AIM Workshop - Contact topology in higher dimensions - (May 21-25, 2012): Questions and open problems

Warning

The following notes, written by Sylvain Courte with the help of Patrick Massot, describe the questions that were addressed during the open problem sessions at the meeting. Many of them are well-known and part of the "contact geometry folklore". Others can be found in the literature. They are associated with names here not for credit purposes but just to give a more vivid account of the discussion.

The organizers.

1 Monday

1.1 Flexible contact structures

E. Murphy: Is there a class of contact structures on closed manifolds abiding to an *h-principle*? Meaning a class of contact structure for which homotopy through almost contact structure implies isotopy. Such a class would provide a generalization of *overtwisted* contact 3-manifolds to higher dimensions [12].

1.2 Test cases for flexibility

Since the preceding question may be very hard, one can rather try to prove by ad hoc methods that some operations do not change contact structures once they look flexible.
C. Wendl: Given a contact manifold (V, ξ) there exists an operation $\mathcal{L}$ called *Lutz-Mori twist* (see [28, 31]) that takes ξ to another contact structure $\mathcal{L}(\xi)$ on V in the same homotopy class of almost contact structures. (It can be performed on any contact 5-manifold and at least on some examples in higher dimensions). It is likely to kill the contact homology of V [5, 6]. If you apply the twist twice you get another contact structure $\mathcal{L}^2(\xi)$ on V, is it contactomorphic to $\mathcal{L}(\xi)$?
J. Etnyre: There is another kind of twist $\mathcal{L}'$ due to Etnyre and Pancholi [16] which applies to any dimensions.
K. Niederkrüger: To complete the list, there is also the *negative stabilization* process $\mathcal{L}''$ by E. Giroux (see [22, 7]). It starts with a supporting open book for a given contact manifold (V, ξ), adds a critical Weinstein handle to the page along a Legendrian sphere bounding a Lagrangian disk and composes the monodromy with a left-handed Dehn twist along the Lagrangian sphere obtained by capping the disk with the handle core. In dimension 5, this preserves the homotopy class of the almost contact structure. Somebody mentioned that in dimension greater than 5, applying it twice also preserves the homotopy class of the almost contact structure. In the same vein as C. Wendl's question, denoting $\mathcal{N}$ the process of applying $\mathcal{L}''$ twice, do we have $\mathcal{N}^2 = \mathcal{N}$?
Many people: What is the relationship between $\mathcal{L}$, $\mathcal{L}'$ and $\mathcal{N}$? J. Etnyre and P. Massot remark that they all produce contact structures that are non fillable, have vanishing contact homology and all their Reeb vector fields have a contractible closed orbit.

P. Massot: How to find bLobs or other remarkable $n+1$-dimensional submanifolds in negatively stabilized contact manifolds?

1.3 Convex hypersurface theory

A. Mori: What would be a useful theory of *convex hypersurfaces* in high dimension? There is a definition [20] as a hypersurface that is transverse to a contact vector field but E. Giroux points out that in opposition to the 3-dimensional case, this is not generic in higher dimensions. Indeed, E. Giroux says one should be able to construct examples of hypersurfaces whose characteristic foliation admits a closed orbit which is neither repelling nor attracting but rather has a hyperbolic type dynamic. This will remain after perturbation and is an obstruction to convexity. A. Mori mentioned that [32] gives an explicit example of this phenomenon.
E. Murphy: Are there other obstructions for perturbing a given hypersurface to a convex one and are there conditions that guarantee the existence of such perturbations?
J. Etnyre: Is every hypersurface at least homologous to a convex hypersurface? This is a potentially easier question and is relevant to the Thurston-Bennequin question below.
K. Honda: What is a good notion of a *bypass* in high dimension [23]? P. Massot remarks that there is a natural definition involving topologically canceling contact handles [20, 38] (but non trivial regarding contact topology), and asks if this is a good one?

