

**Quiz 4**  
 Correction.

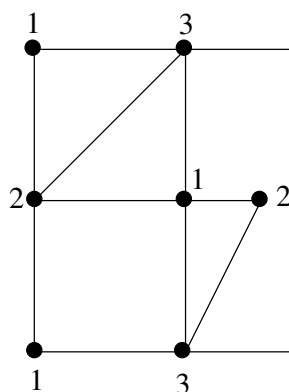
1. (a) A local cafeteria offers a choice of 5 meats, 6 vegetables, and 3 salads. A complete dinner includes 1 meat, 1 vegetable, and 1 salad. How many different dinners can be created?

**Answer.** There are  $5 \times 6 \times 3 = 90$  possible dinners.

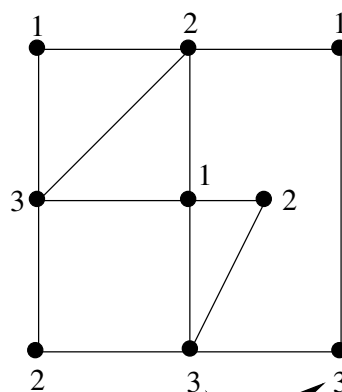
(b) An online banking service requires its customers to select a password that is four characters long. The password is case-sensitive, so upper-case letters are considered to be different than lower-case letters. The first character of the password must be an upper-case letter and the second character must be a digit. The remaining two characters may be a digit, an upper case letter or a lower-case letter. What is the number of possible passwords?

**Answer.** There are 26 possible choices for the first character, and 10 possible choices for the second one; for each of the remaining ones there are  $26 + 26 + 10 = 62$  possible choices. So the total number of possible passwords is  $26 \times 10 \times 62 \times 62 = 999440$ .

2. Which of the following is a correct vertex coloring of the given graph?

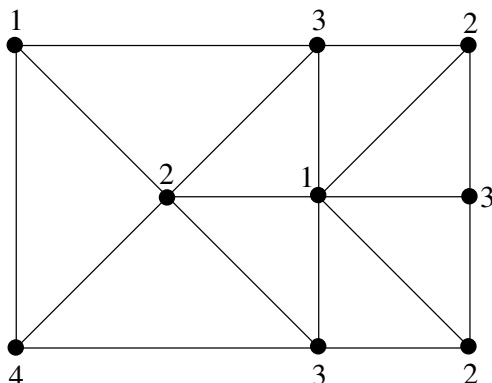


Correct



Incorrect

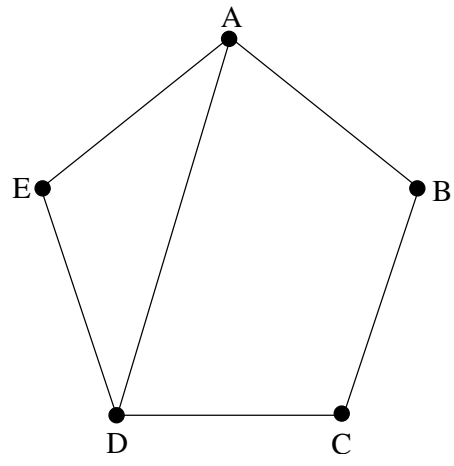
3. Find the chromatic number of the graph below, and give a vertex-coloring that uses a minimum number of colors.



**Answer.** The chromatic number of this graph is 4, as shown by the vertex-coloring above (one can check that no coloring with 3 colors or less works).

4. The table below represents species of plants which have competing light or water requirements (for instance the cross at AB means that A and B cannot be in the same habitat). Draw the graph that would be useful in determining the minimum number of different habitats that would be needed to display all these plants in a garden, and find this number.

	A	B	C	D	E
A		X		X	X
B	X		X		
C		X		X	
D	X		X		X
E	X			X	



**Answer.** The graph is above, and one can see that its chromatic number is three, so that the minimum number of habitats needed is 3.

5. A group of thirteen students have to decide among three types of pizza : Sausage (S), Mushroom (M), and Beef (B). Their preference rankings are shown below.  
S S

	Number of Students				
	4	3	2	2	2
First choice	M	B	S	B	S
Second choice	B	M	M	S	M
Third choice	S	S	B	M	B

Determine which choice the group will make (or if they won't be able to make a choice) if the students apply the following voting systems :

(a) Plurality voting

**Answer.** B has 5 votes, M has 4 and S has 4 too, so with plurality voting the students will eat Beef.

(b) Condorcet method

**Answer.** M beats B 8 to 5, and beats S 7 to 6; so with the Condorcet method the students would eat Mushroom.

(c) Borda count.

**Answer.** This time M has 15 points, B has 14 points and S has 10 points, so again the students eat Mushroom.

(d) Sequential pairwise voting with the agenda B,M,S.

Since Mushroom is the Condorcet winner, it must also be the winner for sequential pairwise voting.