We consider the equivalence relations arising from $GL(2, \mathbb{Z})$-action, $SL(2, \mathbb{Z})$-action, and continued fractions. Real numbers $x$ and $y$ in the unit interval are said to be continued-fraction-equivalent if there exist nonnegative integers $n$ and $m$ such that $T^n(x) = T^m(y)$ where $T$ is the continued fraction map (Gauss map). It is well-known that this relation holds if and only if there exists a matrix $A \in GL(2, \mathbb{Z})$ such that $y = Ax$. Now we introduce the notion of $\alpha$-continued fractions, $0 < \alpha < 1$. Then it is shown that the equivalence relation associated to $\alpha$-continued fractions for a fixed $\alpha$ is not the same as $GL(2, \mathbb{Z})$-equivalence if $\alpha < \sqrt{2} - 1$. We discuss the detail of the relation between these equivalence relations.