Entanglement scaling in 1d critical states (slightly) beyond the usual CFT

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Entanglement in Quantum Systems, Florence 2018

Outline

Work based on

JMS [unpublished]

JMS [1707.06625, J. Stat. Mech 2017]

J. Dubail, JMS & P. Calabrese [1705.00679, Scipost 2017]
J. Dubail, JMS, J. Viti & P. Calabrese [1606.04401, Scipost 2017]
N. Allegra, J. Dubail, JMS & J. Viti [1512.02872, J. Stat. Mech 2016]
JMS & J. Dubail [1105.4846, J. Stat. Mech 2011]

M. Brockmann & JMS [1705.08505, J. Phys. A 2017]
JMS, G. Misguich & V. Pasquier [1104.2544, PRB 2011]
JMS, S. Furukawa, G. Misguich & V. Pasquier [0906.1153, PRB 2009]

General wisdom for Entanglement scaling in 1+1d

- Ground state of a gapped Hamiltonian with local interactions.

  Area law: $S(\ell) \sim \ell^{d-1}$ [Srednicki 1993; Hastings, 2004]

- There can be mild (log) violations for critical systems

  1+1d CFT $S(L) \sim \frac{c}{3} \log L$
  [Holzhey, Larsen & Wilczek 1994; Calabrese & Cardy 2004]

  Systems with a Fermi surface $S(L) \sim L^{d-1} \log L$
  [Wolf 2006; Gioev & Klich 2006]

After a quantum quench (still critical)

- Local quench $S(t) \sim \frac{c}{3} \log t$. [Calabrese, Cardy 2007]
- Global quench $S(t) \sim t$. [Calabrese, Cardy 2005]

NB: Those are pure CFT calculations.

Chaotic systems: random circuits calculations [Nahum, Ruhman, Vijay & Haah 2017] also give $S(t) \sim t$, but have no notion of local quench.

This talk

Try to look for examples where the previous considerations do not apply.

1. What happens in inhomogeneous systems?

2. Do all gapless states in 1d scale at least like log?

1. Inhomogeneous systems

Examples of inhomogeneous systems in $1+1=2$

- Free Fermi gas in a trap

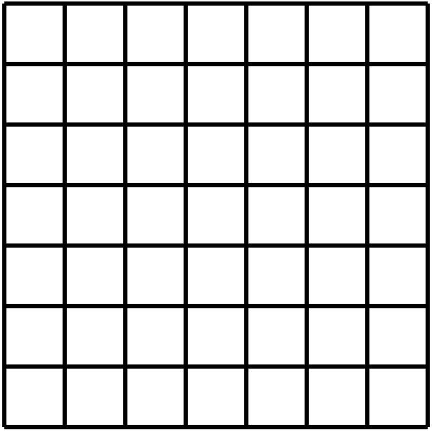
$$H = \int dx \psi^\dagger(x) \left[-\frac{\partial^2}{\partial x^2} - \mu + V(x) \right] \psi(x)$$

- Spin chain with position dependent couplings (e. g. Heisenberg).

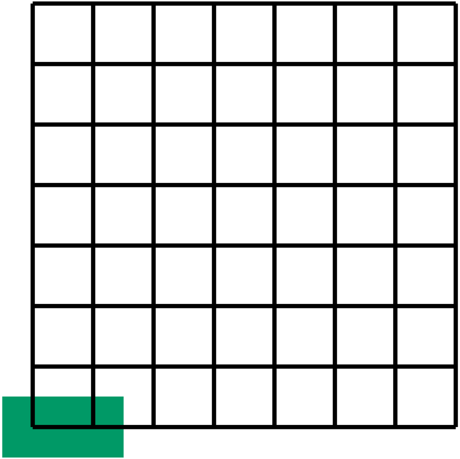
$$H = \sum_{j=1}^{L} J(j/L)\, \mathbf{S}_j . \mathbf{S}_{j+1}$$

- Entanglement Hamiltonians of homogeneous systems.
- Inhomogeneous quenches.
- Arctic circle (2d statistical mechanics).

What is the arctic circle?



