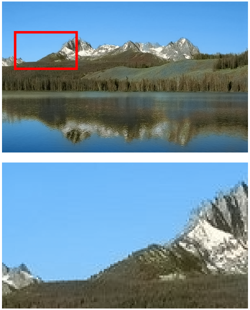
source Image



Bugeau et al



without post-processing



Our model

Comparisons

Source | Target | Our result | Gupta et al | Welsh et al

Results

Overview

Technical recalls.

Primal problem:

$$\min_{x\in X}\left[F(Kx)+G(x)\right] \ ,$$

where $G : X \to [0,+\infty]$; $F^* : Y \to [0,+\infty]$ are convex, proper, lower semi-continuous; K is a continuous linear operator.
Associated saddle point problem:

$$\min_{x\in X}\max_{y\in Y}\left[\langle Kx|y\rangle + G(x) - F^*(y)\right].$$

with F^* convex conjugate of F.

Convergence of the primal-dual algorithm to a saddle-point of the functional.

Algorithm 1 Primal-dual algorithm of Chambolle and Pock 2011.

1: Initialization τ, $\sigma > 0$, $\theta \in [0,1]$, $(x^0, y^0) \in X \times Y$ et $\overline{x}^0 = x^0$.
2: **for** $n \geq 0$ **do**
3: $\quad y^{n+1} = \mathrm{prox}_{\sigma F^*} \left(y^n + \sigma K \overline{x}^n\right)$
4: $\quad x^{n+1} = \mathrm{prox}_{\tau G} \left(x^n - \tau K^* y^{n+1}\right)$
5: $\quad \overline{x}^n = 2x^{n+1} - x^n$.
6: **end for**

$$\text{where } \mathrm{prox}_f(\tilde{u}) = \mathrm{argmin}_u \frac{\|\tilde{u} - u\|_2^2}{2} + f(u).$$

New model.

Non-convex model:

$$\min_{x\in X,\ w\in W} F(Kx) + G(x) + h(x,w) + H(w) \ ,$$

and associated saddle point problem:

$$\min_{x\in X} \min_{w\in W} \max_{y\in Y} \langle Kx|y\rangle - F^*(y) + G(x) + h(x,w) + H(w) \ ,$$

where $G : X \to [0,+\infty)$, $F^* : Y \to [0,+\infty)$, $H : W \to [0,+\infty)$ and $h : (X \times W) \to [0,+\infty)$ are proper, lower semi-continuous, F^*, G , H are convex, h is convex with respect to each variable.

Moreover $\forall w \in \mathbb{R}^n$, $G + h(.,w)$ and $\forall x \in \mathbb{R}^n$, $H + h(x,.)$ are proper.

Algorithm 2 Computing a solution.

1: **for** $n \geq 0$ **do**
2: $\quad y^{n+1} \leftarrow \text{prox}_{\sigma F^*}\left(y^n + \sigma K \overline{x}^n\right)$
3: $\quad w^{n+1} \leftarrow \text{prox}_{\rho H + \rho h(\overline{x}^n,.)}\left(w^n\right)$
4: $\quad x^{n+1} \leftarrow \text{prox}_{\tau G + \tau h(.,w^{n+1})}\left(x^n - \tau K^* y^{n+1}\right)$
5: $\quad \overline{x}^{n+1} \leftarrow 2x^{n+1} - x^n$
6: **end for**

Theorem (Pierre *et al.* 2014)

Assume $\tau\sigma\|K\|^2 < 1$ *and* $\rho > 0$.

- *The sequence* (x^n, y^n, w^n) *generated by the algorithm is bounded.*
- $x^{n+1} - x^n \to 0$, $y^{n+1} - y^n \to 0$, *and* $w^{n+1} - w^n \to 0$.
- *If the cluster points of* (x^n, y^n, w^n) *are isolated, then* (x^n, y^n, w^n) *converges to a fixed point.*

Minimization algorithm.

Algorithm 3 Applied to colorization.

