What is our universe now ?

For the century of the formula 15 written by de Sitter

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For *[1]

Then the line-element becomes *

$$(15) \quad ds^2 = \mathrm{R}^2\{d\omega'^2 - \sinh^2\omega'(d\zeta'^2 + \sinh^2\zeta'[d\psi^2 + \sin^2\psi d\theta^2])\}.$$

Résumé

Starting with the Friedmann-Lemaître (FL) metric g_{FL} of an isotropic universe, we give the radialy inertial form of this metric which is a generalized Gullstrand-Painlevé form of metric g_{GP}. Then, for g_{GP}, the equivariant stress-energy tensor is convenient because it has straightforward interpretation in term of velocity and potential. The energy and the entropy are well-defined. For each model for the universe, the osculating manifold is a de Sitter model. Moreover if the universe is the open cone of the future of a point (a big bang event) then this de Sitter model is an open, accelerated, one. So we could easily confront this model with local observations through the osculating de Sitter model, taking into account the observed SNIa and the Hubble parameter H(z) for redshifts $z \leq 2$. The recent data about H(z) provide a tool to estimate cosmological parameters for the de Sitter models and their Milne limits ; we find : $H_o = 65 \pm 2\, km/s/Mpc$, $\Omega_o = 0.05 \pm 0.02$ and an age $= 15.2 \pm 0.3\, Gyr$. In other words, our universe contains uniqualy baryonic matter.

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1. Read Addendum after the conclusion.

1 Introduction

We describe the Universe by a Friedmann-Lemaître universe model $(\mathcal{U}, g_{FL})$, i.e. spatially homogeneous and isotropic. Most often, g_{FL} is expressed in its Robertson-Walker form

$$g_{FL} = \ ds^2_{FL} = d\tau^2 - R^2\ (\tau)\ \ (dx^2 + f^2_k(x)\ d\omega^2), \tag{1}$$

where $f_k(x) = x,\ \ \sin(x),\ \ \sinh(x)$ according to the sign $k = 0,\ 1$ or -1 of the spatial curvature, and where $d\omega^2$ is the element of spherical angle.

We will give different expressions of *the same* metric g_{FL} defined at the event E_0 =(here, today)$\equiv \{\tau = \tau_o,\ x = \theta = \phi = 0\}$.

It is convenient to start from the $l$ocally inertial form relative to E_0, defined after defining $\rho := R\ (\tau_o)\ x = R_o\ x$, as

$$ds^2_{FL} = d\tau^2 - \frac{R^2(\tau)}{R^2(\tau_o)}\ \left(d\rho^2 + R^2(\tau_o)\ f^2_k(\frac{\rho}{R(\tau_o)})\ d\omega^2\right)\ . \tag{2}$$

The slight change with respect to (1) emphasizes that, rigorously, an inertial metric must have the Minkowskian form, and that, in (1), x is an *angular* coordinate.

The model is characterized by a scale factor $R(\tau)$, τ being the cosmic time. We have the Hubble parameter $H(\tau) \equiv \dot{R}(\tau)/R(\tau)$, the deceleration parameter $q(\tau) \equiv -\frac{\ddot{R}\ \ R}{\dot{R}^2}$ that we assume negative (corresponding to an accelerating universe), and the Einstein equation

$$H(\tau)^2 + \frac{k}{R(\tau)^2} = \frac{8\pi G\rho(\tau)}{3} \equiv \Omega(\tau)\ H^2(\tau). \tag{3}$$

We include in the density parameter Ω the contribution of the cosmological constant. *Present* values of these quantities are written with a zero index : $\Omega(\tau_0) = \Omega_0$, etc.

In section 2, the generalized Gullstrand-Painlevé form is given and the case of the de Sitter models is developped. The section 2.1 address the global problems, without considering an hypothetic equation of state, but only the entropy equation. The section 2.2 address the de Sitter models. In section 2.3 we address the de Sitter model as an osculating manifold to a flat ΛCDM model. The local observations developped in section 3 underline the usefulness and the suitability of the accelerated de Sitter models and also of the limit case of the Milne's models. After a discussion in section 4, the conclusion ends with revisiting some classical problems.

2 The radially inertial form of a FL metric

In order to obtain a radially inertial form, the basic equation (2) is important and avoids a mathematical mistake that is too often writtened in papers and books.

2.1 The general case

We start from the usual form (2) of the metric, which is expressed in the locally inertial frame for the event E_0 = (today, here). The change of variables $(\tau, \rho) \mapsto$

$(\tau, r \equiv R(\tau) \quad f_k[\frac{\rho}{R(\tau_o)}])$; with the help of the Einstein's equation (3) this change of variables leads straightforwardly to the form ([3]) :

$$g_{GP} = ds^2 = d\tau^2 - \frac{(dr - H(\tau)\ r\ d\tau)^2}{1 + (1 - \Omega(\tau))\quad H^2(\tau)\quad r^2} \quad - \ r^2\ d\omega^2. \tag{4}$$

The metric g_{GP} looks like a generalised Gullstrand-Painlevé metric but with an hyperbolic space part (k=-1).

