

# Level lines based disocclusion

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## Abstract

*Object recognition, robotic vision, occluding noise removal or photograph design require the ability to perform disocclusion. We call disocclusion the recovery of hidden parts of objects in a digital image by interpolation from the vicinity of the occluded area. It is shown in this paper how disocclusion can be performed by means of level lines structure, which offers a reliable, complete and contrast-invariant representation of image, in contrast to edges. Level lines based disocclusion yields a solution that may have strong discontinuities, which is not possible with PDE-based interpolation. Moreover, the proposed method is fully compatible with Kanizsa's theory of "amodal completion".*

## 1 Introduction

Disocclusion or "amodal completion" is a very common process in human vision. In a natural scene, an object is seldom totally visible. It is generally partially hidden by other objects. But our perception is under certain geometric conditions able to "reconstruct" the whole object by interpolating the missing part. In the example illustrated in Figure 1, one generally "sees" the same black rectangle in both drawings, despite the fact that this rectangle is never totally visible.

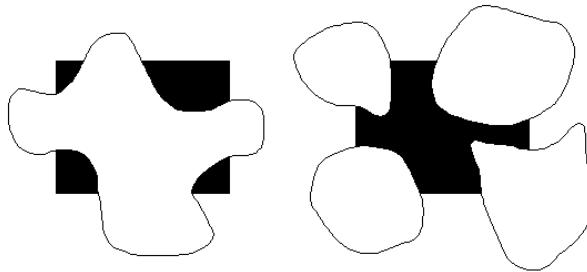


Figure 1: The same rectangle can be seen in both figures by amodal completion.

This ability of human vision has been widely studied by psychophysicists, particularly by Gaetano

Kanizsa [5]. It appears that continuation of objects boundaries plays a central role in the disocclusion process. This continuation is performed between T-junctions, which are points where image edges form a "T". We call "amodal completion" the process by which our perception extends visible edges "behind" occluding objects. According to psychophysicists, the continuation process is such that restored edges must be as smooth and straight as possible and the shapes as convex as possible.

## 2 Disocclusion as a variational process

In [8], Nitzberg, Mumford and Shiota present a method for detecting and removing occlusions in a fixed image within the framework of a segmentation and depth computing algorithm. Their method is based on the detection of image edges and T-junctions followed by a variational continuation process. Roughly speaking, this process consists in linking together T-junctions of approximatively the same level by a new edge with minimum length and curvature. This method appears however to be very fragile. It can only be applied to highly segmented images with few T-junctions and few possible continuations between them, so that disocclusion of natural images is not directly possible. Moreover, it is now well-known that edges are not a reliable information, are very sensitive to contrast changes and do not offer a complete representation of the image.

The method we propose in this paper is an adaptation of the Mumford, Nitzberg and Shiota variational continuation framework to the level lines structure, which is more precise and reliable than edges.

## 3 Level lines continuation

Recent works [3] have emphasized the importance of level lines for image understanding and representation. Let  $u(x)$  denotes the gray level of an image  $u$  at point  $x$ . We define level lines as boundaries of upper level sets, defined at each gray level  $\lambda$  by  $X_\lambda u = \{x, u(x) \geq \lambda\}$ . An image and some of

its level lines are shown in Figure 2. In contrast to edge representation, the family of level lines is a complete representation of  $u$ , from which  $u$  can be reconstructed [1, 3]. In addition, this representation is in-

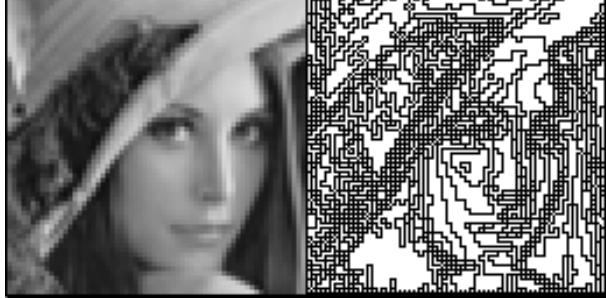


Figure 2: An image and its level lines multiple of 15.

variant with respect to any increasing contrast change. This point is crucial since, according to Gestalt school (Wertheimer), human vision is essentially sensitive to the only ordering of gray levels in an image. The intensity difference between two pixels is not a reliable characterization of an image since it arbitrarily depends on the used sensor as well as illumination conditions. To be reliable, any natural image processing must involve the only ordering of gray levels which remains identical by increasing contrast change. Unlike the edge representation, the level lines representation allows to perform the contrast invariance.

In the following, we denote by  $\Gamma$  the part of the image plane  $\Omega$  occupied by the occluding object. We shall assume that  $\Gamma$  has no hole and we denote by  $\partial\Gamma$  its boundary which we assume to be a simple curve. Our problem is to find some interpolant  $v$  of  $u$  which coincides with  $u$  on  $\partial\Gamma$ . It was shown in [2] that every interpolation method yielding a smooth interpolant reduces under some reasonable stability and invariance assumptions to the only interpolating operator  $D^2v(\frac{Dv}{|Dv|}, \frac{Dv}{|Dv|}) = 0$ . This method can therefore not be used for edges continuation. In addition, this operator does not allow to recover a strongly discontinuous function like most natural images are.