$W \leftarrow 1/N$

$u^0 \leftarrow \sum_i w_i c_i$

$Z^0 \leftarrow \nabla u^0$

for $n \geq 1$ **do**

$$Z^{n+1} \leftarrow P_B \left(Z^n + \sigma \nabla \overline{u}^n \right)$$

$$W^{n+1} \leftarrow P_\Delta \left(W^n - \rho \left(\left(\| \overline{u}^n - c_i \|^2 \right)_i \right) \right)$$

$$u^{n+1} \leftarrow P_{\mathcal{R}} \left(\frac{u^n + \tau \left(\operatorname{div}(Z^{n+1}) + \lambda \sum_i w_i^{n+1} c_i \right)}{1 - \delta \lambda} \right)$$

$$\overline{u}^{n+1} \leftarrow 2u^{n+1} - u^n$$

end for

- P_Δ projection onto the simplex;
- $P_{\mathcal{R}}$ is a projection onto a rectangle.

Convergence

$$F(u,W) := \quad TV_{\mathcal{C}}(u) + \frac{\lambda}{2}\int_\Omega \sum_{i=1}^N w_i ||u - c_i||^2$$

$$+\chi_{u\in R} + \chi_{W\in\Delta} + \alpha\|w\|_2^2.$$

Theorem (Pierre *et al.* 2014)

Assume $\tau\sigma\|K\|^2 < 1$ *and* $\rho > 0$. *Assume the candidates* C *are all different. Then if* $\alpha > 0$ *is small enough, the sequence* (w^n) *generated by the previous algorithm converges to* w^*. *Moreover,* w^* *belongs to the finite set:*

$$\mathcal{W} := \{(1,0,\ldots,0),\ldots,(1/2,1/2,0,\ldots,0),\ldots,(1/N,\ldots,1/N)\}.$$

Convergent algorithm

Algorithm 4 Minimization of the functional

for until convergence of w^n to w^* **do**

$\quad y^{n+1} \leftarrow \text{prox}_{\sigma F^*}\left(y^n + \sigma K \overline{x}^n\right)$

$\quad w^{n+1} \leftarrow \text{prox}_{\rho H + \rho h(\overline{x}^n,.)}\left(w^n\right)$

$\quad x^{n+1} \leftarrow \text{prox}_{\tau G + \tau h(.,w^{n+1})}\left(x^n - \tau K^* y^{n+1}\right)$

$\quad \overline{x}^{n+1} \leftarrow 2x^{n+1} - x^n$

end for

for until convergence of (x^n, y^n) to (x^*, y^*) **do**

$\quad y^{n+1} \leftarrow \text{prox}_{\sigma F^*}\left(y^n + \sigma K \overline{x}^n\right)$

$\quad x^{n+1} \leftarrow \text{prox}_{\tau G + \tau h(.,w^*)}\left(x^n - \tau K^* y^{n+1}\right)$

$\quad \overline{x}^{n+1} \leftarrow 2x^{n+1} - x^n$

end for

$$\begin{aligned}&\operatorname{prox}_{\rho H+\rho h(\overline{x}^n,.)}(w^n)\\ =\ &\operatorname{argmin}_w \|\tilde{w^n}-w|_2^2+\rho\left(\int_\Omega \sum_i w_i\|x-c_i\|_2^2+\alpha\|w\|_2^2+\chi_\Delta(w)\right)\end{aligned}$$

becomes, if $\rho \to +\infty$:

$$\operatorname{argmin}_W \int_\Omega \sum_i w_i\|x-c_i\|_2^2+\alpha\|w\|_2^2+\chi_\Delta(w).$$

$\Longrightarrow w^* \in \mathcal{W}$ with

$\mathcal{W} := \{(1,0,\dots,0),\dots,(1/2,1/2,0,\dots,0),\dots,(1/N,\dots,1/N)\}.$

Fast algorithm

$$\min_w \min_x \max_y \langle Kx|y\rangle - F^*(y) + \int_\Omega \sum_i w_i \|x-c_i\|_2^2 + \chi_{\mathcal{W}}(w) + \chi_R(x).$$

Algorithm 5 Minimization

1: $W = 1/8$ and $x = \sum_i w_i c_i$.
2: $y \leftarrow \nabla u$
3: **for** $n \geq 0$ **do**
4: $\quad y \leftarrow P_B\left(y + \sigma \nabla x^n\right)$
5: $\quad x^{n+1} \leftarrow P_{\mathcal{R}}\left(\dfrac{x^n + \tau\left(\operatorname{div}(y) + \lambda S\right)}{1+\tau\lambda}\right)$
6: **end for**

where $S = \sum_i w_i^* c_i$ stands for the closest candidate to x^n.

