

# Solution du TP Maple n°3

MG2 - automne 2010

## Exercice 1

```

> restart;
> A:=X^n-n*X^7+1; B:=X^5+2*X^4-6*X^3-4*X^2+13*X-6;
      A:=X^n - nX^7 + 1
      B:=X^5 + 2X^4 - 6X^3 - 4X^2 + 13X - 6
> eq:=A-B*Q(X)+R;
> R:=add(a[i]*X^i,i=0..4);
      R:=a_0 + a_1X + a_2X^2 + a_3X^3 + a_4X^4
> racines:=solve(B);
      racines:= -3, -2, 1, 1, 1
> eqs:=subs(X=-3,eq),subs(X=-2,eq),subs(X=1,eq),subs(X=1,diff
(eq,X)),subs(X=1,diff(eq,X$2));
eqs:= (-3)^n + 2187n + 1 = a_0 - 3a_1 + 9a_2 - 27a_3 + 81a_4, (-2)^n + 128n + 1 = a_0 - 2a_1
+ 4a_2 - 8a_3 + 16a_4, 2 - n = a_0 + a_1 + a_2 + a_3 + a_4, -6n = a_1 + 2a_2 + 3a_3 + 4a_4, n^2
- 43n = 2a_2 + 6a_3 + 12a_4
> vars:=coeffs(R,X);
      vars:= a_0, a_1, a_4, a_3, a_2
> sols:=solve({eqs},{vars});
sols:= {a_0 = 1/9 (-1)^n 2^n - 1465/24 n + 553/288 + 1/32 (-1)^(1+n) 3^n + 1/4 n^2, a_1 = 20551/144 n
- 7/24 n^2 + 377/1728 + 5/64 (-1)^n 3^n + 8/27 (-1)^(1+n) 2^n, a_2 = -1/8 n^2 - 3427/48 n - 101/576
+ 1/64 (-1)^(1+n) 3^(1+n) + 1/9 (-1)^n 2^(1+n), a_3 = 1/64 - 627/16 n + 1/64 (-1)^(1+n) 3^n
+ 1/8 n^2, a_4 = 1/64 (-1)^n 3^n + 4019/144 n + 37/1728 + 1/24 n^2 + 1/27 (-1)^(1+n) 2^n}
> Rsol:=subs(sols,R);
Rsol:= 1/9 (-1)^n 2^n - 1465/24 n + 553/288 + 1/32 (-1)^(1+n) 3^n + 1/4 n^2 + (20551/144 n - 7/24 n^2
+ 377/1728 + 5/64 (-1)^n 3^n + 8/27 (-1)^(1+n) 2^n) X + (-1/8 n^2 - 3427/48 n - 101/576
+ 1/64 (-1)^(1+n) 3^(1+n) + 1/9 (-1)^n 2^(1+n)) X^2 + (1/64 - 627/16 n + 1/64 (-1)^(1+n) 3^n
+ 1/8 n^2) X^3 + (1/64 (-1)^n 3^n + 4019/144 n + 37/1728 + 1/24 n^2 + 1/27 (-1)^(1+n) 2^n) X^4
> # vérification pour n=100 :
> R1:=rem(subs(n=100,A),B,X); R2:=subs(n=100,Rsol);

```

```

R1:= -16105547522875353954039343613150175716751618623
+ 8052773761437677000494682921912669367574018225 X^4
- 8052773761437677047444705152587832385970432700 X^3
- 24158321284313030860633982073712519047532808400 X^2
+ 40263868807188384861623347917537857782680841400 X
R2:= -16105547522875353954039343613150175716751618623
+ 8052773761437677000494682921912669367574018225 X^4
- 8052773761437677047444705152587832385970432700 X^3
- 24158321284313030860633982073712519047532808400 X^2
+ 40263868807188384861623347917537857782680841400 X
> R1-R2;
      0
>

```

## Exercice 2

```

> restart;
> product(X-r,r=RootOf(X^7-2,X));
      X^7 - 2
> H:=product(product(X-r-s,r=RootOf(X^7-2,X)),s=RootOf(X^17
-3));
> H:=sort(H);
H:= X^119 - 34X^112 + 544X^105 - 21X^102 - 5440X^98 - 10295880X^95 + 38080X^91
- 11865431928X^88 + 189X^85 - 198016X^84 - 1753404217656X^81 - 958322232X^78
+ 792064X^77 - 73058971757184X^74 + 10251916815168X^71 - 2489344X^70
- 945X^68 - 1171409550359328X^67 - 5650506012497976X^64 + 6223360X^63
- 10593209592X^61 - 8407568031198336X^60 + 441668319396677856X^57
- 12446720X^56 - 176998340024352X^54 - 29134921153548672X^53 + 2835X^51
- 7338824689392054432X^50 + 19914752X^49 - 87673540330647936X^47
- 50194522347443712X^46 - 22874426568X^44 + 31059905300189006976X^43
- 25346048X^42 - 3346477961073716448X^40 - 42660269259761664X^39
+ 156757268001600X^37 - 35439911729774568576X^36 + 25346048X^35 - 5103X^34
- 14193305989102745568X^33 - 17061635654784000X^32 - 15692214152659968X^30
+ 10451514240106845696X^29 - 19496960X^28 - 8687592144X^27
- 7277809809405502080X^26 - 2897877384880128X^25 + 49170413590869312X^23
- 679128586744172544X^22 + 11141120X^21 - 4271574360816X^20
- 388388753217360000X^19 - 170567675228160X^18 + 5103X^17
- 5655170032317792X^16 + 6775977758957568X^15 - 4456448X^14
- 5054275195200X^13 - 1308242376709632X^12 - 2228765736960X^11