Fun with dimers



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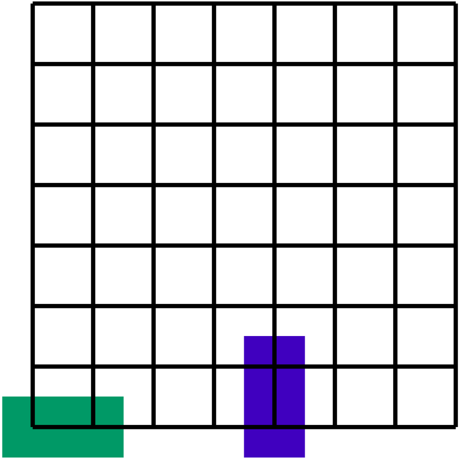

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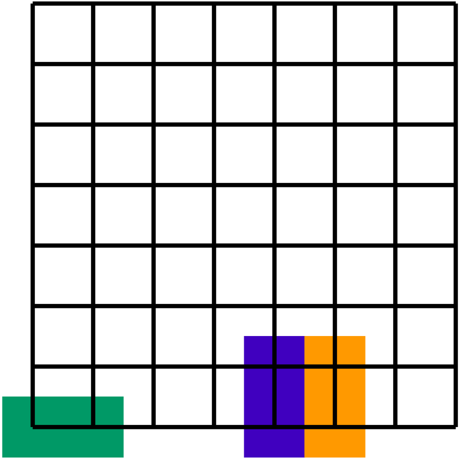

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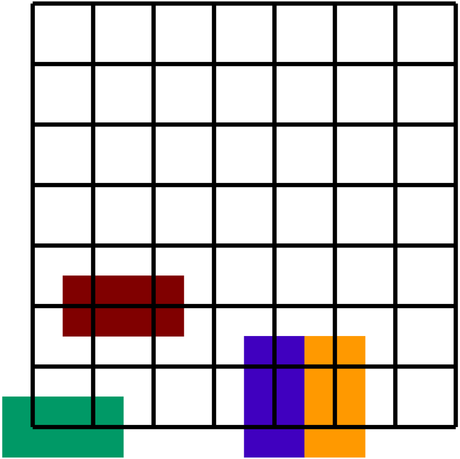

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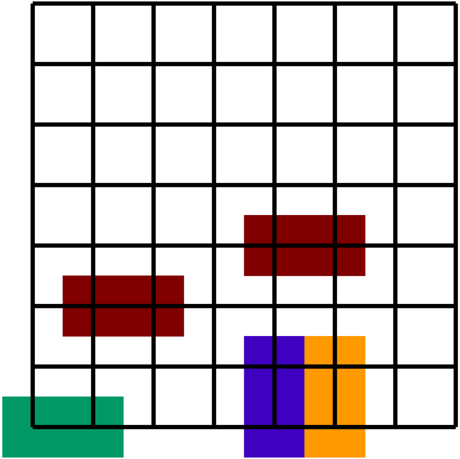

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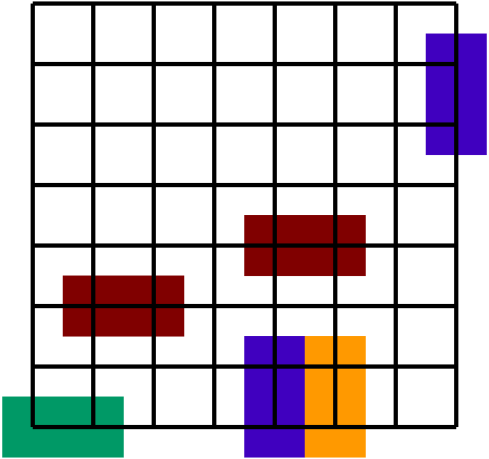

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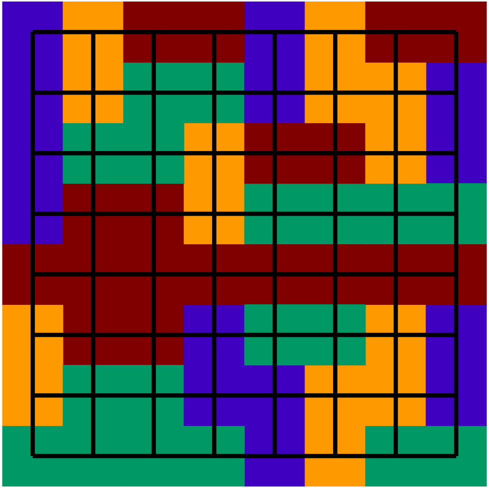

