UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN — Spring 2007
Math 181 — Group F1

Quiz 8

Friday, April 20th.

NAME ____

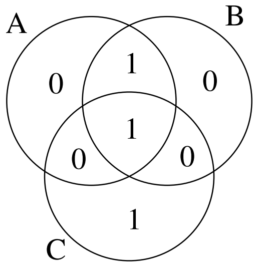
1. Remember that the check-digit for an ISBN number $a_1a_2\ldots a_10$ is chosen in such a way that $10a_1 + 9a_2 + 8a_3 + 7a_4 + 6a_5 + 5a_6 + 4a_7 + 3a_8 + 2a_9 + a_{10}$ is evenly divisible by 11. Is the number $0 - 2435 - 6411 - 3$ a correct ISBN number?
Answer. The sum $10a_1+9a_2+8a_3+7a_4+6a_5+5a_6+4a_7+3a_8+2a_9+a_{10}$ is here equal to $10.0+9.2+8.4+7.3.+6.5+5.6+4.4+3.1+2.1+3 = 154$, and 155 is not evenly divisible by 11. So the number $0-2435-6411-3$ is not a correct ISBN number.

2. The check-digit a_{10} for a nine-digit ZIP+4 code $a_1a_2\ldots a_9$ is chosen in such a way that $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_10$ ends with a 0.
(a) Find the check-digit for the ZIP+4 code $55811 - 2742$.
Answer. One has $5 + 5 + 8 + 1 + 1 + 2 + 7 + 4 + 2 = 35$, so one must choose 5 as the check digit
(b) Is the ZIP+4 code (including its check-digit) 5431275001 correct (explain)? If not, can you correct it? If you know that only the third digit is incorrect, can you recover the correct code?
Answer. This time the sum is $5+4+3+1+2+7+5+0+0+1 = 31$, so the number is not correct; we cannot correct the error unless we know which digit is incorrect (we don't have enough information). However, if we know that a_3 is the only incorrect digit, then we know that it must be such that $5+4+a_3+1+2+7+5+0+0+1 = 25+a_3$ ends with a 0, so we obtain $a_3 = 5$. So, with this additional information, we obtain that the correct number was 5451275001.

3.(a) Find the code words for the binary messages 0100, 0101, 0011 and 1101 obtained by adding three check digits using the parity-check sums $a_2 + a_3 + a_4$, $a_2 + a_4$, and $a_1 + a_2 + a_3$.
Answer. For 0100, the first sum is 1, the second is 1, and the third is 1; so the code word is 0100111. For 0101, the first sum is 0, the second is 0, and the third is 1; so the code word this time is 0101001. Similarly, one obtains that the other two code words are 0011011 and 1101001.
(b) If you were asked to find the code words for all binary messages of length 4 using the above method, how many code words would you have to compute?
Answer. There are $2^4 = 16$ binary messages of length 4, so we would have had to compute 16 code words.

Please turn the page.

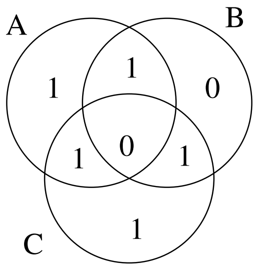
4. (a) Use the Venn diagram method to determine the code word of message 1010.



We first write 1010 in the relevant positions, then we add digits in such a way that the number of 0's and 1's in each circle is even (see the figure above). Then we read the code word: 1010001.

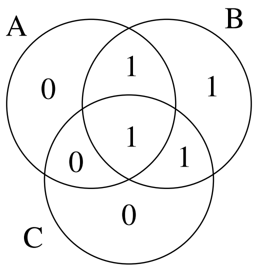
(b) Use the Venn diagram method to decode each of the following received words (if there is an error in the message, say so; say if you can correct the error or not)

1101101



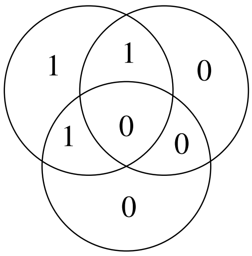
Circle A looks OK, but both B and C are incorrect; so we assume that the error comes from the digit that is in both B and C, and this leads us to decode the message as 1001 (when decoding, we no longer care about the error−detection digits).

1011010



This time, every circle looks OK; so we assume that no transmission error was made, and we decode the message as 1011.

1100100 (see the last page, for some unfathomable reason the software I use to include pictures refuses to include this one where it should...)



This time, all three circles are incorrect; so we assume that the incorrect digit is the one that belongs to all three circles, and decode the message as 1110.