1.4 Thurston-Bennequin inequality

A. Mori: formulated a Thurston-Bennequin type inequality [13] in any dimension that generalizes the 3-dimensional case (see [32]) to some hypersurfaces whose boundary is a contact submanifold. Does it hold for the standard contact sphere S^{2n+1}? He points out that the Lutz-Mori twist $\mathcal{L}$ produces contact structure that violate this inequality. He also says there is an absolute version of this inequality that trivially holds for the standard contact sphere.
P. Massot: Is there a Thurston-Bennequin inequality which holds for closed hypersurfaces in fillable or tight contact manifolds? The first case to look at would be: is there any constraint on the Chern class of a 5-dimensional fillable contact manifold?

1.5 Characterization of the standard contact sphere

K. Niederkrüger: Are there properties that uniquely determine the standard contact S^{2n+1}? For example, are there other contact structures on S^{2n+1} that are filled by a symplectic manifolds diffeomorphic to the ball D^{2n+2}? M. Abouzaid and M. McLean answer yes: there are examples constructed by McLean in [30] (see also [2]) of Stein manifolds diffeomorphic to $\mathbb{C}^n$ with non standard contact boundary .
M. McLean: What constraint on the symplectic structure of D^{2n+2} would imply that its boundary is the standard contact S^{2n+1}? He suggests symplectic balls D^{2n+2} that are symplectomorphic to a smooth affine variety of negative log-Kodaira dimension.
P. Massot: If ξ is a contact structure on S^{2n+1} with $CH_*(S^{2n+1}, \xi) \simeq CH_*(S^{2n+1}, \xi_{std})$, are ξ and ξ_{std} contactomorphic?

1.6 Contact structure on $M \times S^2$

F. Presas: Given a contact manifold (M, ξ), can you build a contact structure on $M \times S^2$? We know there is no homotopy obstruction to this. If yes can you require in addition that for some $p \in S^2$, $M \times \{p\}$ is a contact submanifold contactomorphic to (M, ξ)?

1.7 Contact fibration and orderability

E. Giroux: From the paper of Eliashberg and Polterovich introducing the notion of orderability of a contact manifold [15] we can get the following statement: (M, ξ) is non-orderable if and only if there exists a contact fibration $M \times S^2 \to S^2$ with fiber contactomorphic to (M, ξ). What about topologically non trivial bundles? For 3-manifold, the problem of constructing a contact structure transverse to a given circle bundle is related to the Milnor-Wood inequality and to quasimorphisms on $\mathrm{Diff}(S^1)$ (see [21]). Analogously, the higher dimensional question is probably related to the existence of quasimorphisms on the group of contactomorphisms of (M, ξ).

1.8 Generalized Giroux torsion

What is the generalization of Giroux torsion for higher dimensional contact manifolds? In the morning, P. Massot proposed the notion of Giroux domain introduced recently in [28]. There is a model case $S^1 \times M \times D^1$ with $M \times D^1$ a Liouville manifold, which allows to produce non fillable manifolds, yet not flexible and having Reeb vector fields without contractible Reeb orbits.

There is some definition of algebraic 1-torsion in SFT [25]. It is conjectured that geometric torsion implies algebraic torsion.

1.9 Fillability and cobordisms

Recall the general picture about fillable contact manifolds:

$$\begin{array}{ccccccc} & & \{\text{Exact}\} & & & & \\ & \subset & & \subset & & & \\ \{\text{Stein=Weinstein}\} & & & & \{\text{Strong}\} & \subset & \{\text{Weak}\} \\ & \subset & & \subset & & & \\ & & \{\text{Holomorphic}\} & & & & \end{array}$$

J. Latschev: Are there obstructions to exact or Weinstein cobordisms between contact manifolds? For example, are there strongly fillable manifolds with no exact filling?
Y. Eliashberg: $(\mathbb{R}P^{2n+1}, \xi_{std})$ is holomorphically fillable but not Stein fillable ([14, 39]). They probably do not have exact fillings. Are $(T^{2n+1}, \xi_{Bourgeois})$ exactly fillable? He says they are not Stein fillable ([14]) but according to P. Massot they are weakly fillable ([28]). Y. Eliashberg expects these manifolds not to be strongly fillable.
C. Wendl: In dimension 3 and 5, there are examples of weakly but not strongly fillable manifolds (see [28]). What about dimension greater than 5?

