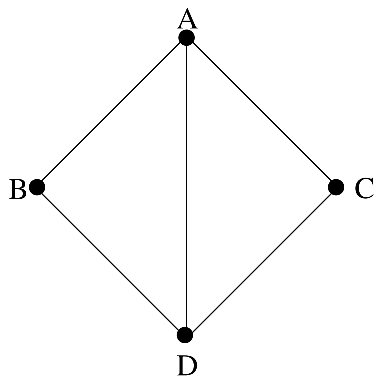
University of Illinois at Urbana-Champaign — Spring 2007
Math 181 — Group F1

Midterm 1 : Correction.

Friday, Feb. 23.

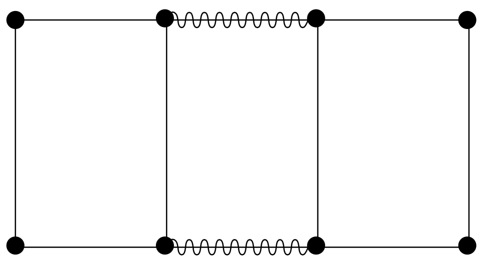
1. (a) Draw a graph with vertices A, B, C and D in which the valence of vertices A and D is 3 and the valence of vertices B and C is 2.



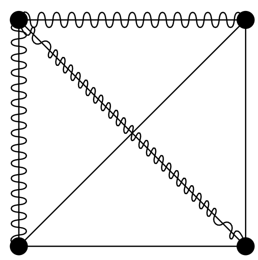
(b) Is it possible to draw a graph on the same vertices in which A, B and C have valence 2 and D has valence 3 ? (explain).

Answer. It is impossible, because the number of odd-valent vertices is always even.

2. For each of the graphs below, determine the minimal number of edges that need to be removed to disconnect it.

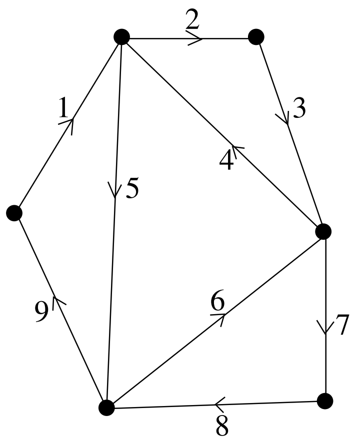


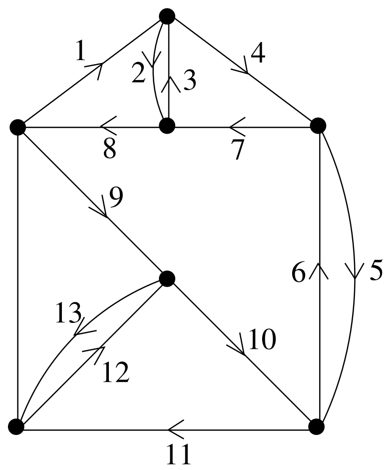
One must remove 2 edges to disconnect this graph

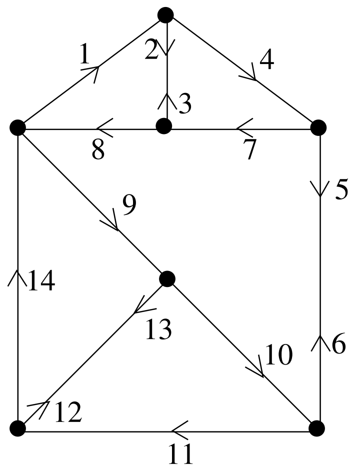


Removing three edges is needed in order to disconnect this graph.

3. For each of the graphs below, say whether it admits an Euler circuit or not. If it does, draw an Euler circuit on the graph ; if it doesn't, find an efficient Eulerization of the graph and use it to obtain a circuit that reuses a minimum number of edges.



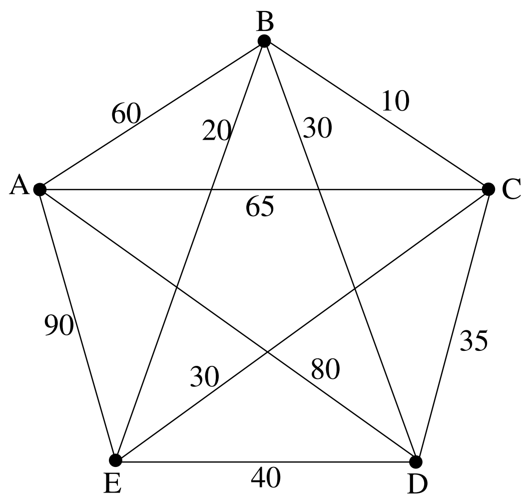




This graph admits an Euler circuit, as verified by the one that is represented above.

