

R. William Gosper and His Identities

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Abstract

I will say something about Bill Gosper and his work. I will present some of the identities he discovered and where they come from and the mathematical concepts behind them. Here are some sample formulas:

$$\text{(Ramanujan)} \quad \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{\pi}{n + \sqrt{n^2 + 1}}\right) = ??$$

and

$$1 + \sum_{n=1}^{\infty} (x^{1/2} - c)(x^{1/4} - c) \cdots (x^{2^{-n}} - c) = ??$$

for $|1 - c| < 1$, $x \notin (-\infty, 0)$. Your home work is to fill in the ?? on the right-hand sides.

The second identity is not very difficult to prove but the first is harder.