UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN — Fall 2006
Math 380 — Group G1

Graded Homework XI.
Due Friday, December 1.

1. Compute the following surface integrals :
(a) $\iint_S \vec{F}\cdot\vec{n}\,d\sigma$, where S is the triangle with vertices $(1,0,0)$, $(0,2,0)$, $(0,0,3)$, $F(x,y,z) = (xy, y+z, z-x)$ and the normal vector is pointing away from the origin.
(b) $\iint_S (x^2+y-z)\,d\sigma$, where S is the portion of the cylinder of equation $x^2+y^2=1$ that is below the plane $z=1$, and above the plane $x+z=0$ (viewed as a closed surface).

2. Compute the following line integrals :
(a) $\int_C yzdx + zxdy + xydz$, where C is the arc of helix $x = R\cos(t)$, $y = R\sin(t)$, $z = \frac{t}{2\pi}$, with $0 \le t \le 2\pi$ and C is oriented in the direction of increasing t.
(b) $\int_C xdx + ydy + (x+y-1)dz$ where C is the straight line segment from $(1,1,1)$ to $(2,3,4)$.

3. Compute in two different ways the integral $\iint_S (\vec{F}\cdot\vec{n})\,d\sigma$ (following the definition of a surface integral, and using the Divergence theorem) :
(a) $\vec{F}(x,y,z) = (x,y,z)$ and S is the surface of the cube of equation $0 \le x \le l$, $0 \le y \le l$, $0 \le z \le l$.
(b) $\vec{F}(x,y,z) = (x^2,y^2,z^3)$ and S is the surface of the quarter-cylinder of equation $x^2+y^2=R^2$, $0 \le x, y$, and $0 \le z \le H$.
(c) $\vec{F}(x,y,z) = (xz, yz, 3z^2)$ and S is the surface bounded by the paraboloid of equation $z = x^2+y^2$ and the plane $z=1$.

4. (a) Verify Stokes's theorem for the vector field $F(x,y,z) = (z^2+x, -y^2, z-y)$, if C is the boundary of the square $0 \le x \le 1$, $0 \le y \le 1$ oriented counterclockwise, and the capping surface of C is a cube.
(b) Let C be the intersection of the hyperbolic paraboloid of equation $z = y^2 - x^2$ and of the cylinder of equation $x^2+y^2=1$. Find a parametrization of C, and verify Stokes's Theorem for the vector field $F(x,y,z) = (x^2y, \frac{1}{3}x^3, xy)$ and the curve C (you will need to produce your own surface!)