This graph doesn't have any Euler circuit; on the left you see an Eulerization with three added vertices (which is optimal since there are six odd−valent vertices in the graph) and an Euler circuit in the Eulerized graph;
on the right you see a circuit that covers all edges while reusing a minimal number of them, obtained by "squeezing" the Euler circuit from the Eulerized graph.

4.



a) For the graph above, find the hamiltonian circuit obtained by applying the nearest neighbor algorithm, starting at A. What is the cost of that circuit ?
Answer. One obtains the circuit ABCEDA, the cost of which is $60 + 10 + 30 + 40 + 80 = 220$.

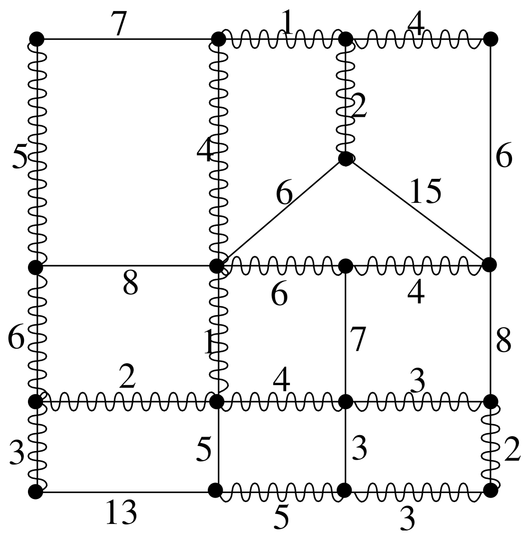
b) Same question, replacing the nearest-neighbor algorithm by the sorted-edges algorithm.
Answer. This time one obtains the circuit AEBCDA, the cost of which is $10 + 35 + 80 + 90 + 20 = 235$.

c) How many hamiltonian circuits would one have to consider in order to apply the brute force method here ?
Since there are 5 vertices, the number of hamiltonian circuits in this graph (the complete graph with 5 vertices) is $(5 - 1)!/2 = 4!/2 = 12$.

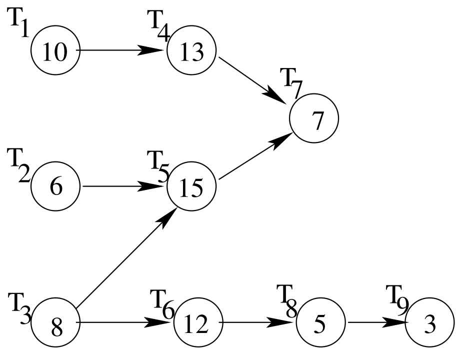
5.a) In some states, license plates use a mixture of letters and numerals. How many possible plates could be constructed using three letters followed by two numerals ?
Answer. There are $26 \times 26 \times 26 \times 10 \times 10$ different possibilities.

b) You own a chain of nine appartment complexes (including the one you live in), and you plan to visit each of your properties. If it takes 1/2 minute to compute the total length of a tour, how long will it take (in minutes) to apply the brute force algorithm to find the optimal tour ?
Answer. The total number of tours is $(9 - 1!)/2 = 8!/2$. Since each tour requires a half-minute of computation time, applying the brute force algorithm in this situation will take $(1/2) \times (8!/2)$ minutes, or 10080 minutes.

6. Apply Kruskal's algorithm to the graph below.

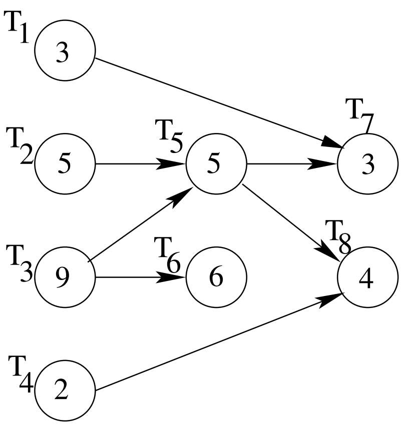


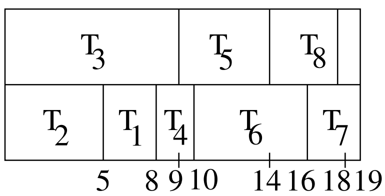
7. Consider the order-requirement digraph below.