A comoving galaxy is defined by $d\rho = 0$, which implies $dr = r\ H(\tau)\ d\tau$. This formulation of the Hubble law remains exact at any time. We note $v = r\ H(\tau)$ the radial velocity. In the denominator the total energy $(1 - \Omega(\tau))\quad H^2(\tau)\quad r^2$ gives the potential Φ defined by $2\ \Phi = \Omega(\tau)\quad H^2(\tau)\quad r^2$ so we have the well defined accelerating equation :

$$\gamma = \frac{dv}{d\tau} = \frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r} = \frac{\partial \Phi}{\partial r} + \frac{\partial \Phi}{v\,\partial \tau}\ . \tag{5}$$

The potential Φ being well defined, we can compute the stress-energy tensor ; but the inertial metric g_{GP} has a cross term, so if we want a thermodynamic interpretation of this tensor (cf. Weinberg chap. 2.11) [2], we must compute the equivariant form of the stress-energy tensor which is not symetric (it is an ordinary mathematical fact which comes from the cross term).

First, the components of the non null equivariant Einstein's tensor $G^{\mu}_{\ \nu}$ with v and Φ, and using the identity (5) are given by

$$\begin{aligned}
G^0_{\ 0} &= -2\,\frac{\Phi + r\frac{\partial}{\partial r}\Phi}{r^2} \\
G^1_{\ 0} &= 2\,\frac{\frac{\partial}{\partial \tau}\Phi}{r} \\
G^1_{\ 1} &= -2\,\frac{vr\frac{\partial}{\partial r}\Phi + v\Phi + r\frac{\partial}{\partial \tau}\Phi}{r^2 v} \\
G^2_{\ 2} = G^3_{\ 3} &= -\frac{2\,v^2\frac{\partial}{\partial r}\Phi + r\left(\frac{\partial^2}{\partial r^2}\Phi\right)v^2 + v\frac{\partial}{\partial \tau}\Phi + rv\frac{\partial^2}{\partial r\partial \tau}\Phi - r\left(\frac{\partial}{\partial r}v\right)\frac{\partial}{\partial \tau}\Phi}{rv^2}
\end{aligned} \tag{6}$$

We must underline that $G^0_{\ 1} = 0$ is equivalent to $\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r} = \frac{\partial \Phi}{\partial r} + \frac{\partial \Phi}{v\,\partial \tau}$, cf. (5).
We can notice also that the $G^1_{\ 0}$ term is related to the entropy, and the conservation law $G^{\mu}_{\ \nu;\mu} = 0$ is verified.

Otherwise, the non null terms of the equivariant Einstein's tensor $G^{\mu}_{\ \nu}$ with the 3 cosmological parameters $H(\tau)$, $\Omega(\tau)$ and $q(\tau)$, is even shorter, but it hides the accelerating equation (5) :

$$\begin{aligned}
G^0_{\ 0} &= -3\,H\left(\tau\right)^2\Omega\left(\tau\right) \\
G^1_{\ 0} &= H\left(\tau\right)^3 r\left(\Omega\left(\tau\right) + q\left(\tau\right)\right) \\
G^1_{\ 1} = G^2_{\ 2} = G^3_{\ 3} &= -H\left(\tau\right)^2\left(\Omega\left(\tau\right) - 2\,q\left(\tau\right)\right)
\end{aligned} \tag{7}$$

After this study of the geometric tensor, the first member of Einstein's equations, let us pass to the second member of the equations $G^{\mu}{}_{\nu} = -\kappa T^{\mu}{}_{\nu}$ where $\kappa = 8\pi G_N/c^4$ where G_N is the Newton constant ; the non-zero elements are :

$$\begin{aligned} &T^0{}_0 = \rho \\ &T^1{}_0 = v(\rho + p) \\ &T^1{}_1 = T^2{}_2 = T^3{}_3 = -p, \end{aligned} \tag{8}$$

where ρ denotes a matter-energy density and p a "pressure". Note that p and ρ are always defined as the pressure and energy density measured by an observer in a locally inertial frame that happens to be moving with the fluid at the instant of measurement. The cross term $T^1{}_0$ whose writing follows from the identity $G^1{}_0 = v\,(G^0{}_0 - G^1{}_1)$, would translate to an "energy dissipation or exchange" of this thermodynamic fluid ; the nullity of $T^1{}_0$ is equivalent to the constancy of the entropy for a universe model, (cf. [2] formula 15.6.13).

The fundamental consequence of these three forms of the same tensor is coming from the following query : What does the nullity of the $G^1{}_0$, or $T^1{}_0$ terms imply ?

We have two answers : first $\frac{\partial}{\partial\tau}\Phi = 0$ i.e. $H^2(\tau)\,\Omega(\tau)$ is constant, and second $\rho + p = 0$. Moreover, if, by analogy with thermodynamic, $G^1{}_0 = 0$ translates to a constant entropy then there exits no dissipation because the universe has no exterior part. So it may exists only an exchange between two or more fluids in the universe.

The are two cases :

i) there exists only one fluid, moving with the inertial frame and we obtain all the de Sitter universe models,

ii) there exists another non co-moving fluid which exchange energy with the co-mowing one ; this is possible, a radiative fluid is then a convenient choice.

We shall consider the first case in the next subsection, but before, we want to point out that the so called dust fluid, with a null pressure, does not fulfill this entropy condition. It is the case for the ΛCDM models which contain such a fluid. Where is the problem ? Too often a dust fluid is taken, a priori, based on a strong analogy with thermodynamic. It is taken with the covariant tensor coming from the diagonal Robertson-Walker metric ; this form of metric is not inertial, leading to a misunderstanding ; but a Gullstrand-Painlevé like form of this metric is inertial so we can only then apply the thermodynamic analogy with the equivariant stress-energy tensor as a consequence of the general relativity.