```

```

- 198486288 X^10 + 13829136531840 X^9 - 4436523012096 X^8 + 1114112 X^7
- 25744920768 X^6 - 56418840576 X^5 - 1666990080 X^4 + 13880160 X^3
- 30844800 X^2 + 4386816 X - 133259
> # vérification :
> irreduc(H);
true
> simplify(subs(X=2^(1/7)+3^(1/17),H));
0
> # autre méthode :
> e:=eliminate({X^7-2,(Y-X)^17-3},{X}):
> Hbis:=sort(op(e[2]));
Hbis:=Y^119 - 34 Y^112 + 544 Y^105 - 21 Y^102 - 5440 Y^98 - 10295880 Y^95 + 38080 Y^91
- 11865431928 Y^88 + 189 Y^85 - 198016 Y^84 - 1753404217656 Y^81 - 958322232 Y^78
+ 792064 Y^77 - 73058971757184 Y^74 + 10251916815168 Y^71 - 2489344 Y^70
- 945 Y^68 - 1171409550359328 Y^67 - 5650506012497976 Y^64 + 6223360 Y^63
- 10593209592 Y^61 - 8407568031198336 Y^60 + 441668319396677856 Y^57
- 12446720 Y^56 - 176998340024352 Y^54 - 29134921153548672 Y^53 + 2835 Y^51
- 7338824689392054432 Y^50 + 19914752 Y^49 - 87673540330647936 Y^47
- 50194522347443712 Y^46 - 22874426568 Y^44 + 31059905300189006976 Y^43
- 25346048 Y^42 - 3346477961073716448 Y^40 - 42660269259761664 Y^39
+ 156757268001600 Y^37 - 35439911729774568576 Y^36 + 25346048 Y^35 - 5103 Y^34
- 14193305989102745568 Y^33 - 17061635654784000 Y^32 - 15692214152659968 Y^30
+ 10451514240106845696 Y^29 - 19496960 Y^28 - 8687592144 Y^27
- 7277809809405502080 Y^26 - 2897877384880128 Y^25 + 49170413590869312 Y^23
- 679128586744172544 Y^22 + 11141120 Y^21 - 4271574360816 Y^20
- 388388753217360000 Y^19 - 170567675228160 Y^18 + 5103 Y^17
- 5655170032317792 Y^16 + 677597758957568 Y^15 - 4456448 Y^14
- 5054275195200 Y^13 - 1308242376709632 Y^12 - 2228765736960 Y^11
- 198486288 Y^10 + 13829136531840 Y^9 - 4436523012096 Y^8 + 1114112 Y^7
- 25744920768 Y^6 - 56418840576 Y^5 - 1666990080 Y^4 + 13880160 Y^3
- 30844800 Y^2 + 4386816 Y - 133259
> # vérification :
> Hbis-subs(X=Y,H);
0
>

```

### Exercise 3

```

> restart;
> S:=series(1/((1-X)*(1-X^2)*(1-X^4)),X=0,401);
> coeff(S,X,400);

```

```

10201
> L:=[seq(coeff(S,X,i),i=0..40)];
L:= [1, 1, 2, 2, 4, 4, 6, 6, 9, 9, 12, 12, 16, 16, 20, 20, 25, 25, 30, 30, 36, 36, 42, 42, 49, 49, 56,
56, 64, 64, 72, 72, 81, 81, 90, 90, 100, 100, 110, 110, 121]
> with(genfunc):
> eq:=rgf_findrecur(20,L,u,n);
eq:=u(n)=u(n-1)+u(n-2)-u(n-3)+u(n-4)-u(n-5)-u(n-6)+u(n-7)
> y:=rsolve({eq,seq(u(i)=L[i+1],i=0..6)},u);
y:= 5/32 (-1)^n + (1/16 n + 1/16) (-1)^n + (1/16 - 1/16 I) I^n + (1/16 + 1/16 I) (-I)^n
+ 1/8 (n+1) (1/2 n + 1) + 1/4 n + 17/32
> assume(n,integer): z:=evalc(Re(y));
z:= 5/32 (-1)^n + (1/16 n + 1/16) (-1)^n + 1/16 n^2 + 7/16 n + 21/32 + 1/8 cos(1/2 n pi)
+ 1/8 sin(1/2 n pi)
> # vérification pour n=400 :
> simplify(subs(n=400,z));
10201
>

```