Overview

No current method is effective all situations:

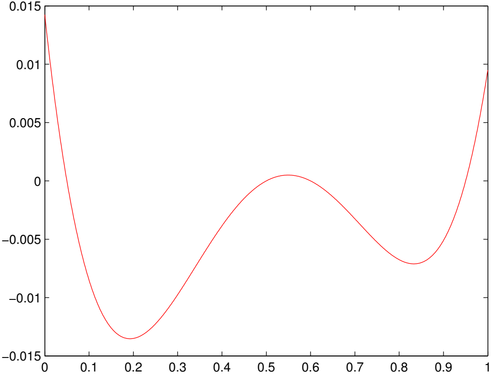
- The selection of an image for the exemplar-based method is complex.
- The manual colorization is long and tedious.
- The exemplar-based method often provides good result, but small defects are very common.

Previous functional:

$$F(u,W) := \quad TV_{\mathcal{C}}(u) + \frac{\lambda}{2}\int_\Omega \sum_{i=1}^N w_i ||u - c_i||^2$$

$$+\chi_{u\in R} + \chi_{W\in\Delta}.$$

Use of the non-convexity.



Gradient descent algorithm: $x_{n+1} \leftarrow x_n - \gamma \nabla f(x_n)$.
The result depends of the initialization.

W is initialized as the inverse of the geodesic distance for the candidate corresponding to the scribble.

Initial scribble Geodesic distance. Diffusion.

Example of diffusion of color with geodesic distance.

Choice of the initialization.

Source.

Exemplar-based.

With one scribble.

3 scribbles.

Final result.

Choice of the initialization.


Source.


Target.


With example.


Scribbles.

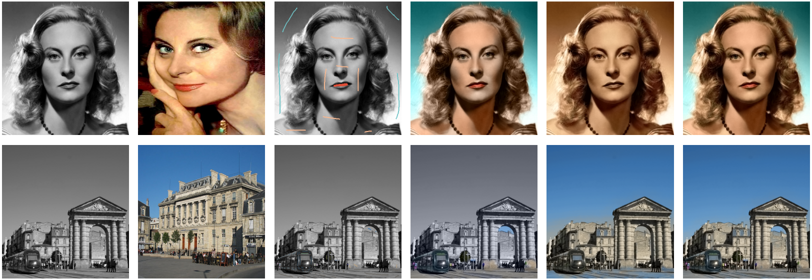

Scribbles.


Scribbles.


Scribbles.

Results.

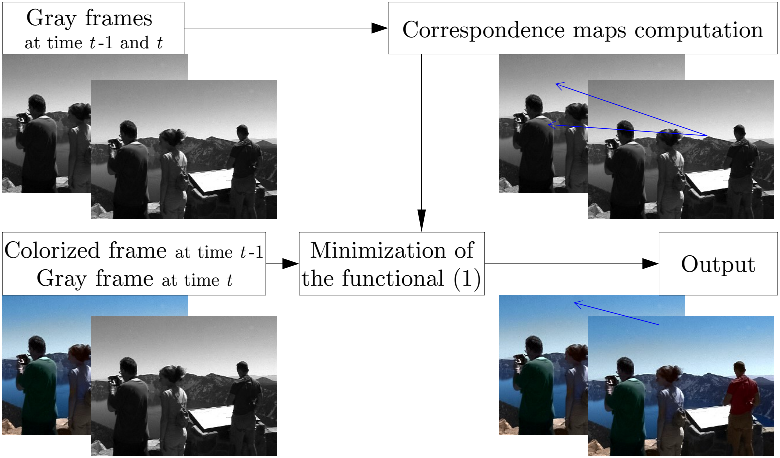
Target Source Scribbles Manual Exemplar Both