So, if we have a dust fluid which is comoving then it has a pressure p and a density ρ such that $p + \rho = 0$; we think this fact to come from a basic principle of the general relativity which asserts that the gravitational mass is equal to the inertial one. Indeed the de Sitter expanding models fulfill the Mach principle which asserts that the inertial mass of a body comes from all the content of the universe ; if so, then we have a proof for main results about fluids in general relativity without the analogy with the thermodynamics but with the use of what is inherited from an inertial frame. Starting from another theoretical point of view, the conservation of global energy for the Universe, H. Telkamp [4] found the same result : the open, flat or closed de Sitter models.

2.2 The de Sitter case

As it appears that the de Sitter models for isotropic universes models are basic we developp this case but only for de Sitter models coming from a big bang. The study of the de Sitter manifold was made in 1917 ([1]) and for the model of the expanding universe, de Sitter gives the metric which is very near of the Robertson-Walker form, it was long before Friedman or Lemaître, here his formula 15 with R=1/λ, $\omega' = \lambda\,\tau$, etc. :

$$ds^2 = d\tau^2 - \frac{sinh^2\lambda\tau}{\lambda^2}(d\alpha^2 + sinh^2\alpha\, d\omega^2)\,, \tag{9}$$

where the curvature λ is a non-negative real number, so we have an infinity of de Sitter manifold. Each inertial form is given by :

$$ds^2 = d\tau^2 - \frac{(dr - r\ H(\tau)\ d\tau)^2}{1 + \frac{r^2\lambda^2}{\sinh^2\lambda\tau}} - r^2 d\omega^2. \tag{10}$$

These two forms of the same metric are defined on the same open part of a de Sitter manifold which is isomorphic to the SO(1,4)/SO(1,3) manifold, where SO(1,4) is the ten dimensional de Sitter group and SO(1,3) is the Lorentz group, see [25] for details. The generalized Gullstrand-Painlevé form (10) is introduced in ([3]). For these de Sitter models the function $H^2(\tau)\,\Omega(\tau)$ is constant, more precisely :

$$\lambda^2 = H^2(\tau)\,\Omega(\tau) = -\,H^2(\tau)\,q(\tau). \tag{11}$$

Let us take the initial conditions at the event (now,here) for the three cosmological parameters H_o, q_o, Ω_o ; as $\Omega_o = -q_o$, then the set of de Sitter models is two dimensional. We choose H_o and q_o as parameters in order to confront these models with observations. The time $\tau = 0$ is the time of the big-bang event, so τ_o denotes "now".

The Milne models can be viewed as the limit of these de Sitter models when the curvature λ, tends to 0, H_o being fixed ; one Milne model for each value of H_o, one de Sitter model for each pair (λ, H_o).

As the redshift z is a useful cosmological observable, we will provide here some formulas.

Lemma : Let us take the two parameters $(H_o, q_o = -\Omega_o)$ characterizing one of these models of de Sitter with negative curvature (0<$\Omega_o < 1$), we have :

i) The time of the emission of a photon received at τ_o with a redshift z :

$$\tau(z) = \frac{1}{H_o\sqrt{-q_o}}\,arcsinh(\frac{\sqrt{-q_o}}{\sqrt{1+q_o}(1+z)}) \tag{12}$$

and for z=0, the age of the universe

$$\tau_o = \frac{1}{H_o\sqrt{-q_o}}\,arcsinh(\frac{\sqrt{-q_o}}{\sqrt{1+q_o}})\ . \tag{13}$$

ii) The Hubble function $z -> H(z)$ is merely :

$$H\,(z) = H_o\,\sqrt{q_o\,z^2 + 2\,q_o\,z + z^2 + 2\,z + 1} \tag{14}$$

iii) The angular distance is given by :

$$dA_{dS}(z) = \frac{1}{-H_o q_o (1+z)} \left((1+z) - \sqrt{1+(1+q_o)z(2+z)}\right). \tag{15}$$

These are classical formulas [5] formula 237, [6];

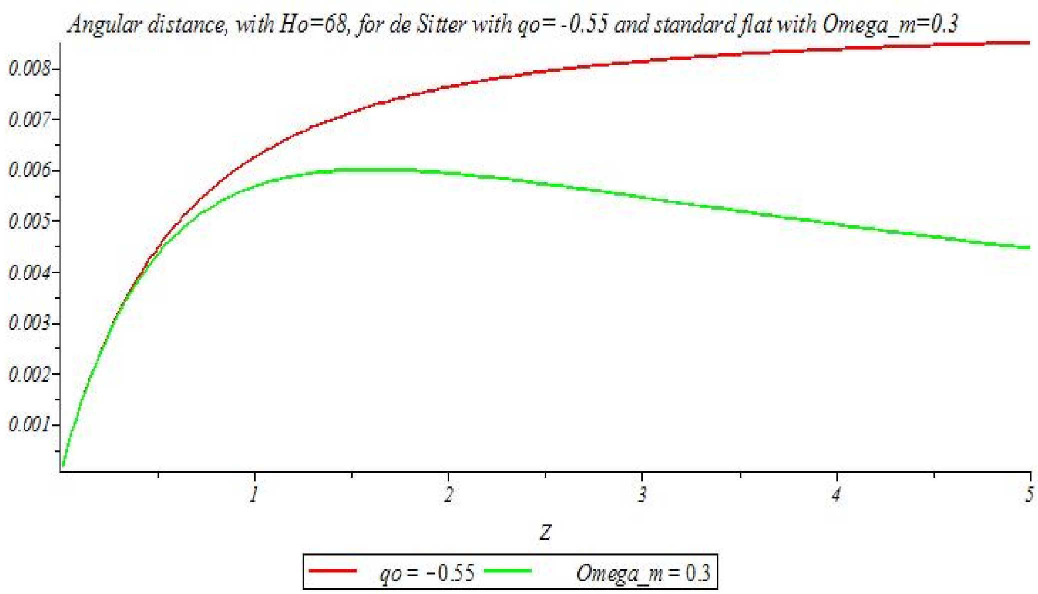
2.3 The osculating de Sitter model to a flat ΛCDM one

A flat ΛCDM has two initial parameters : the Hubble parameter H_o and the density parameter Ω_m of the cocomoving fluid (dust without pressure and dark matter).