1.10 Lefschetz fibration

O. Plamenevskaya: Does every Weinstein domain admit a Lefschetz fibration over the disc D^2? E. Giroux says he has a proof in mind using Donaldson's approximately holomorphic techniques [11], it should also work in the case of Stein domains (requiring the projection to be holomorphic) thanks to Hörmander's L^2-theory, but it is not yet written.
C. Wendl: Is there a hyperplane pencil decomposition of contact manifolds? By this we mean a kind of open book decomposition but with 2-dimensional pages (instead of codimension 2 pages). One interest for this comes from making these pages holomorphic while applying holomorphic curves techniques. For instance, it would probably allow to prove the Weinstein conjecture in some cases. F. Presas says there exists a notion for this and they can be constructed using approximately holomorphic techniques but resulting maps will have singularities modeled on singularities of maps from $\mathbb{C}^{n+1}$ to $\mathbb{C}^n$ which are very complicated.

1.11 Open book decomposition

O. Plamenevskaya: Can you say anything about contact structure using monodromy data? For example, do we have Stein fillable $\Rightarrow$ there is a supporting open book whose monodromy is a product of positive Dehn twists along Lagrangian spheres? (it would be a corollary of the existence of Lefschetz fibration on Stein domains). K. Honda remarks that in low dimension there are examples of open book decomposition of Stein fillable manifolds whose monodromy is not a product of Dehn twists so one cannot hope for the stronger result that any supporting open book has such kind of monodromy.
Y. Eliashberg: How to read strong fillability on monodromy data? P. Massot says that it is not even clear in dimension 3.
F. Presas: For (M, ξ) an exact fillable contact manifold, does there exist an open book decomposition whose monodromy is a product of positive and negative Dehn twists? He remarks that it is a non trivial condition since there are symplectomorphisms which are not products of Dehn twists.
M. Abouzaid: For example take $T^*\mathbb{C}P^n$ with the associated "Dehn twist" (not to confuse with a Dehn twist along a Lagrangian sphere, indeed it is not a product of Dehn twists because there are no Lagrangian spheres), is the contact manifold with the corresponding open book exactly fillable? E. Giroux remarks that there are other examples of symplectomorphisms which are not products of Dehn twists, the so-called fibered Dehn twists. For example, take $T^*\mathbb{R}P^n = \mathbb{C}P^n \setminus \text{Quadric}$, the corresponding fibered Dehn twist is not a product of Dehn twists since there are no Lagrangian spheres.
C. Wendl: Take the negative "Dehn twist" on $T^*\mathbb{C}P^n$, the associated contact manifold has probably zero contact homology (this should follow from the strategy of [7]). How does this relate to notions of overtwistedness?

1.12 Fillings

E. Murphy: What can you say about contact structures that are filled by a subcritical or flexible Weinstein manifold? Can you classify their fillings? Y. Eliashberg underlines that we have to study the topological type but also the symplectic type of fillings. A naive question would be: are they all symplectomorphic?
Y. Eliashberg: To give a concrete example, take the standard contact sphere S^{2n+1}, we know that the fillings are all diffeomorphic to the ball D^{2n+2} [29], but are they all symplectomorphic?

1.13 Contact manifolds with a lot of symmetries

Y. Karshon: Two families of contact manifolds with a lot of symmetry are

- Contact toric manifolds [26];
- Prequantization circle bundles of coadjoint orbits of Lie Groups.

Every compact contact manifold that admits a transitive action of a compact Lie group by coorientation preserving contactomorphisms lies in the second family above [4].