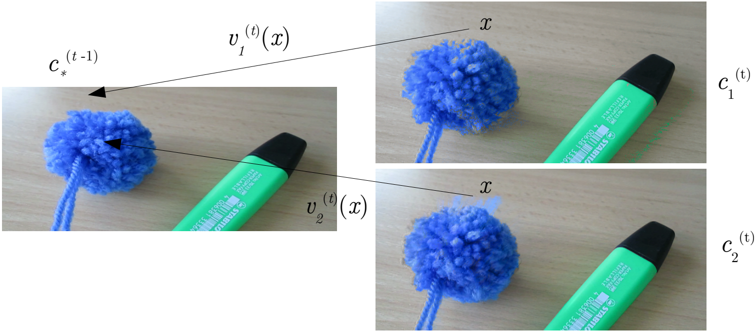
Target Source Scribbles Manual Exemplar Both

Overview

Overview of the Approach



Color propagation



From a given result at time $t-1$, two possible propagation results are provided at time t from the two maps.

PatchMatch

TVL1 optical flow

Propagation result

Fusion of correspondence maps

- advantages of TVL1 optical flow and PatchMatch;
- reducing the number of propagation mistakes.

Basic data:

- Chrominance c_1 and correspondence v_1 from PatchMatch;
- c_2 and v_2 from TV-L1 optical flow.

Luminance-chrominance Propagation Model

Propagation functional:

$$(\hat{u}^{(t)}, \hat{w}^{(t)}) = \mathrm{argmin}_{u^{(t)}, w^{(t)}} \lambda \int_\Omega \sum_{i=1}^{2} w_i^{(t)}(x) \| u^{(t)}(x) - c_i^{(t)}(x) \|_2^2 \, dx$$
$$+ \alpha \, \mathrm{TV}_{\mathcal{C}}(u^{(t)}) + \beta \, \mathrm{TV}(v^{(t)}) + \chi_{v^{(t)} = w_1^{(t)} v_1^{(t)} + w_2^{(t)} v2^{(t)}} + \chi_{\mathcal{R}}(u) + \chi_{\Delta}(w), \tag{1}$$

where $\mathrm{TV}_{\mathcal{C}}(u)$ is the coupled total variation:

$$\mathrm{TV}_{\mathcal{C}}(u) = \int_\Omega \sqrt{\gamma |\nabla Y|^2 + |\nabla U|^2 + |\nabla V|^2}.$$

$u^{(t)}$: chrominance channels (U, V) at time t; $w^{(t)}$: auxiliary weight variable; $\mathrm{TV}(v^{(t)})$ enforces the regularisation of the propagation map; Δ is the probability simplex.

The Model is rewritten in the primal-dual form:

$$\min_{u^{(t)}\in\mathbb{R}^{3\times N\times M}, w^{(t)}\in\mathbb{R}^{N\times M}} \max_{p\in\mathbb{R}^{6\times N\times M}, z\in\mathbb{R}^{4\times N\times M}}$$

$$\lambda \sum_{x\in\mathbb{R}^{N\times M}} w(x)\|u(x)-c_1(x)\|_2^2 + (1-w(x))\|u(x)-c_2(x)\|_2^2$$

$$+\langle p(x)|\nabla u\rangle_{\mathbb{R}^{6\times N\times M}} - \chi_{B(0,\alpha)^{N\times M}}(p)$$

$$+\langle Aw|z\rangle_{\mathbb{R}^{4\times N\times M}} + \langle \nabla v_2|z\rangle_{\mathbb{R}^{4\times N\times M}} - \chi_{B(0,\beta)^{N\times M}}(z)$$

$$+\chi_{\mathcal{R}^{N\times M}}(u) + \chi_{[0,1]^{N\times M}}(w).$$

where $Aw = (v_1 - v_2)\otimes \nabla w + w(\nabla v_1 - \nabla v_2)$.

A General Model

Let us consider the general model :

$$\min_{u\in\mathcal{U},w\in\mathcal{W}}\ \max_{p\in\mathcal{P},z\in\mathcal{Z}}\ h(u,w) + \langle p|Ku\rangle - F^*(p) + \langle Aw|z\rangle - J^*(z) + G(u) + H(w)\ ,$$

where $G:\mathcal{U}\to[0,+\infty)$, $F^*:\mathcal{P}\to[0,+\infty)$, $J^*:\mathcal{Z}\to[0,+\infty)$, $H:\mathcal{W}\to[0,+\infty)$ and $h:(\mathcal{U}\times\mathcal{W})\to[0,+\infty)$ are proper lower semi-continuous functions. F^*, J^*, G , H are convex, h is convex *w.r.t.* each of its variables separately. K and A are a continuous linear mapping.