For the flat ΛCDM the angular distace is given by an integral formula :

$$dA_{\Lambda CDM}(z) = \frac{1}{(1+z)\, H_o} \int_0^z \frac{1}{\sqrt{\Omega_m\ (1+x)^3 + 1 - \Omega_m}}\, \mathrm{d}x\ . \tag{16}$$

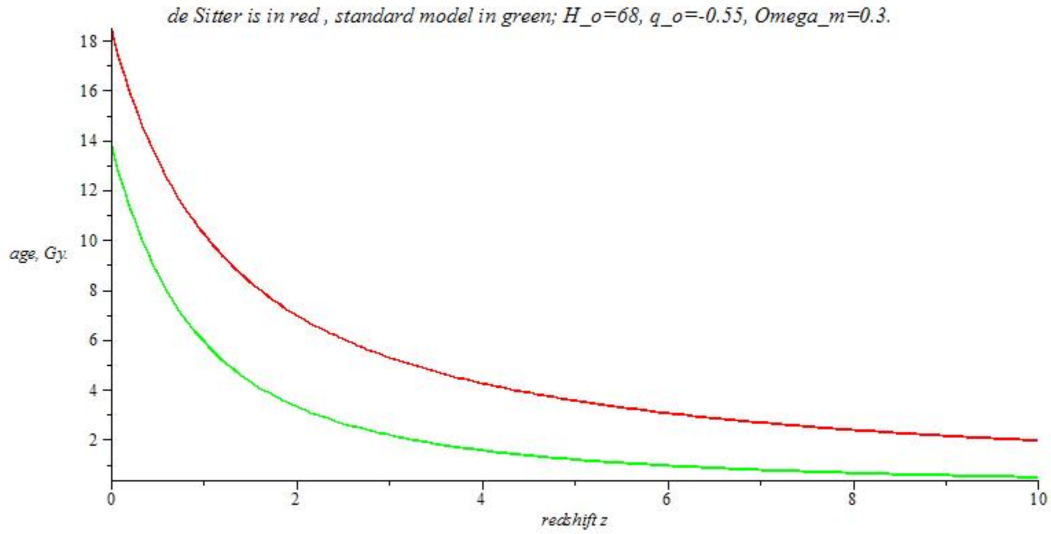
If we want that $dA_{\Lambda CDM}(z) \equiv dA_{dS}(z)$ near z=0, then $q_o = \frac{3}{2}\Omega_m - 1$. For example, for the recent Planck ΛCDM model with $\Omega_m = 0.3$ and $H_o = 68$, cf. ([7]), we have the de Sitter osculating model today defined by $q_o = -0.55$ and the same Hubble value, here figure 1 :



We have also an integral formula for the flat ΛCDM model :

$$\tau(z) = 1/H_o \int_0^{1/(1+z)} \frac{1}{\sqrt{\Omega_m/x + (1-\Omega_m) * x^2}}\, dx\,. \tag{17}$$

For the figure 2 the same initial values are taken.



The theoretical error is very big for the age because we made an integration for all z and not only for z less than 0.6. Thus the misunderstanding about frames (comoving and inertial) became a big mathematical mistake if we consider the universe model as a whole. The problems are the same for the values coming from the WMAP collaboration ([8]). Two facts seem weird or puzzling : first the fact that the angular distance is decreasing after $z > 1.5$ and, second, the very accurate values given by the each team with, for example, ages with more three exact digits.

What confidence can then be given to studies which, based on the a priori of a flat ΛCDM model, give extremely accurate results both with respect to BAO (Baryon Acoustic Oscillations) and to cosmic background radiation because they involve large redshifts ? If we compare the functions $z \to H(z)$ for de Sitter models on the one hand and flat ΛCDM models on the other hand, the latter is of order $\sqrt{z}$ times the former. Another example near the redshift $z = 1100$ of the CMB : $\tau(1100) = 0.5\,10^6$ light years for the standard model and $\tau(1100) = 16\,10^6$ light years (32 times more), for the osculating de Sitter universe (but without radiation, so it's just an approximate calculus).

Is it also worth recalling that one of the consequences of the Machian nature of de Sitter models is, ipso facto, no resort to a hypothetical period of inflation is necessary to account for the isotropy of the cosmic background radiation and also for the primordial baryogenesis ([5]).

