University of Illinois at Urbana-Champaign — Fall 2006
Math 380 — Group G1

Graded Homework I

Due Friday, September 8.

1. Let $f(x,y) = \dfrac{y^2}{2y - x^2}$.
- What is the domain of definition of f ? Give a graphical representation of it.
- Determine the level curve $f = 1$, and represent it on the same graph.

2. Determine the domain of definition of f, its Jacobian matrix and its Jacobian determinant (when it makes sense) in the following cases :
- $f(x,y) = (2x^3 + xy^2, \cos(x+y))$;
- $f(x,y,z) = \dfrac{x}{y} - \dfrac{y}{z}$;
- $f(x,y,z) = \dfrac{yz}{x^2 + yz}$;
- $f(x,y) = \sqrt{1 + \ln(1 + x^2 + y^2)}$;
- $f(r,\theta,\varphi) = (r\cos\varphi\, cos\theta, r\cos\varphi\, \sin\theta, r\sin\phi)$.

3. Use linear approximation (differentials) to find an approximate value of
$\sin\left((\sqrt{\frac{\pi}{2}} + 0.1)(\sqrt{\frac{\pi}{2}} - 0.1)\right)$ and $e^{1.01^2 0.98^2}$.

4. Compute $\dfrac{dz}{dt}$ when $z = x^2 + \dfrac{y}{x}$ and $x = e^t + t$, $y = \sin(t^2)$.

5. Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ be a differentiable function, and $u(x,y,z) = f(x-y, y-z, z-x)$.
Prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.