Solution du TP Maple n°1

MG2 - automne 2010

Exemple 2

```
> restart:
> y:=sqrt(x+sqrt(x))-sqrt(x);
```

$$y := \sqrt{x+\sqrt{x}} - \sqrt{x}$$

```
> limit(y,x=infinity);
```

$$\frac{1}{2}$$

```
> y:=(sqrt(1+3*x)-1-x)/sin(x);
```

$$y := \frac{\sqrt{1+3x}-1-x}{\sin(x)}$$

```
> limit(y,x=0);
```

$$\frac{1}{2}$$

```
> y:=2/x-ln(exp(2)+x)/abs(x);
```

$$y := \frac{2}{x} - \frac{\ln(e^2+x)}{|x|}$$

```
> limit(y,x=0);
```

undefined

```
> limit(y,x=0,left);
```

$$-\infty$$

```
> limit(y,x=0,right);
```

$$-e^{-2}$$

```
>
```

Exemple 4

```
> evalf(Pi,100);
```

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706\
8

```
> y:=(exp(Pi*sqrt(163))-744)^(1/3);
```

$$y := \left(e^{\pi\sqrt{163}} - 744\right)^{1/3}$$

```
> evalf(y,20);
```

$$6.4032000000000000035\ 10^{5}$$

```
> evalf(y,100);
```

6.4031999999999999999999999999939031735231947012650283553902603366392189960936\
1481245830388535411536 9 10^{5}

```
> evalf(y,20);
```

$$6.4032000000000000000\ 10^{5}$$

Maple ne donne pas la même réponse.

```
>
```

Exemple 5

```
> restart:
> u:=sqrt(8)/9801*(4*n)!*(1103+26390*n)/((n!)^4*396^(4*n));
```

$$u := \frac{2}{9801}\,\frac{\sqrt{2}\,(4n)!\,(1103+26390n)}{n!^4\,396^{4n}}$$

```
> simplify(subs(n=0,u));
```

$$\frac{2206}{9801}\sqrt{2}$$

```
> seq(u,n=0..10);
```

$$\frac{2206}{9801}\sqrt{2},\ \frac{27493}{5021227463472}\sqrt{2},\ \frac{209545}{4573270344735880482816}\sqrt{2},$$

$$\frac{14047775}{345055306579562196311295783075 84}\sqrt{2},$$

$$\frac{8493041375}{22627584049811059597548711058942459316797 44}\sqrt{2},$$

$$\frac{9125262601}{25761146019778542344625182369022816283618307368 5504}\sqrt{2},$$

$$\frac{5281694627573}{15549520527125399719943002030279050539454438618110698369056768}\sqrt{2},$$

$$\frac{5832837933325}{17702882815727117914823543914320539419008333408411319219708964508794 88}$$

$$\sqrt{2},\ 907330013871125\,/$$

$$2814294342568707159758498994015650623934360272543431823780149420674115710 0\backslash$$

$$126208\sqrt{2},\ 20030778771093033125\,/$$

$$6306491325064404474153163655322516783916816762262199919451011393160814680 6\backslash$$

$$389926086389530624\sqrt{2},\ 1626459255411957032885\,/$$

$$5169485144709529541032684489540117235872875821849760270034741704307046289 5\backslash$$

$$5412526957061959877864194048\sqrt{2}$$

```
> S:=sum(u,n=0..10);
```

$$S :=$$

$$1163542955478442004796255409627053054450310104983883070628575192906877848 7\backslash$$

$$1920308555177681218916232885\,/$$

$$5169485144709529541032684489540117235872875821849760270034741704307046289 5\backslash$$

$$5412526957061959877864194048\sqrt{2}$$

```
> evalf(1/S-Pi,100);
```

$$4.36960080905\ 10^{-88}$$

Donc : très bonne approximation de π.

```
>
```

Exemple 6

```
> restart:
> Z:=z^2/(z+I);z:=x+I*y;evalc(Z);
```

$$Z := \frac{z^2}{z+\mathrm{I}}$$

$$z := x + \mathrm{I}\,y$$

$$\frac{(x^2-y^2)\,x}{x^2+(y+1)^2} + \frac{2\,x\,y\,(y+1)}{x^2+(y+1)^2} + \mathrm{I}\left(\frac{2\,x^2\,y}{x^2+(y+1)^2} - \frac{(x^2-y^2)\,(y+1)}{x^2+(y+1)^2}\right)$$

```
> simplify(evalc(Re(Z)));simplify(evalc(Im(Z)));
```

$$\frac{x\,(x^2+y^2+2\,y)}{x^2+y^2+2\,y+1}$$

$$\frac{x^2\,y-x^2+y^3+y^2}{x^2+y^2+2\,y+1}$$

```
> j:=exp(2*I*Pi/3);
```

$$j := -\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\,\sqrt{3}$$

```
> y:=(a+b)*(a+b*j)*(a+b*j^2);
```

$$y := (a+b)\left(a+b\left(-\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\,\sqrt{3}\right)\right)\left(a+b\left(-\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\,\sqrt{3}\right)^2\right)$$

```
> expand(y);
```

$$a^3 + b^3$$

```
>
```

Exercice 1

```
> restart:
> L:=[seq(k^2+k+41,k=0..100)];
```

$L :=$ [41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601, 1681, 1763, 1847, 1933, 2021, 2111, 2203, 2297, 2393, 2491, 2591, 2693, 2797, 2903, 3011, 3121, 3233, 3347, 3463, 3581, 3701, 3823, 3947, 4073, 4201, 4331, 4463, 4597, 4733, 4871, 5011, 5153, 5297, 5443, 5591, 5741, 5893, 6047, 6203, 6361, 6521, 6683, 6847, 7013, 7181, 7351, 7523, 7697, 7873, 8051, 8231, 8413, 8597, 8783, 8971, 9161, 9353, 9547, 9743, 9941, 10141]

```
> P:=select(isprime,L); nops(P);
```

$P :=$ [41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601, 1847, 1933, 2111, 2203, 2297, 2393, 2591, 2693, 2797, 2903, 3011, 3121, 3347, 3463, 3581, 3701, 3823, 3947, 4073, 4201, 4463, 4597, 4733, 4871, 5011, 5153, 5297, 5443, 5591, 5741, 6047, 6203, 6361, 6521, 7013, 7351, 7523, 7873, 8231, 8597, 8783, 8971, 9161, 9547, 9743, 9941, 10141]

87

```
>
```

Exercice 2

```
> restart:
> u[0]:=3:u[1]:=0:u[2]:=2:
  for n from 3 to 1000 do u[n]:=u[n-3]+u[n-2] od:
> u[1000];length(u[1000]);
```

1328689313406067435318416601959683287866715714172702822904753842943337079165\
9

749605799581300930607309368646727264843529312 5

123

```
> L:=[$2..1000]: # liste des entiers de 2 à 1000
> L1:=select(isprime,L):L2:=select(n->irem(u[n],n)=0,L):
> evalb(L1=L2);
```

true

```
>
```