UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Math 181

Spring 2007
Group F1

Midterm 1.

Friday, Feb. 23.

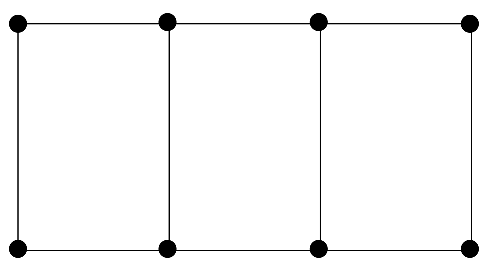
No documents allowed. Mobile phones, mp3 players, etc., are also forbidden. The one and only piece of equipment you may use is a basic calculator- and you won't need it.

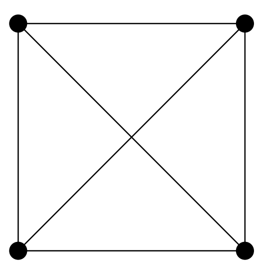
NAME ____

1. (a) Draw a graph with vertices A, B, C and D in which the valence of vertices A and D is 3 and the valence of vertices B and C is 2.

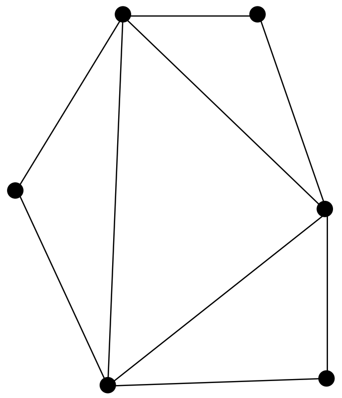
(b) Is it possible to draw a graph on the same vertices in which A, B and C have valence 2 and D has valence 3 ? (explain).

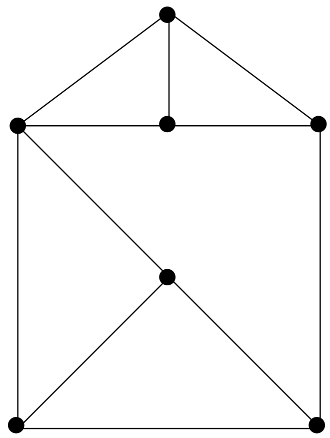
2. For each of the graphs below, determine the minimal number of edges that need to be removed to disconnect it.



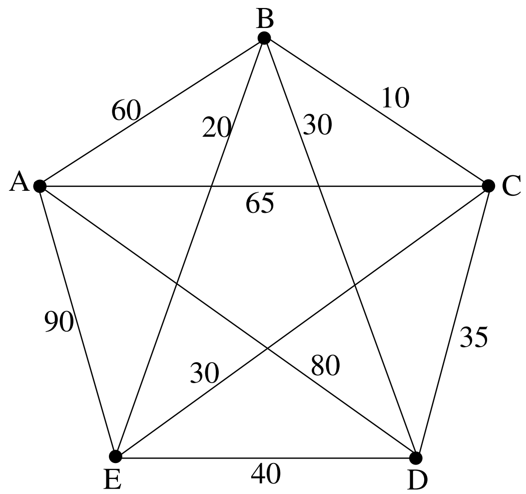


3. For each of the graphs below, say whether it admits an Euler circuit or not. If it does, draw an Euler circuit on the graph ; if it doesn't, find an efficient Eulerization of the graph and use it to obtain a circuit that reuses a minimum number of edges.





4.



a) For the graph above, find the hamiltonian circuit obtained by applying the nearest neighbor algorithm, starting at A. What is the cost of that circuit ?

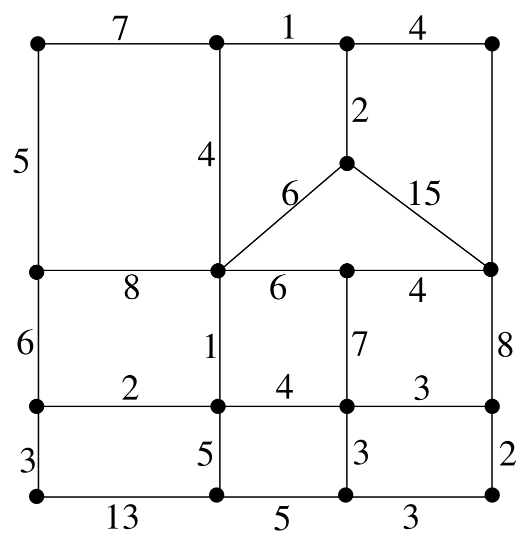
b) Same question, replacing the nearest-neighbor algorithm by the sorted-edges algorithm.

c) How many hamiltonian circuits would one have to consider in order to apply the brute force method here ?

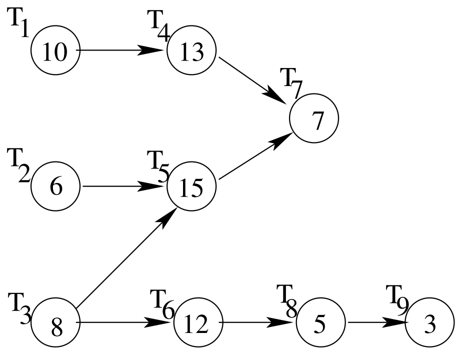
5.a) In some states, license plates use a mixture of letters and numerals. How many possible plates could be constructed using three letters followed by two numerals ?

b) You own a chain of nine appartment complexes (including the one you live in), and you plan to visit each of your properties. If it takes 1/2 minute to compute the total length of a tour, how long will it take (in minutes) to apply the brute force algorithm to find the optimal tour ?

6. Apply Kruskal's algorithm to the graph below.



7. Consider the order-requirement digraph below.



(a) What is (are) the critical path(s) ?

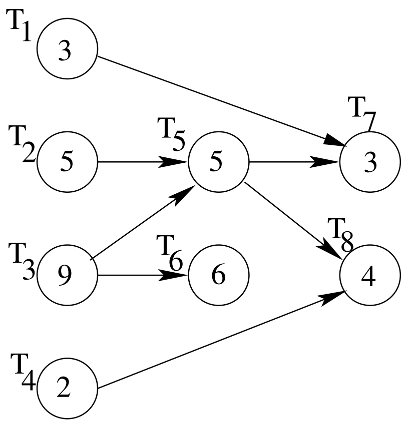
(b) Give an estimate of the minimal amount of time needed to finish the job in the following situations :
(i) You don't know the number of processors.

(ii) There is one processor.

(iii) There are two processors

(iv) There are three processors.

8. For the order-requirement digraph below, find the schedules (on two processors) obtained by applying the critical-path scheduling method and the decreasing-time list scheduling method.



9. You must pack the following weights into bins that can hold no more than 9lbs : 5lbs, 7lbs, 1lb,2lbs, 4lbs, 5lbs, 1lb, 1lb, 3lbs, 6lbs, 2lb, 3lbs, 4lbs.

(a) Use the next-fit algorithm to pack the weights ; represent your solution below

(b) Same question, using this time the first-fit decreasing algorithm

(c) Again the same question, using the worst-fit decreasing algorithm.

10. True or false ?

(a) A heuristic algorithm does not necessarily produce optimal results.

(b) The path produced by the sorted-edges algorithm when solving the traveling salesman problem may be dependent on the starting city.

(c) A spanning tree of a graph must contain every edge of a graph.

(d) When scheduling tasks using the list-processing algorithm, decreasing the time required by each task may increase the completion time.

(e) The worst-fit algorithm never uses more boxes than the first-fit algorithm.