3 The de Sitter models and astronomic observations

For the observations which concern the de Sitter and Milne models, three very interesting papers exist, based on studies of supernovae (SNIa) : in 1998, A. Riess and his team [9] ; in 2010, F. Farley [11] and in 2016, V. Lukovic and all [12] ; see also in 1999 [10]. They pointed that the better values for the Hubble parameter are about from 63 to 65 km/s/Mpc for these models :

i) in his historic paper of 1998, A. Riess said : *The Hubble constants as derived from the MLCS method,* 65.2 $\pm$ 1.3 *km/s/Mpc, and from the template fitting approach,* 63.8 $\pm$ 1.3 *km/s/Mpc, are extremely robust and attest to the consistency* [9], see also their emblematic figure that highlights many elements ;

ii) in 2010, F. Farley gives us a nice proof that the Milne models explain the kinematic face of the SNIa observations and that H_o is so around 63 km/s/Mpc ; see also [13] and [14].

iii) let us present only the wonderfull figure given by V. Lukovic and all in 2016 :

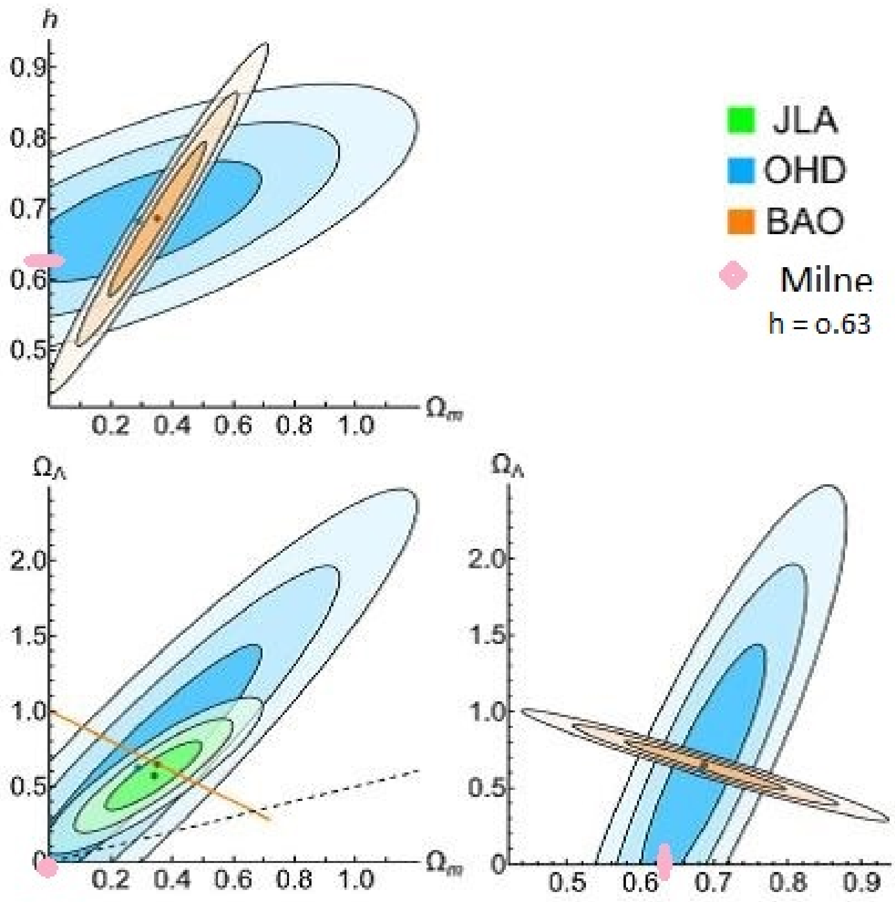


Fig. 2. 1σ, 2σ and 3σ confidence regions resulting from the fit of $k\Lambda$CDM model to the single datasets as indicated in the top right panel. The dashed line in the $\Omega_m - \Omega_\Lambda$ plane represents the transition from the decelerating (below) to the accelerating (above) models.

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We have added, on the three panels of this figure, where are the Milne models, using pink color ; the de Sitter models are defined, on these plots, by $\Omega_m = 0$ and $0 < \Omega_\Lambda < 1$. Let us remark that the BAO results are definitely not relevantt for our study, cf. the red line on the ($\Omega_m =$, Ω_Λ) plot, bottom left panel. Thus our Ω_o must be less than 0.3 and $60 \leq H_o \leq 68$ (top panel), cf. also the paper of Buchert, Coley and others [15].

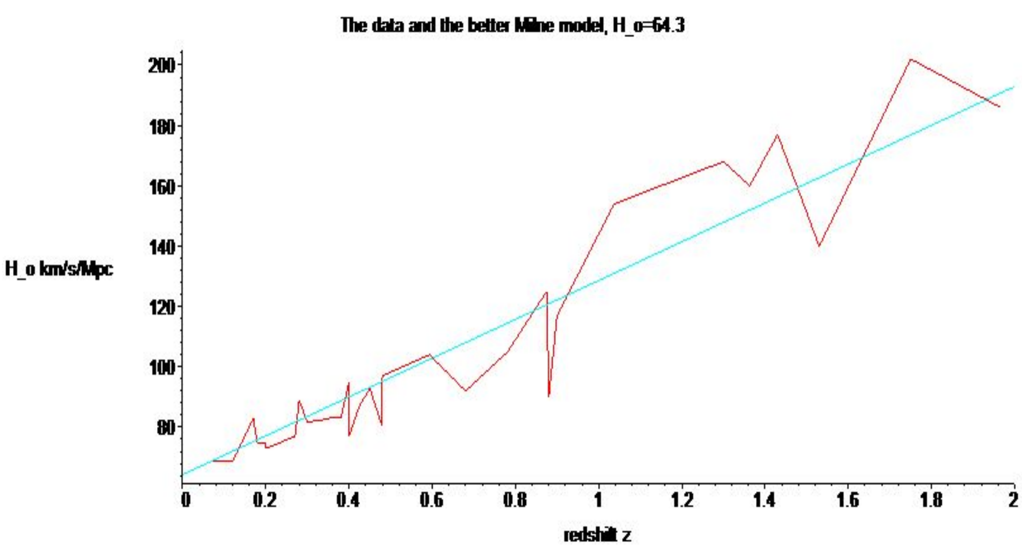
What is now the observational problem ? The comoving density Ω_o must be less than 0.3 and, more likely, even near 0 as Farley suggests. For a theoretical reason we have no need for dark energy as explained above via the inertial frame ; we have also no need for dark matter to explain the flat curves of spiral galaxies (a big mathematical mistake about the "exponential disk profile"), this is developed by many astrophysicists, see ([5]) and a bibliography in ([16]), ([18]) and also Y. Sofue ([19]) formula (38). A remark, the conformal gravity and the Einstein gravity are equivalent for the de Sitter universes models. Therefore, the 4% of baryonic matter is allowable.