The method we present here allows to recover functions with strong discontinuities. Let us describe an image as a function  $u$  of bounded variation (BV) such that  $\int_{\Omega} |u| dx < \infty$  and the total variation of  $u$

$$TV_{\Omega}(u) = \sup \left\{ \int_{\Omega} u \operatorname{div} \phi dx : \phi \in C_c^1(\Omega, \mathbb{R}^2), |\phi| \leq 1 \right\}$$

is finite. The reader may wish to refer to [9] for more details.  $TV_{\Omega}(u)$  may also be denoted as  $\int_{\Omega} |Du| dx$ .

A set  $E$  is said of finite perimeter if the quantity  $P(E) = TV_{\Omega}(\mathbb{1}_E)$  is finite, where  $\mathbb{1}_E$  denotes the characteristic function of  $E$ . If  $E$  has Lipschitz boundary then  $P(E) = H^1(\partial E)$  is interpreted as the length of the boundary of  $E$ . The Coarea formula states that

$$TV_{\Omega}(u) = \int_{-\infty}^{\infty} P(X_{\lambda}u) d\lambda$$

and thus establishes a connection between the total variation of  $u$  and the length of its level lines.

Following Kanizsa and Mumford *et al.*, it seems reasonable to perform the continuation of level lines joining the occluding object at T-junctions by minimizing the functional

$$\int_{\Omega} |Dv|(1 + |\operatorname{curv} v|) dx$$

subject to the constraint  $v|_{\Gamma^c} = u|_{\Gamma^c}$

where  $\operatorname{curv} v = \operatorname{div} \frac{Dv}{|Dv|}$ . In view of Coarea formula, the first term of the functional forces restored level lines to have minimum length when the second term forces the angle total variation to be minimum along the lines. Therefore, the solution to this problem consists of a function of bounded variation whose level lines within the occlusion  $\Gamma$  are geodesic paths joining T-junctions with minimum angle total variation. The reader may wish to refer to [7] for more details about the functional relaxation within a subset of the BV-space.

#### 4 A practical algorithm

Let us now examine how disocclusion can be performed on a digital image. It must be emphasized that this method can be applied to any image without preprocessing and independently from the number of T-junctions. Due to the properties of level lines, there is an even number of lines arising at  $\partial\Gamma$  (Figure 3). These level lines can be continued with respect to the same rules holding for amodal completion. Each level line is oriented according to the direction of the image gradient, yielding two possible orientations, positive and negative. At each level, the number of lines with positive orientation equals the number of lines with negative orientation. Let  $L_1$  and  $L_2$  be two lines arising at  $\partial\Gamma$ .  $L_1$  and  $L_2$  can be connected only if they have the same level and the same orientation. Since level lines can never cross, a global disocclusion will be valid if and only if this condition is satisfied. Our method makes an optimal connection of all pair of level lines. It must be emphasized that at least one (non optimal) connection exists which is given by the initial occluded image.

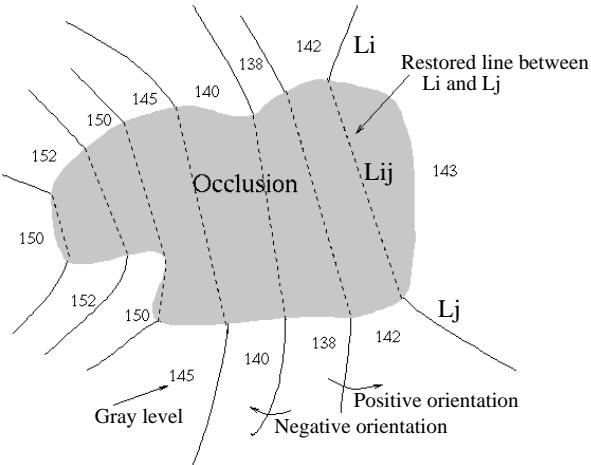


Figure 3: An occlusion and a possible connection of level lines two by two.

Like in the previous section, we compute the optimal disocclusion by minimizing the cost

$$\mathcal{C} = \sum \int_{L_{i,j}} (1 + |\sigma(s)|) ds$$

where  $\int_{L_{i,j}} |\sigma(s)| ds$  denotes the angle total variation along each restored level line  $L_{i,j}$  and the sum goes over all restored level lines. The impossibility for level lines to intersect introduces a spatial causality which makes possible the use of dynamic programming. Indeed, the computation of an optimal set of connections within the arc  $[t_1, \dots, t_{2n}]$  where  $t_1, \dots, t_{2n}$  are T-junctions, consists of an optimal completion of the optimal connections within the arcs included in  $[t_1, \dots, t_{2n}]$ . Geodesic paths joining compatible T-junctions are computed by means of Hershberger and Snoeyink's funnel algorithm [4] after a triangulation of the polygonal line to which the occlusion boundary reduces. Once the best level lines configuration has been computed, the disocclusion process is completed by means of a simple geodesic propagation within the occlusion of the gray level values of the restored level lines.

