

Midterm 3.
Friday, April 27th.

*No documents allowed. Mobile phones, mp3 players, etc., are also forbidden. The one and only piece of equipment you may use is a basic calculator- and you won't need it.
You must provide explanation for all your answers.*

NAME _____

1. Consider the following binary linear code :

0000	→	0000000		0110	→	0110010
0001	→	0001011		0101	→	0101110
0010	→	0010111		0011	→	0011100
0100	→	0100101		1110	→	1110100
1000	→	1000110		1101	→	1101000
1100	→	1100011		1011	→	1011010
1010	→	1010001		0111	→	0111001
1001	→	1001101		1111	→	1111111

- (a) What is the weight of this code?
- (b) How many errors could this code detect? How many could it correct?
- (c) Using nearest-neighbor decoding, decode (or explain why you cannot decode) the message 1011011.

2. One creates a code for five-digit binary strings by using the parity-check sums $a_1 + a_2 + a_3$, $a_3 + a_4 + a_5$ and $a_2 + a_4$.

- (a) How many code words would you have to compute if you were to give the code in full?
- (b) Write down the code words for 01010 and 11000.

3.(a) Use the Venn diagram method to encode the string 1001.

(b) Use the Venn diagram method to decode 1000011 and 1100001 (each time, say whether the code word is correct or not).

4. Recall that the check-digit a_{10} is added to a nine-digit ZIP+4 code in such a way that $a_1 + a_2 + a_3 + \dots + a_{10}$ ends with a 0.

(a) Find the check digit for the ZIP+4 code 61820-1309.

(b) Is the number 61801-1405-2 valid? If not, can you correct the error? If you know that the fourth digit is incorrect, can you correct the error?

5. Consider the following voting system : $[54:45,43,7,5,1]$

(a) List minimal winning coalitions.

(b) Is there a dictator? Voters with veto power? Dummy voters? (explain your answers)

6. Are the voting systems $[11:10,9,2]$ and $[2:1,1,1]$ equivalent? (explain)

7. A weighted voting system has five members.

(a) How many (distinct) coalitions are there?

(b) How many (distinct) coalitions are there in which exactly two voters vote YES?

8. Consider the voting system $[8:5,2,2,2,2,2]$.

(a) What are the winning coalitions in which A is critical? (**describe** them, don't give a list!)

(b) Compute the Banzhaf power index of A.

9. Given the weighted voting system $[10:5,5,3,2]$, give the Shapley-Shubik power index for each voter.

10. Recall that, for practical purposes, ZIP+4 codes (along with their check-digit) are printed using bar codes, with the following correspondence between bar code and digits :

Decimal digit	1	2	3	4	5	6	7	8	9	0
Bar code										

Is the code below correct? If not, can you recover the correct ZIP+4 code?

|||| ||| ||| ||| ||| ||| ||| ||| ||| |||