A General Algorithm

Primal-dual like algorithm:

1: **for** $n \geq 0$ **do**
2: $\quad p^{n+1} \leftarrow \text{prox}_{\sigma_u F^*}\left(p^n + \sigma_u K \overline{u}^n\right)$
3: $\quad z^{n+1} \leftarrow \text{prox}_{\sigma_w J^*}\left(z^n + \sigma_w A \overline{w}^n\right)$
4: $\quad w^{n+1} \leftarrow \text{prox}_{\tau_w H + \tau_w h(u^{n+1},.)}\left(w^n - \tau_w A^* z^{n+1}\right)$
5: $\quad u^{n+1} \leftarrow \text{prox}_{\tau_u G + \tau_u h(.,w^{n+1})}\left(u^n - \tau_u K^* p^{n+1}\right)$
6: $\quad \overline{u}^{n+1} \leftarrow 2u^{n+1} - u^n$
7: $\quad \overline{w}^{n+1} \leftarrow 2w^{n+1} - w^n$
8: **end for**

Parameters τ_u, τ_w, σ_u and σ_w are the time steps.

Properties of the Algorithm

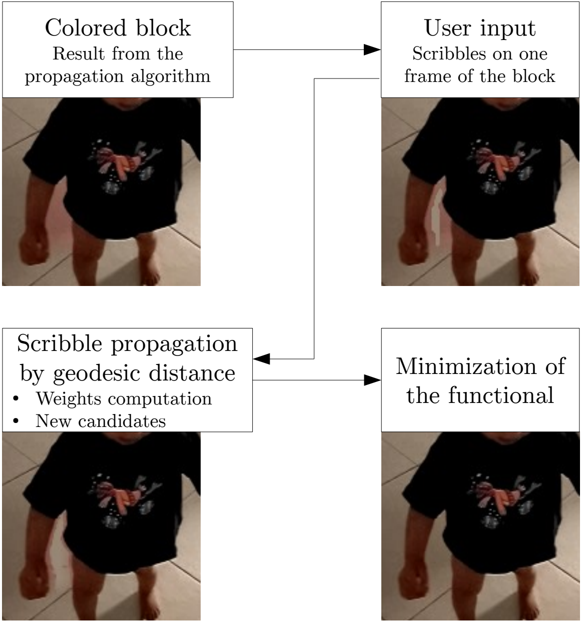
Theorem (Pierre *et al.*)

Let $L = \|K\|$, $Q = \|A\|$, *choose* $\tau_u \sigma_u L^2 < 1$, $\tau_w \sigma_w Q^2 < 1$. *Assume that* $\mathcal{U}$, $\mathcal{P}$, $\mathcal{W}$*and* $\mathcal{Z}$ *are of finite dimension.*

- *Then, the sequence* (u^n, p^n, w^n, z^n) *is uniformly bounded.*
- *There exists a cluster point which is a fixed-point of the Algorithm.*

Under additional technical hypothesis, the sequence converges.

Overview of the Correction Model



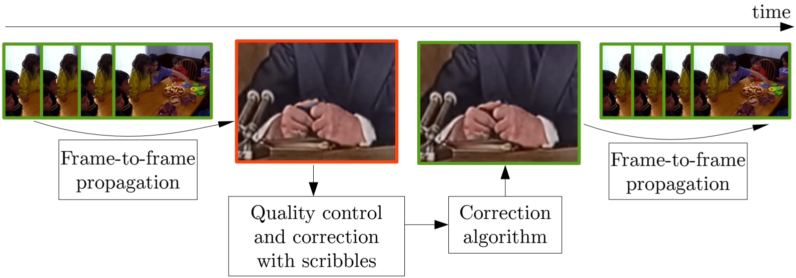
$$(\hat{u}^{(t)}, \hat{w}^{(t)}) = \operatorname{argmin}_{u \in \mathbb{R}^{\Omega\times[0,n]\times 2}, w \in \mathbb{R}^{\Omega\times[0,n]\times 2}} \alpha TV_{[0,n]}(u)$$
$$+ \lambda \int_{\Omega\times[0,n]} \sum_{i=1}^{2} w_i^{(t)}(x) \| u^{(t)}(x) - \tilde{c}_i^{(t)}(x)\|_2^2 \, dx + \chi_{\mathcal{B}}(u) + \chi_{\mathcal{E}}(w),$$