For the respect of general relativity, we want to test the de Sitter models with other data than these coming from SNIa data. One year ago Duan and all give us several recent data about the function $z \to H(z)$ ([17]) ; thirty eight data, thanks to astronomers. But eight are coming from the erroneous mathematical use of the BAO. So we have thirty data coming from local measurements, in the table of the values (z,H(z)) :

table := [[.7e-1, 69.], [.9e-1, 69.], [.12, 68.6], [.17, 83.], [.179, 75.], [.199, 75.], [.20, 72.9], [.27, 77.], [.28, 88.8], [.352, 83.], [.38, 83.], [.4, 95.], [.40, 77.], [.425, 87.1], [.45, 92.8], [.478, 80.9], [.48, 97.], [.593, 104.], [.68, 92.], [.781, 105.], [.875, 125.0], [.88, 90.0], [.9, 117.], [1.037, 154.], [1.3, 168.], [1.363, 160.], [1.43, 177.], [1.53, 140.], [1.75, 202.], [1.965, 186.]].

The accuracy of these measures, although not shown here, is small (uncertainties from 5 to 25% for most points), but the number of measures has been well developed in recent years. A first small problem about these data, the smaller value of H(z) is $H(0.12) = 68.6$, but the functions $\tau(z)$ given by 12 and also 17 are increasing, statistics are valid if H_o is less than 68.6, stricto sensu.

As the function $H(z)$ is near linear for $0.07 \leq$ z ≤ 2, and for the small densities Ω_o, we just use the least-square method for some values of H_o to compute the values of Ω_o. Before for the Mine models we have :



The linear regression is given by H(z) = 59.8 ($\pm$ 1.2) + 70.0 ($\pm$ 1.6) z, so a mathematical minimum error around 2% on $H(0)$; and for the Milne models by

$H(z) = 64.32\ (\pm\ 0.75)\ (1+z)$, so with a 1.2% error. These error bars are independant of the uncertainties on the data. For the de Sitter models we must have Ω_o = - q_o non negative so $H(z)\ \leq\ H_o\,(1*z)$, the H_o Milne model. Moreover $\Omega_o \geq 0.04$ the baryonic density, that gives a lower bound to Ω_o and with the data, a lower limit to H_o.

These first results from the data about the function z->H(z) confirm those of F. Farley ([11]) about the Milne universe and agree with the analysis of V. Lukovic (3).

The results

H_o =64.3 km/s/Mpc :	Milne, $\Omega_o = 0$	Age(Milne, 64.3)= 15.2 Gyr
H_o =65 km/s/Mpc :	0.03<= Ω_o <= 0.07	15.2 Gyr <= Age(65)<= 15.4 Gyr
H_o =67 km/s/Mpc :	0.06<= Ω_o <= 0.17	14.9 Gyr <= Age(67)<= 15.5 Gyr
H_o =69 km/s/Mpc :	0.12<= Ω_o <= 0.26	14.8 Gyr <= Age(69)<= 15.6 Gyr

For a fixed H_o among the set {63, 65, 67, 69}, by the least-square fit method, the H_o-Milne curve is computed (in cyan color on the two following figures), after in black on figures, the development at order two of $H(z)$ at $z = 0$ is computed also by the least-square fit method, the de Sitter $H(z)$ tangent curve at zero is in green and the $H(z)$-de Sitter curve such that $H(0) = H_0$ and $q_0 = -0.04$, the minimum baryonic density, is in pink. As for H_o = 63 km/s/Mpc, the better fit is obtained for $\Omega_o = 0.02$, this value is at the borderline and as for H_o = 69 km/s/Mpc, the best fit is obtained for $\Omega_o = 0.256$, this value is to be rejected because the de Sitter models contain neither dark energy nor dark matter. Even if the case H_o = 67 km/s/Mpc is also borderline the figure looks interesting.

