Abstracts of the Student Seminar

June 24, 2011

Pedro Acosta ***(University of Michigan)*** **: Orbifold Groupoids**

I will present a brief introduction to the notion of orbifold groupoids. Intuitively, orbifolds are topological spaces that locally look like the quotient of $\mathbb{R}^n$ by a finite group. However, this definition of orbifold is not the most appropiate when it comes to defining homotopy groups and other constructions on a given orbifold. In this talk, I will present an alternative definition of orbifold using the concept of topological groupoids, and I will show how to define homotopy groups using this alternative definition.

Roman Budylin ***(Moscow State University)*** **: Moduli of irreducible representations.**

Let G be some group. We want to define some algebraic structure M_G on the set of irreducible G-representations. It is natural to consider families of representations for this purpose. Let S be some scheme. Then family of representations is a locally free sheaf $\mathcal{E}$ with homomorphism of abstract groups $G \to Aut_S(\mathcal{E})$. We want to find a fine moduli space for such families. It doesn't exist as a scheme, but does exist as a stack. I consider examples of finite groups and reductive groups. In this cases moduli space comes out to be an algebraic stack. Another interesting question is functoriality of M_G on G. The talk is based on materials from Moscow seminar of A.N. Parshin.

Emily Clader ***(University of Michigan)*** **: Localization Computations in Gromov-Witten Theory.**

The technique of localization provides a powerful tool for computing integrals over a scheme equipped with a torus action by reducing them to integrals over the components of the fixed-point locus. In particular, the moduli space of stable maps into projective space admits such an action, and via localization one can exploit this action to compute Gromov-Witten invariants. We will present the statement of the localization theorem as well as some of its basic applications in Gromov-Witten theory.

Bashar Dudin ***(Université Joseph Fourier)*** **: The ELSV model of the Hurwitz stack.**

Lando, Ekhedal, Vahenstein and Shapiro defined a cone over $\overline{M}_{g,n}$ parametrizing in some sense covers of the projective line with given ramification over a fixed point. We give a modular construction of this stack-cone, and study its geometry, in particular the relationships to the Hurwitz stacks and the stack of stable maps.

Sara Angela Filippini ***(Università degli Studi dell'Insubria)*** **: A rigid Calabi-Yau threefold.**

Abstract to come.

Patricio Gallardo ***(Stony Brook University)*** **: Maximal Semistability on Plane Curves and Variation of GIT Quotients.**

Maximal semistable plane curves are the strictly semistable curves (in the GIT Sense) that become stable under any perturbation. In this talk, we discuss the combinatorics and singularities of maximal semistable plane curves. We describe their Newton polytope, their Hilbert-Mumford numerical function, and algorithms for computing both objects. As an application, we give bounds for the Milnor number of maximal semistable plane curve singularities, and address the problem of counting their monomial configurations. Finally, we discuss the GIT stability of pairs of curves. This construction depends on a choice of linearization, we discuss an algorithm to find the critical linearizations (those choices where the stability changes) and the maximal semistable configurations in this relative situation.

Alisa Knizel and Alexander Neshitov ***(St. Petersburg State University)*** **: Algebraic analogue of Atiyah's theorem.**

In topology there is a well known theorem of Atyiah which states that for a connected Lie group G there is an isomorphism $\widehat{R(G)} \cong K_0(BG)$ where BG is the classifying space of G. In the present paper we consider an algebraic analogue of this theorem. In the paper of B.Totaro [Tot] there is a computation of $\varprojlim K_0(BG_i)$ for specially selected sequence BG_i. However, to compute $K_0(BG)$ one needs to prove that $\varprojlim^1 K_1(BG_i)$ vanishes. For split reductive groups we present another aproach and prove that the Borel construction induces a ring isomorphism $\widehat{R(G)}_{I(G)} = K_0(BG)$, where BG is an étale classifying space introduced by Voevodsky and Morel in [VM]. Our approach makes possible to compute $K_i(BG)$, which we expect to provide in a next paper.