These families of manifolds constitute a good playground for contact topology. It can be interesting to compute their contact topological invariants.
Y. Eliashberg: Take a complex line bundle L over an integral symplectic manifold (M, ω) with first Chern class $c_1(L) = n\,\omega$. The associated circle bundle V is a contact manifold. Suppose n is large, is V Stein fillable? F. Presas asks why n is supposed to be large in this problem, and Y. Eliashberg explains that it corresponds somehow to the fact that we need very ample divisor instead of just ample. Without n being large, it might be only symplectically fillable but not

Stein fillable. E. Giroux suggests that in the case where M is a torus, the cohomology ring of V can be an obstruction to fillability (compare [39]).

M. Abouzaid asks if there are obstruction for an algebraic variety to be realized as a divisor. After some discussion, it turns out that the question has already been considered at least for hyperplane sections in [46].

1.14 Symplectization

F. Presas: Suppose two contact manifolds have symplectomorphic symplectizations, are they contactomorphic? Maybe, assume in addition that the manifolds are simply connected.

1.15 Sasakian manifolds

K. Honda: Is there anything symplectic geometry can say about Sasakian manifolds [8]? R. Komendarczyk says that many things are known in dimension 3. In all dimensions, Sasakian manifolds are fillable (see [40, 35]). A. Mori points out that there is also a theorem of D. Martínez Torres [27] that every Sasakian manifold M^{2n+1} admits a contact immersion into (S^{4n+3}, ξ_{std}) which pulls back the standard open book of S^{4n+3} to a supporting open book of M.

F. Presas: A simply-connected closed manifold is *formal* over rational (resp. real) numbers if its rational (resp. real) homotopy type can be recovered from its cohomology ring. Simply-connected closed orientable manifolds of dimension ≤ 6 are formal, and there are examples of non-formal simply-connected manifolds in any dimension ≥ 7 (see [17] and the references therein). It is known [42] that Sasakian manifolds are formal (for real homotopy type). This leads us to the question of a producing non-formal *contact* manifolds. For example, are there non-formal simply-connected closed *contact* manifolds (of dim necessarily ≥ 7)? (see [18, 3] for related work).

1.16 Exotic spheres and contact geometry

P. Massot: Let Σ^n be an exotic sphere. Is $(ST^*\Sigma^n, \xi_{std})$ contactomorphic to (ST^*S^n, ξ_{std})? Compare this also to the related result for cotangent bundles [1].

2 Tuesday

2.1 Metrics on contactomorphism group

M. Sandon: There is an integer-valued biinvariant metric on the universal cover of the contactomorphism group of any contact manifold, which was recently constructed by M. Sandon and V. Colin in [10]. It is called the *discriminant metric*. Can you find examples of contact manifolds for which this metric is unbounded? We already know that it is bounded for standard S^{2n+1} and $\mathbb{R}^{2n+1}$ and unbounded for $\mathbb{R}P^{2n+1}$ and $\mathbb{R}^{2n} \times S^1$. Are there necessary or sufficient conditions for this metric to be unbounded? Having a 1-periodic Reeb flow is not sufficient, but maybe one only needs to add the hypothesis that Reeb orbits are non contractible.

Is it compatible with the partial order constructed in [15]?

V. Colin: If there are no contractible Reeb orbits, are Reeb flows geodesics in the contactomorphism group with respect to the discriminant metric (meaning length-minimizing path)?

There is also a metric for Legendrian isotopies (in fact the metric on contactomorphism group comes from this.) In the case of $T^2 \times [-\frac{\pi}{2}, \frac{\pi}{2}]$, with contact structure $\ker\big(\cos(t)\mathrm{d}x - \sin(t)\mathrm{d}y\big)$, take the Legendrian circle $\{y = 0\}$ in $T^2 \times \{0\}$ and the isotopy that rotates this in the y direction n times. The length of this Legendrian isotopy with respect to the discriminant metric is exactly

n (see [10]). What happens for the length of this isotopy if we replace $T^2 \times [-\frac{\pi}{2}, \frac{\pi}{2}]$ by $T^2 \times \mathbb{R}$? Intuitively, it should be the same result but there is no proof at present.

