

Solution du TP Maple n°2

MG2 - automne 2010

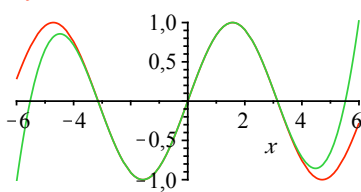
Exercice 1

```
> restart;
> eq:=cos(x)=1-x^2/2+x^4/24*cos(theta*x);
      eq:=cos(x)=1-1/2*x^2+1/24*x^4*cos(theta*x)
> s:=solve(eq, theta);
      s:=arccos(12(2*cos(x)-2+x^2)/x^4)/x
> limit(s, x=0, right);
      1/15*sqrt(15)
```

Exercice 2

```
> restart;
> y:=(a*x+b*x^3+c*x^5)/(1+d*x^2+e*x^4);
      y:=ax+bx^3+cx^5/(1+dx^2+ex^4)
> t:=taylor(y-sin(x), x=0, 12);
      t:=(a-1)x+(b-ad+1/6)x^3+(c-ae+(-b+ad)d-1/120)x^5+(1/5040+(-b+ad)e+(-c+ae+db-ad^2)d)x^7+((-c+ae+db-ad^2)e+(eb-2ead+dc-d^2b+ad^3)d-1/362880)x^9+((eb-2ead+dc-d^2b+ad^3)e+(ec-ae^2-2edb+3ead^2-d^2c+d^3b-ad^4)d+1/39916800)x^11+O(x^12)
> eqs:=seq(coeff(t, x, i)=0, i=0..10);
      eqs:=0=0, a-1=0, 0=0, b-ad+1/6=0, 0=0, c-ae+(-b+ad)d-1/120=0, 0=0, 1/5040+(-b+ad)e+(-c+ae+db-ad^2)d=0, 0=0, (-c+ae+db-ad^2)e+(eb-2ead+dc-d^2b+ad^3)d-1/362880=0, 0=0
> vars:=a,b,c,d,e;
      vars:=a, b, c, d, e
```

```
> s:=solve({eqs}, {vars});
      s:={a=1, e=5/11088, d=13/396, b=-53/396, c=551/166320}
> ysol:=subs(s, y);
      ysol:=x-53/396*x^3+551/166320*x^5/(1+13/396*x^2+5/11088*x^4)
> taylor(ysol-sin(x), x=0, 12);
      11/457228800*x^11+O(x^12)
> plot({sin(x), ysol}, x=-6..6, view=[-6..6, -1..1]);
```



Exercice 3

```
> restart;
> eq:=x^2-3*x*y+y^2-1;
      eq:=x^2-3xy+y^2-1
> s:=isolve(eq);
      s:=
Pas de résultat, alors que le couple (0,1) est solution évidente.
Montrons que les couples (F_{2k}, F_{2k+2}) sont solutions, par récurrence sur k, sachant que c'est vrai pour k=0 :
> hyp:=subs(x=F[2*k], y=F[2*k+2], eq); # hypothèse de récurrence
      hyp:=F_{2k}^2-3F_{2k}F_{2k+2}+F_{2k+2}^2-1
> eq2:=subs(x=F[2*k+2], y=F[2*k+4], eq); # il faut montrer que eq2=0
      eq2:=F_{2k+2}^2-3F_{2k+2}F_{2k+4}+F_{2k+4}^2-1
> simplify(eq2, {hyp, seq(F[2*k+i+2]=F[2*k+i]+F[2*k+i+1], i=0..2)});
      0
# QQFD
```

Exercice 4

```
> restart;
On met le problème en équations (voir dessin à la fin) :
> AB:=6:CD:=4:EH:=2:
> eqs:=(AB^2-AC^2)/AC^2=EH^2/(AC-HC)^2, (CD^2-AC^2)/AC^2=
```

EH²/HC²;

$$eqs:=\frac{36-AC^2}{AC^2}=\frac{4}{(AC-HC)^2},\quad \frac{16-AC^2}{AC^2}=\frac{4}{HC^2}$$

> `s:=solve({eqs},{AC,HC});`

$$s:=\left\{AC=2\operatorname{RootOf}(_Z^8-22_Z^6+163_Z^4-454_Z^2+385,\text{label}=_LI),HC=-\frac{1}{5}\operatorname{RootOf}(_Z^8-22_Z^6+163_Z^4-454_Z^2+385,\text{label}=_LI)^5+\frac{9}{5}\operatorname{RootOf}(_Z^8-22_Z^6+163_Z^4-454_Z^2+385,\text{label}=_LI)^3-\operatorname{RootOf}(_Z^8-22_Z^6+163_Z^4-454_Z^2+385,\text{label}=_LI)\right\}$$

> `lsol:=subs(s,AC):`

> `a:=allvalues(lsol): # on n'affiche pas car c'est monstrueux evalf(a);`

2.462371447, 3.746327110, 5.830229976 + 0.2017462782 I, -5.830229976 + 0.2017462782 I, -2.462371447, -3.746327110, -5.830229976 - 0.2017462782 I, 5.830229976 - 0.2017462782 I

Seule la 1ère valeur est plausible :

> `solution:=simplify(a[1]);`

`solution:=`

$$\begin{aligned} & \frac{1}{15} \left(\sqrt{3} \sqrt{10} \left(33 \cdot 5^{1/3} (24\sqrt{3} + \sqrt{13})^{1/3} (37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3}))^{1/3} \right. \right. \\ & + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} + 1225)^{1/4} - \sqrt{3} (37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} \\ & + 48\sqrt{13}\sqrt{3})^{1/3} + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} + 1225)^{3/4} \\ & \left. - \sqrt{3} (74 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3}))^1 \right) \\ & \sqrt[3]{37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} + 1225} \\ & - 5 \sqrt{37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} + 1225} \cdot 5^{1/3} \\ & (1741 + 48\sqrt{13}\sqrt{3})^{2/3} \\ & - 1225 \sqrt{37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} + 1225} \\ & + 240\sqrt{3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} (24\sqrt{3} + \sqrt{13})^{1/3} \left. \right)^{1/2} \cdot 5^{1/3} \left. \right) / \left((24\sqrt{3} + \sqrt{13})^{1/6} (37 \cdot 5^{2/3} (1741 + 48\sqrt{13}\sqrt{3})^{1/3} + 5 \cdot 5^{1/3} (1741 + 48\sqrt{13}\sqrt{3})^{2/3} \right. \end{aligned}$$

+ 1225)^{1/8})

On fait le dessin :

> `with(plots):g1:=plot([[0,0],[2.47,5.48],[2.47,0],[0,3.15],[0,0],[2.47,0]],[[0.9,0],[0.9,2]]],axes=None,scaling=constrained,color=[blue,red]);g2:=textplot({[-0.2,-0.5,'A'],[2.5,-0.5,'C'],[0.9,-0.5,'H'],[1.2,2,'E'],[-0.3,3.15,'D'],[2.7,5.47,'B']});display(g1,g2);`