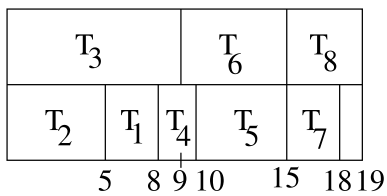
(a) What is (are) the critical path(s) ?
Answer There are two critical paths here : $T_1T_4T_7$ qnd $T_3T_5T_7$.
(b) Give an estimate of the minimal amount of time needed to finish the job in the following situations :
(i) You don't know the number of processors.
Answer. If we don't know the number of processors, the best estimate we can give is that the job will require at least 30 minutes (length of the critical path).
(ii) There is one processor.
Answer. With one processor, the minimal amount of time is equal to the total task time, which is 79 minutes here.
(iii) There are two processors
Answer. Total task time divided by 2 is $79/2 = 39.5$ minutes, so the minimal amount of time needed is at least 40 minutes.
(iv) There are three processors.
For three processors, we get $79/3 = 26.333\ldots$; since this is lower than the length of a critical path, we again can only say in this case that the job will require at least 30 minutes.

8. For the order-requirement digraph below, find the schedules (on two processors) obtained by applying the critical-path scheduling method and the decreasing-time list scheduling method.





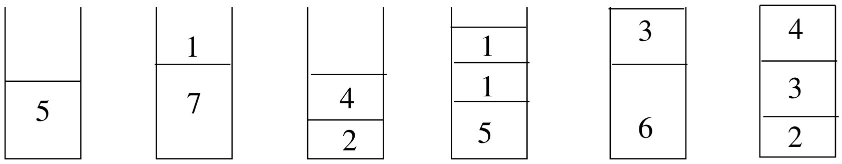
Schedule obtained with the critical–path scheduling method.



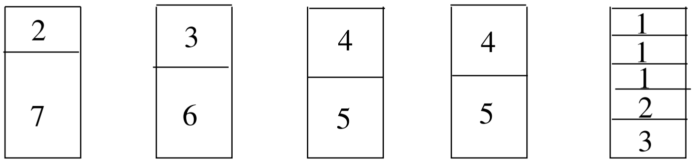
Schedule obtained with the decreasing–time list method .

Answer. The priority list obtained by the critical-path analysis is $T_3T_2T_5T_1T_4T_6T_8T_7$, while the priority list from the decreasing-time list is $T_3T_6T_2T_5T_8T_1T_7T_4$. These lists yield the schedules represented above.

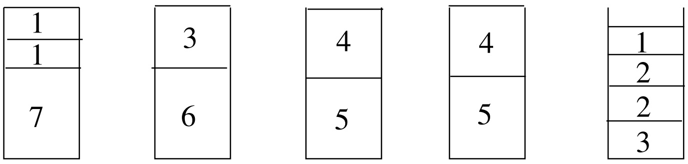
9. You must pack the following weights into bins that can hold no more than 9lbs : 5lbs, 7lbs, 1lb,2lbs, 4lbs, 5lbs, 1lb, 1lb, 3lbs, 6lbs, 2lb, 3lbs, 4lbs.
(a) Use the next-fit algorithm to pack the weights ; represent your solution below



(b) Same question, using this time the first-fit decreasing algorithm
Answer. The list of weights in nonincreasing order is : $7, 6, 5, 5, 4, 4, 3, 3, 2, 2, 1, 1, 1$.
Applying the first-fit algorithm to it gives the following packing :



(c) Again the same question, using the worst-fit decreasing algorithm.
Answer. We use the list from question (b), and obtain :



10. True or false ?
(a) A heuristic algorithm does not necessarily produce optimal results. **TRUE**
(b) The path produced by the sorted-edges algorithm when solving the traveling salesman problem may be dependent on the starting city. **FALSE**
(c) A spanning tree of a graph must contain every edge of a graph. **FALSE**
(d) When scheduling tasks using the list-processing algorithm, decreasing the time required by each task may increase the completion time. **TRUE** ; see textbook page 87.
(e) The worst-fit algorithm never uses more boxes than the first-fit algorithm. **FALSE**