where

$$TV_{[0,n]}(u) = \int_{\Omega\times[0,n]} \left(\|\Lambda\nabla U\|_2^2 + \|\Lambda\nabla V\|_2^2 + \|\gamma\Lambda\nabla Y\|_2^2 \right)^{\frac{1}{2}},$$

with $\nabla = (\partial_x, \partial_y, \partial_t)$, $\Lambda := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu \end{pmatrix}$, and γ a coupling parameter.

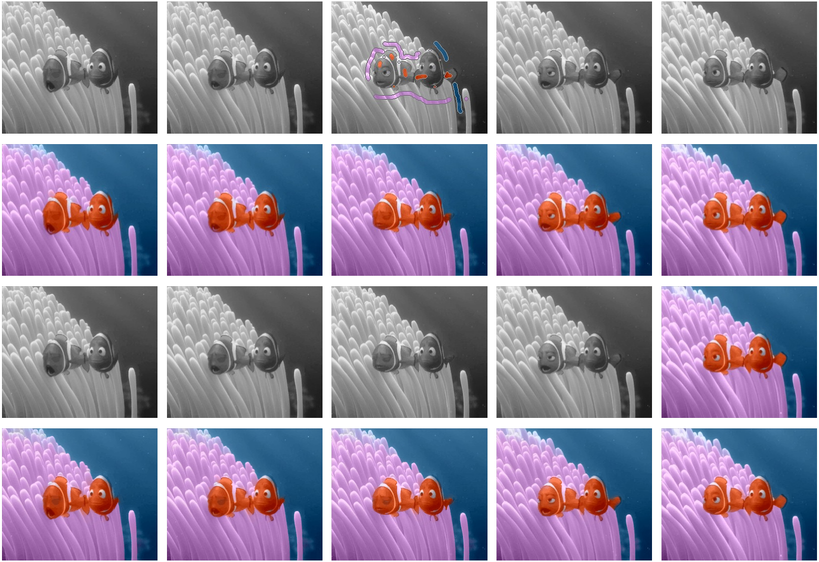
Workflow


time
Frame-to-frame propagation
Quality control and correction with scribbles
Correction algorithm
Frame-to-frame propagation

Results

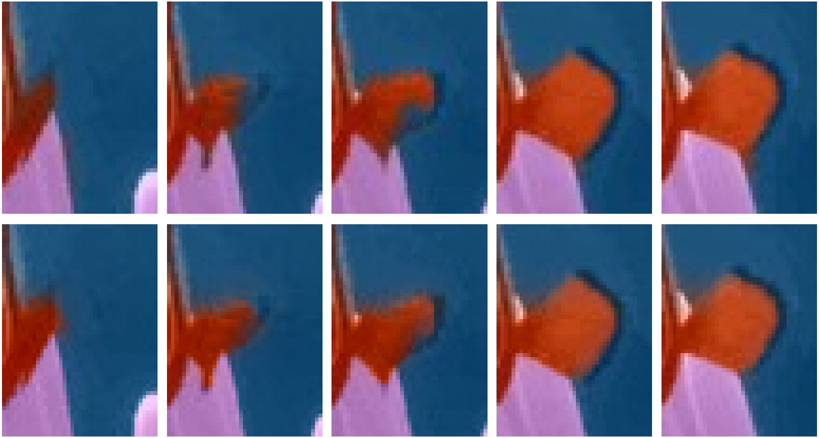
Top: Yatziv and Sapiro 2006; down: ours.

Results



Top: Yatziv and Sapiro 2006; down: ours.

Results



Top: Yatziv and Sapiro 2006; down: ours.

Results

Top: Levin et al 2004 (more then 50 well chosen scribbles); down: ours (1 scribble).

Results

Top: Levin et al 2004 (more then 400 well chosen scribbles); down: ours (20 coarse scribbles).

Results

See videos on player.

Questions ?

More details at:
http://www.math.u-bordeaux.fr/∼jaujol/