References

[VM] V. Voevodsky, F. Morel A^1-homotopy theory of schemes. Publications Mathématiques de l'IHES(90)

[Tot] The Chow Ring of a Classifying Space. Proc. Symp. Pure Math. (K-Theory, 1997)

Uros Kuzman ***(University of Ljubljana)*** **: On the theory of J-holomorphic discs.**

Let (M, J) be a smooth manifold with a smooth almost complex structure J. In general the existence of complex submanifolds is a problem of an overdetermined PDE system, and therefore, generically there are no solutions. A special case are complex submanifolds of complex dimension 1, usualy called pseudo-holomorphic curves. We present the basic notion of pseudo-holomorphic discs and the fundamental theorem of Nijenhuis-Woolf on their existence. The proof is based on the implicit function theorem giving a parametrization of small J-holomorphic discs by holomorphic ones. We also give a brief discussion on the latest results in case of big discs where the linearized Cauchy-Riemann operator turns out to be Fredholm.

Alessio Lo Giudice ***(SISSA)*** **: A compactification of the Moduli space of Principal Higgs bundles over singular curves.**

A principal Higgs bundle (P, ϕ) over a singular curve X is a pair consisting of a principal bundle P and a morphism $\phi : X \to AdP \otimes \Omega^1_X$. We construct the moduli space of Principal Higgs G-bundles over a nodal curve X using the theory of decorated vector bundles, more precisely given a faithful representation $\rho : G \to GL_r(\mathbb{C})$ of G, we consider pairs (E, f) where E is a vector bundle of rank r over X and $f : E_\rho \to L$ is a morphism of bundles, being L a line bundle and

E_ρ a vector bundle depending on the Higgs field ϕ. Moreover, we show that this moduli space coincides with the space of framed bundles for suitable representations.

Gunnar Magnusson *(Université Joseph Fourier)* : Canonical metrics and moduli spaces.

Every complex curve carries a canonical Kähler metric. We will use these metrics to construct the Weil-Petersson metric on the moduli space of curves. If time permits, we will see how properties of the canonical metrics translate into geometric statements about the moduli space of curves.

Clélia Pech *(Université Joseph Fourier)* : Quantum cohomology of the odd symplectic Grassmannian of lines.

Odd symplectic Grassmannians are a generalization of symplectic Grassmannians to odd-dimensional spaces. We will compute the classical and quantum cohomology of the odd symplectic Grassmannian of lines. Although these varieties are not homogeneous, we obtain Pieri and Giambelli formulas that are very similar to the symplectic case. We notice that their quantum cohomology is semi-simple, which enables us to check Dubrovin's conjecture for this case.

Flavia Poma *(SISSA)* : Gromov-Witten invariants in positive and mixed characteristic.

I will describe how to define Gromov-Witten invariants for smooth projective schemes over any field and, more generally, over a regular scheme, focusing on the construction of a virtual fundamental class. I will show that they satisfy the fundamental axioms and some more properties (*e.g.* WDVV equation, reconstruction theorem). If there is time, I will present a result on the comparison of invariants in different characteristics for smooth projective schemes defined in mixed characteristic.

Alexandra Popa *(Stony Brook University)* : Two-point Gromov-Witten formulas for symplectic toric manifolds.

In 2007 A. Zinger computed certain two-point genus zero hyperplane integrals in Gromov-Witten theory. In 2010 A. Zinger and the speaker extended these results to projective complete intersections. Then, the speaker extended most of these results to symplectic toric manifolds. I will explain ideas behind the proof of these formulas in the projective setting and how they extend to the toric case.

Nathan Priddis *(University of Michigan)* : FJRW ring and Mirror symmetry.

Several years ago Fan, Jarvis and Ruan developed a theory that plays an important role in Mirror Symmetry. It is also being used currently to describe a Landau-Ginzburg/Calabi Yau correspondence. In this theory one constructs a state space with a quantum multiplication structure, which yields a Frobenius algebra. I will briefly outline the construction of this ring, with some examples. I will also explain some results relating to mirror symmetry of Landau-Ginzburg models.

Dustin Ross *(Colorado State University)* : Open Gromov-Witten Theory and the Crepant Resolution Conjecture.

The crepant resolution conjecture relates the GW theory of an orbifold to the GW theory of a crepant resolution. We suspect that the CRC for toric Calaby-Yau threefolds can be approached locally via "open" GW theory. In this talk I will lay out the foundations for open GW theory of CY threefolds and describe the recent success of the local approach for the specific orbifold $[\mathcal{O}(-1) \oplus \mathcal{O}(-1)/\mathbb{Z}_2]$. This is joint work with my advisor Renzo Cavalieri.