E. Giroux asks if we know something in the overtwisted case, for example if T^2 is the boundary of a Lutz tube. Again, it is not known. P. Massot and C. Wendl discuss also that there may be higher dimensional analogues of this question.

M. Fraser: There is also a metric constructed by M. Fraser and L. Polterovich, and yet another one by F. Zapolsky [45]. How do they relate to each other? Are they quasi-isometric?

2.2 Lagrangian concordance

Y. Eliashberg: Let $L \subset (M, \xi)$ a Legendrian submanifold. Take an exact Lagrangian concordance Λ in the symplectization SM of M between L at the top and another Legendrian at the bottom. Then the Liouville form restricted to Λ writes $\mathrm{d}f$ for some function $f \colon \Lambda \to \mathbb{R}$ which is constant on L and uniquely defined by imposing that this constant is zero. How large can f be at the bottom? Is there a bound? Is it always unbounded? M. Abouzaid remarks that if the Reeb flow on M is complete (for example if M is a closed manifold), then flowing L along the Reeb flow while moving down in the symplectization yields f as large as we want at the bottom. However the question is interesting for manifolds with non complete Reeb flow, typically the complement of a Legendrian submanifold in a contact manifold. V. Colin points out that this might be related to the previous question about length of contact isotopies with respect to metrics on the contactomorphism group.

2.3 Loose Legendrians

E. Murphy: We know that the space of loose Legendrians is C^0-dense in the space of Legendrians (see [33]). Is it also C^0-open? That is, if we take a loose Legendrian and we C^0-perturb it, is it still loose? V. Colin remarks that in dimension 3, all knots C^0-close to a stabilized one are also stabilized, which is the 3-dimensional analogue of the question.

A. Mori: He explains that Lutz-Mori tubes can be deformed into foliations [31], and asks whether this may give restrictions on loose knots.

2.4 Open book decompositions

C. Wendl: Given a contact manifold (M, ξ), what are the constraints on open books supporting ξ? For example, M. Abouzaid formulates something vague like, given an abstract open book decomposition of a manifold with pages admitting Weinstein structure, can it support a given contact structure? F. Presas points out that it is related to the following problem: given a diffeomorphism of a Weinstein manifold which is the identity near the boundary, can it be deformed into a symplectic diffeomorphism among diffeomorphism that are the identity on the boundary? Of course a positive answer is rather unlikely and would lead to existence of contact structures in higher dimensions.

E. Giroux: gives an example of this problem: the group $\pi_0 \operatorname{Diff}(D^6, \partial D^6)$ has 28 connected components [24, 9], each of which gives an open book for the corresponding exotic 7- sphere. Can these diffeomorphisms be deformed to symplectic diffeomorphisms? We can also ask the question for higher dimensional balls.

O. van Koert: Take T^*S^2 with even multiples of the right-handed Dehn twist τ, it gives an infinite family of contact manifolds $M_k = OB(T^*S^2, \tau^{2k})$ for all $k \geq 1$. These manifolds are diffeomorphic to $S^2 \times S^3$, the contact structures are homotopic as almost contact structures and all have the same contact homology. Are they contactomorphic? E. Giroux also asks the question for negative k.

2.5 Contact structures on S^5

E. Giroux: How many different contact structures do we know on S^5? O. van Koert explains that Brieskorn spheres provide an infinite family [43]. Then it was shown that a connected sum of Brieskorn spheres is no longer a Brieskorn sphere [44], so this produces new ones. M. McLean also points out that there are infinitely many different symplectic balls D^6, and their boundaries are very likely to be non contactomorphic (but it is not yet proved). We know that there is at least one non-standard S^5 in this family because it has exponential growth of periodic Reeb orbits, and therefore cannot be the standard contact sphere. There is also a non-fillable contact structure on any sphere constructed in [37].

3 Friday

3.1 Lagrangian caps

Y. Eliashberg: Explore relation between Lagrangian caps and closed immersed Lagrangians. See the work of D. Sauvaget on closed immersed Lagrangians [41].