## 5 Experimental results

We present below (Figures 4, 5, 6) performances of our algorithm for different types of problems involving occlusion. The first one deals with occlusive (destructive) noise. Occlusions are supposed to be all image subsets that are not invariant with respect to a contrast-invariant and idempotent denoising filter, the so-called *grain filter*, that was described in [6]. Fig-

ure 5 illustrates that our disocclusion algorithm can be used for old photographs restoration. It is here associated with the grain filter which reduces occlusive noise within image. The final experiment (Figure 6) shows that in contrast to PDE-interpolation, our level lines based disocclusion allows to recover image singularities in a way compatible with Kanizsa's perception theory.

In [7] we shall present a method for performing a disocclusion by minimizing

$$\int_{\Omega} |Dv|(1 + |\text{curv } v|^p) dx, \quad p > 1$$

subject to the constraint  $v|_{\Gamma^c} = u|_{\Gamma^c}$

which allows to restore smooth level lines yielding a disocclusion even closer to what the human perception achieves.

## Acknowledgments

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Figure 4: **Left** – Noisy image (impulse noise, frequency = 10%)  
**Right** – Occlusion detection is performed with the grain filter (see [6]); occlusions are non-invariant sets. Disocclusion is then achieved with our level lines based algorithm.

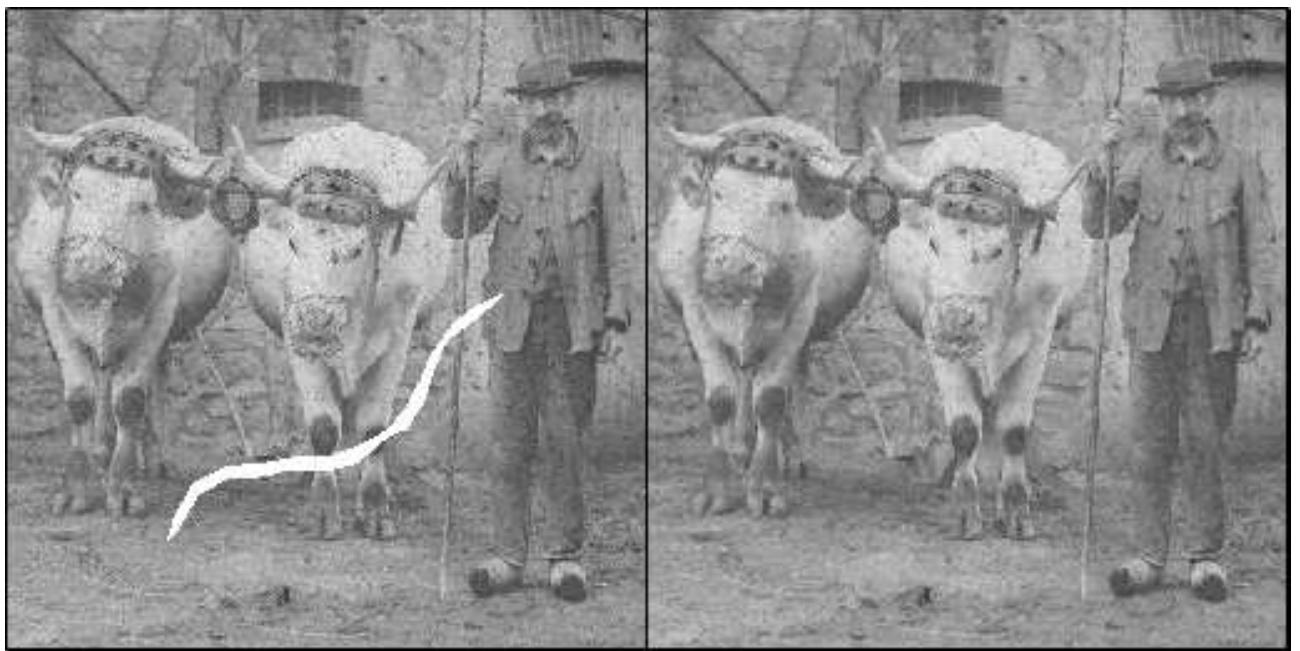


Figure 5: **Left** – An old damaged and noisy photograph (occlusion is in white).  
**Right** – Result of image denoising by the grain filter and disocclusion by our level lines based algorithm.



**Figure 6:**

**Above** Original image where occlusions are in white.

**Below-left** Disocclusion performed by solving equation  $D^2v(\frac{Dv}{|Dv|}, \frac{Dv}{|Dv|}) = 0$  (see [2]). Singularities cannot be restored but regular parts of image are well recovered.

**Below-right** Disocclusion performed by our level lines based algorithm. The singularities are well restored.