Pablo Solis *(Berkeley)* : G-bundles on Curves and Loop Groups.

Recently Frenkel, Teleman, and Tolland have constructed a geometric completion of the stack of maps from stable marked curves to the quotient stack $[pt/GL(1)]$. This completion is used to define gauge-theoretic Gromov-Witten invariants. I'd like to discuss this construction and give an alternative construction using the representation theory of the loop group $LGL(1)$. If time allows I'll present a preliminary results on how to construct a similar completion with $GL(1)$ replaced by a semisimple group G, again using the representation theory of the loop group LG.

Loek Spitz *(University of Amsterdam)* : An ELSV type formula relating completed Hurwitz numbers to the moduli space of spin curves.

Hurwitz numbers count the number of covers of $\mathbb{P}^1$ with given ramification over two special points, and simple ramification elsewhere. It is well known that these numbers are related in several ways to intersection theory on moduli spaces of curves and Gromov-Witten theory. In this talk I will define a type of Hurwitz number where the ramification over the non-special points is given by completed $r+1$-cycles. These completed Hurwitz numbers are conjecturally related to the moduli space of r-spin curves by an analog of the ELSV formula ([Zvo]). I will explain this formula and give you an idea of a proof of a special case, based on joint work with Sergey Shadrin and Dimitri Zvonkine ([SSZ]).

References

[Zvo] D. Zvonkine, A preliminary text on the r-ELSV formula, preprint 2006.

[SSZ] S. Shadrin, L. Spitz, D. Zvonkine, On Double Hurwitz numbers with completed cycles, arXiv:1103.3120.

Zhiyu Tian *(Stony Brook University)* : Symplectic geometry and rationally connected varieties.

A smooth projective variety is called rationally connected if there is a rational curve through any two given points. In the 90's, Kollar conjectured that rational connectedness is preserved under symplectic deformation. I will sketch a proof of the conjecture for 3-folds and explain an idea of Kollar which reduces the conjecture to the study of the symplectic analogues of two properties of rationally connected varieties, the symplectic MRC quotient and the symplectic Graber-Harris-Starr theorem. I will also discuss in more detail the symplectic Graber-Harris-Starr theorem.

Amos Turchet *(Università degli Studi di Udine)* : Algebraic Hyperbolicity and Vojta conjectures in Cubic surfaces.

The problem of determining whether there exist infinitely many integral points on varieties is an important open problem in Arithmetic Geometry. For algebraic surfaces the problem focuses on the density of integer points with respect to Zariski topology. The celebrated Vojta's conjecture asserts that for a surface of log-general type the integral points are not Zariski dense. This still unproved conjecture admits two analogous in complex analysis and in algebraic function field theory. In this latter case it predicts that given a smooth curve C and a finite subset $S \subseteq C$, there should exist a bound for the degree (in a suitable projective embedding) for the images of non-constant morphisms $C \setminus S \to X$, where X is again an algebraic variety of log-general type. In the arithmetic and function field case, starting from their proof of Siegel's theorem via Schmidt's Subspace Theorem, Pietro Corvaja and Umberto Zannier solved several particular cases of Vojta's conjecture in the last years.

In this talk we present some recent results, namely for the complement of the union of a conic and two lines in the projective plane and for a smooth cubic surface minus two completely reducible hyperplane sections. In this latter case we describe our personal work proving that in general the only families of curves where infinite integral points lay are the 21 lines that remain on the surface.

Michel van Garrel *(California Institute of Technology)* : Higher rank Donaldson-Thomas invariants.

Donaldson-Thomas invariants of smooth complex three-folds are defined either via integration of a virtual fundamental class, or as weighted Euler characteristic. The former, earlier version works for moduli spaces of ideal sheaves (rank 1). While the latter applies to moduli spaces of sheaves of any rank, it assumes that the variety is Calabi-Yau. I will discuss elements of these constructions and talk about an attempt to generalize the first one to moduli spaces of higher rank sheaves for a class of varieties that are not Calabi-Yau.