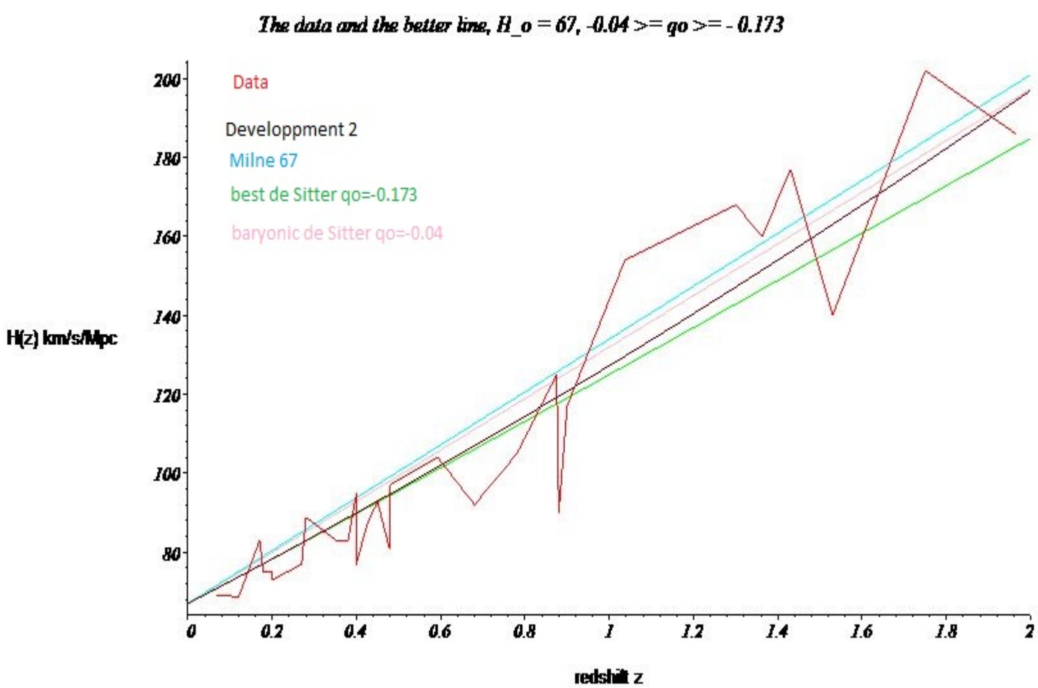


Figure 2 : for H_o = 67 km/s/Mpc.

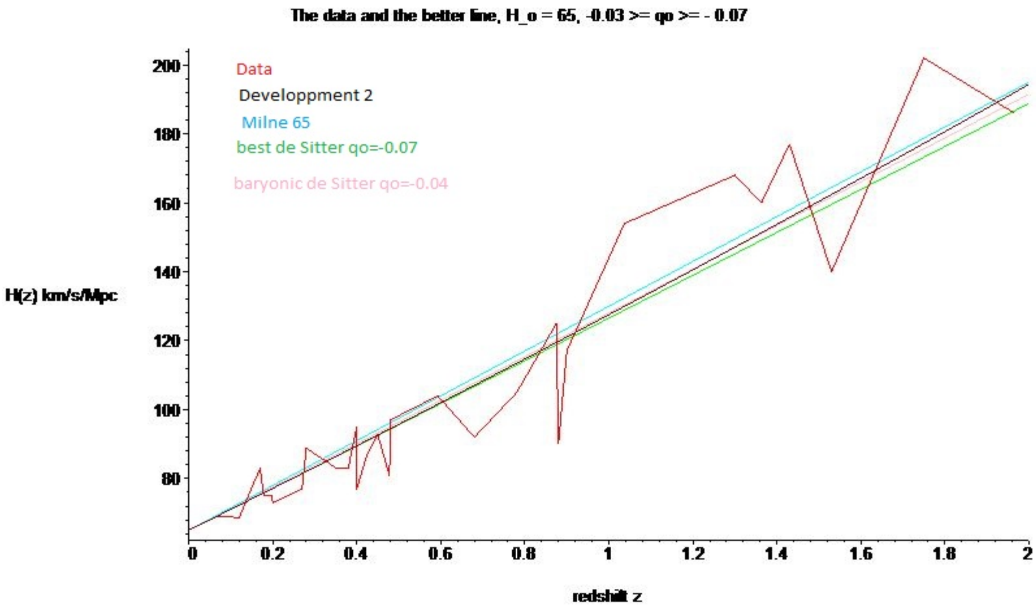


Figure 3 : for $H_o = 65$ km/s/Mpc.

With these only thirty data the final result about the de Sitter universe is the following :

$$H_o = 65 \pm 2\, km/s/Mpc,\ \Omega_o = 0.05 \pm 0.02\,,\ Age = 15.2 \pm 0.3\, Gyr\,. \qquad (18)$$

4 Discussion

But, we do not have take care, for this study, of the uncertainties on the data. In a few years, the data will be more numerous and more accurate and so it will be possibe, with more theoretical statistics, to improve this first result.

Also we don't have to pay attention to the contribution of radiation ; even if this latter is tiny for a small redshift, for big redshift as the redshift 1100 for example, it would be necessary. If we added radiation, the scale factor $R(\tau)$ of the FL metric is equal to $b + c_1\, exp(\lambda\,\tau) + c_2\, exp(-\lambda\,\tau)$, with relations between the four constants ; $R(\tau)$ appears as a light modidification of the de Sitter metric for $z \leq 2$.

The results (18) rest upon the a priori that, for theoretical reasons, it exists only baryonic matter around 4%, but it could be supposed that it exists a little unseen matter or even a little dark matter, but not in halos around galaxies [18], perhaps in clusters of galaxies or elsewhere ; so the density Ω_o would be bigger and, ipso facto, the Hubble value now H_o. This is compatible with the data $H_o \leq 67$ i.e. $\Omega_o \leq 0.17$, see the table of results and figure 2 above.

But a value of H_o as hight as 67 is not compatible with results coming from SNIa based on local methods, cf. for example the works of G. Tammann and B Reindl [20] who found $H_o = 63.7 \pm 2.3$km/s/Mpc or of V. Busti [21] who found $H_o = 64.9 \pm 4.2$km/s/Mpc and also of J.-J. Wei, F. Melia and X.-F. Wu [22].

"We emphasize here that the CMB estimates are highly model dependent" as Planck team said ([23] page 30), it is the same for the BAO.

