We wish to prove by induction that, for all $n \in \mathbb{N}$, $\frac{1}{\sqrt{1}} + \ldots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$. Call this statement $P(n)$.

(a) Initialization: $P(1)$ is the statement $1 \geq 1$, which is true.

(b) Induction: assume that $P(n)$ is true for some n. Then, we have $\frac{1}{\sqrt{1}} + \ldots + \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{1}} + \ldots + \frac{1}{\sqrt{n}} + \frac{1}{n+1}$, so our induction hypothesis yields $\frac{1}{\sqrt{1}} + \ldots + \frac{1}{\sqrt{n+1}} \geq \sqrt{n} + \frac{1}{\sqrt{n+1}}$ (*). To obtain the inequality we're looking for, it is therefore enough to prove that $\sqrt{n} + \frac{1}{\sqrt{n+1}} \geq \sqrt{n+1}$; this is equivalent to $\sqrt{n+1} - \sqrt{n} \leq \frac{1}{\sqrt{n+1}}$.
To show this, we use the conjugate quantity method:

$$\sqrt{n+1} - \sqrt{n} = \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}} \leq \frac{1}{\sqrt{n+1}} .$$

This last inequality is just what we were looking for to deduce from (*) that $\frac{1}{\sqrt{1}} + \ldots + \frac{1}{\sqrt{n+1}} \geq \sqrt{n+1}$; in other words, $P(n+1)$ is true.

Conclusion: (a) shows that $P(1)$ is true, and (b) proves that the property $P(n)$ is *hereditary*, i.e that if $P(n)$ is true then $P(n+1)$ is also true; both statements together imply, by the induction theorem, that $P(n)$ is true for all n.