3.2 Plastikstufe

J. Etnyre: Suppose (M^{2n-1}, ξ) contains a Plastikstufe [34] with some core B^{n-1}, can we also find a Plastikstufe with core T^{n-1}? Can any manifold B' be realized as the core of a Plastikstufe in M?

3.3 Loose knots

K. Niederkrüger: Take $M = N_{ot} \times D^2_R$ with contact form $\alpha_{ot} + r^2\, \mathrm{d}\theta$ where α_{ot} is an overtwisted contact form on N and (r, θ) are polar coordinates on the disc D_R of radius R. What is the influence of R on the looseness of knots in M? (see [36])

3.4 Contact bundles and contactomorphism group

E. Giroux: Let (N, ξ) be a contact manifold and S a surface. We denote by G the contactomorphism group of N and by $\tilde{G}$ its universal cover. Given a contact bundle $M \to S$ with fiber (N, ξ), can we construct a contact structure on M inducing the given contact structure on the fibers? Assuming triviality of the bundle on the 1-skeleton, the bundle is described by an element of $\gamma \in \pi_1(G)$. Then the question becomes: can we find a product of commutators $\prod_{i=1}^{2g}[\varphi_i, \psi_i]$ in $\tilde{G}$ bigger (maybe smaller depending on conventions) than γ? L. Polterovich says it seems possible for S^3 and Y. Eliashberg says it should be true for orderable contact manifold. E. Giroux asks for explicit constructions of big products of commutators in $\tilde{G}$. Is there a bound on the length of such a product of commutators coming from a quasimorphism on $\tilde{G}$?
L. Polterovich: There is a related question in the symplectic case. Can we find a commutator in $\mathrm{Ham}(M, \omega)$ with arbitrary large Hofer norm?

3.5 Convex hypersurfaces

K. Honda: Let N^{2n-1} be a contact submanifold of (M^{2n+1}, ξ) with trivial normal bundle. Can the boundary of a tubular neighborhood of N be perturbed to a convex hypersurface? Y. Eliashberg suggests to try on $S^3 \subset S^5$ and in higher codimension, for example for $S^1 \subset (M^5, \xi)$.

3.6 Submanifolds with Legendrian foliations

K. Niederkrüger: Let (M^{2n+1}, ξ) be a contact manifold, develop tools to find submanifolds N^{n+1} in M with Legendrian foliation in view of applying holomorphic techniques. Y. Eliashberg adds that the foliation has to be given by a closed 1-form to control the behavior of holomorphic curves.

3.7 Liouville domain with disconnected boundary

C. Wendl: A Stein domain of (real) dimension $2n$ admits a handle decomposition with handles only of index n and lower. This is why such a manifold will always have connected boundary if its dimension is at least 4. Liouville manifolds with disconnected boundary however do exist. Examples have been constructed in dimension 4 [29], 6 [19] and then in any dimension [28] but they are still rare. Can we develop methods for finding more Liouville domains with disconnected boundary?

3.8 Contact structures on exotic spheres

Y. Eliashberg: Let $(f_t)_{t\in S^1}$ be a loop of diffeomorphisms of S^{2n-1} based at the identity, and F the diffeomorphism of $U = S^{2n-1} \times [0,1]$ given by:

$$F(t,x) = (f_t(x), t)$$

Let $U_F = U \times [0,1]/_{(x,t,1)\sim(F(x,t),0)}$ be the associated mapping torus, it has two boundary component diffeomorphic to $S^{2n-1}\times S^1$, fill in one of these by attaching $S^{2n-1}\times D^2$ to get M^{2n+1}. Can we construct a contact structure on M? Denoting by ξ the standard contact structure on S^{2n-1}, $f_t^*\xi$ is a loop of contact structure on S^{2n-1}. If this loop is contractible then f_t is isotopic to a loop in $\mathrm{Aut}(S^{2n-1}, \xi)$ and you get a contact open book on M. A related question is how to construct contact structures on homotopy spheres? Do these homotopy sphere bound manifold with half-dimensional homotopy type? (it is an obvious necessary condition to admit a Stein fillable contact structure).

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