5 Conclusion

In a first step, we have underlined a theoretical confusion : within a chart radially inertial, there is no need for dark energy. We have also recall why a huge mathematical mystake implies the needness for dark matter to explain the flatness of the curves of rotation for spiral galaxy as many papers said since a long time (25 years). Thus, working in inertial frame instead comobile one the de Sitter models for the Universe appears the good theoretical models. It is well-known that these models are very good for the interpretation of the supernovae (a kinematic effect).
The recent data about the Hubble parameter $H(z)$ was the ocasion to confront the de Sitter models with these data. The results are beyond all that could be expected ; no conflict with the SNIa approach. No inflation, no problem of stability, no mystery about all which seems dark, but in conflict with the ΛCDM models. The general relativity go on, even it remains many others problems, but the icing on the cake, the star HD140283 can extend his very long life quietly.

Among the problems to address :

i) The BAO and the small fluctuations of the CMB, by using the inertial form of metric with radiation, a difficult problem even for the ΛCDM model [15].

ii) The "Pioneer anomaly" which does not come from the dynamics of the universe model [3] but likely from the kinematics of the Milne model as it is also the case for the SNIa.

iii) The baryogenesis in this inertial frame for the Universe, and particularely the study of baryogenesis of the "lithium problem".

iv) If we want to glue together the quantum mechanics and the general relativity, the invariant de Sitter group is unavoidable ; for this goal, the Lie semigroup of causalty of the de Sitter group is very interesting [24],[25], and maybe a good step for a well-posed problematic to address this question.

Addendum : Let us return to the work of de Sitter : *On the curvature of space* [1] where the main metric (2B) he study is his metric (15), i.e. our metric 9. In this wonderful paper W. de Sitter said among others :

> §7- "If in the future it should be proved that very distant objects have systematically positive apparent radial velocities, this would be an indication that the system B, and not A, would correspond to the truth."

> §8- "We must then for "ρ_o" take the density not within the galactic system, but the average density over a unit of volume which is large cornpared with the mutual distances of the galactic systems. With the numerical data adopted above, this leads to R $= 5\,10^{13}$, and there would then be more than a billion galactic systems."

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6 Annexe 1 : About bayesian methods

In several papers, authors work with bayesian methods to study this cluster of 30 points coming from the data $z \to H(z)$. Is it necessary ? We think that it is no for this problem, why ? We shall not confound "plausible" and "probable". Indeed with a bayesian method we test if an hypothesis is possible, plausible. The answer is yes or no, and if yes then the method give us the better parameters for this hypothesis. Suppose now that we have two very different hypothesis which could furnish a good answer, so we obtain two different explanations to interpret the data. For the two cases we don't obtain a probability concerning each hypothesis, we obtain the better possibility for each hypothesis. Each hypothesis may be wrong and at most one is right.

For our problem let us test the four following hypothesis :

- a) The ΛCDM H(z) is valid.
- a)bis The ΛCDM H(z) is valid and $H_o \approx 64\, km/s/Mpc$.
- b) The de Sitter H(z) is valid.
- b)bis The de Sitter H(z) is valid and $H_o \approx 71\, km/s/Mpc$.

The least square method is enough to test these hypothesis and the answer is straitght-forward : a) and b) are plausible but a)bis and b)bis are not possible. But J.-J. Wey and others, in a recent paper [22], add that two plausible hypothesis are on an equal footing, by a right use of a bayesian method. Now suppose that in two years the astronomers get two new data for two z greater than 2 and obtained by local measures, then we have 3 cases : 1- the new data are in huge favor of the a) hypothesis such that the hypothesis b) became not possible, unlikely ; 2- the new data are in huge favor of the b) hypothesis such that the hypothesis a) became not possible ; 3- the new data don't favor the same hypothesis which is the more likely case. Notice that in the case 1- or 2- we don't have a probability, we have only one possibility, no more.

For the study of the SNIa data it is the same problem, the two hypothesis, flat ΛCDM and de Sitter models, are equally plausible after numerous tests with bayesian methods, c.f. the beautiful paper of J. T. Nielsen and others [14] who found $-q_o = 0.094$. As Einstein wanted always a beautiful and simple solution for a problem, we think that a de Sitter model 18 is welcome for the Universe, inside the general relativity theory, because it solve several problem : no useless dark matter, no enigmatic dark energy, no strange inflation, but an understandable inertial mass via the Mach principle, and the mysterious cosmological constant which translates merely the comobile matter ; moreover the kinematic is well posed. For this we were careful to do confusion nor between inertial frame and comobile frame and nor between plausible and likely.

7 Annexe 2 : News results about the Hubble parameter

The 2017/11/09 Yu and all [26] have published a new study on the $z \to H(z)$ data, with 31 local measures and 6 others measures with BAO method. They study, by bayesian methods the best plausible H_o values with two important options : 1- local data or all data, 2- data with $z \leq 2$ or not. Here their main results :

$H_o = 66 \pm 4\, km/s/Mpc$, $H_o = 64.3 \pm 3.5\, km/s/Mpc$, $H_o = 65.5 \pm 4.8\, km/s/Mpc$, $H_o = 67 \pm 4.4\, km/s/Mpc$.

We notice that our result, $H_o = 65 \pm 2\, km/s/Mpc$ for de Sitter and Milne models, are compatible with their four results ; moreover, they said also that they could not reject the non existence of a transition phenomenon between a non accelerating period and a recent accelerating one for the Universe, a phenomenon which does not exist for de Sitter models.

We could also notice the very recent paper from G. Paturel and others [27] who, starting from the study of the redshift z with the frequential point of view z_ν, instead the usual z_λ, finded the Milne model better to the study of the cepheïds and that give them $H_o \approx 63\, km/s/Mpc$, c.f. [28].