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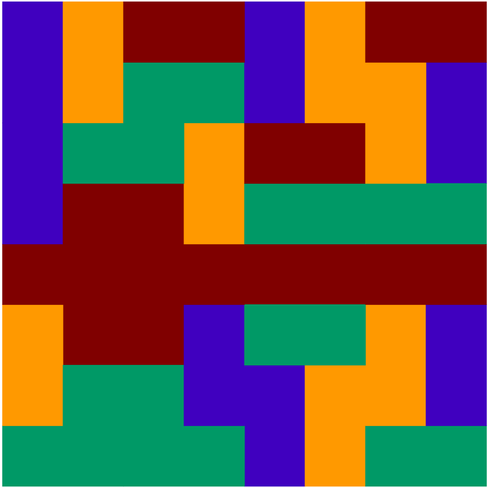

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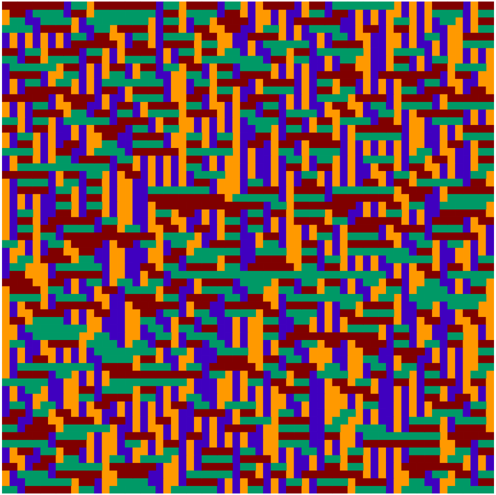

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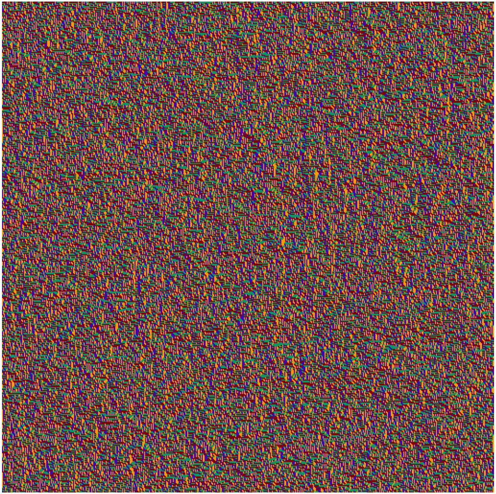
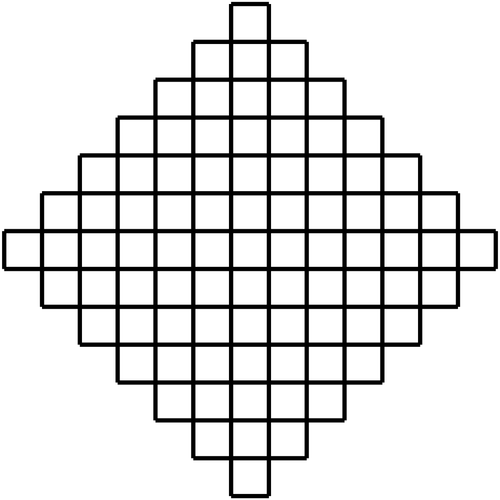
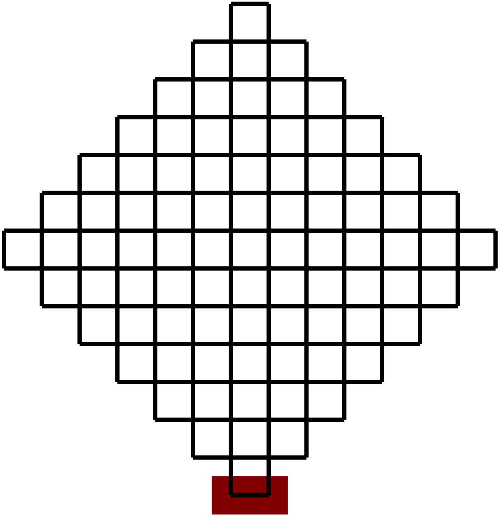


Long range correlations: gaussian free field, or coulomb gas, or free compact boson CFT $(c = 1)$, or euclidean Luttinger liquid.

Dimer coverings on the Aztec diamond



Dimer coverings on the Aztec diamond



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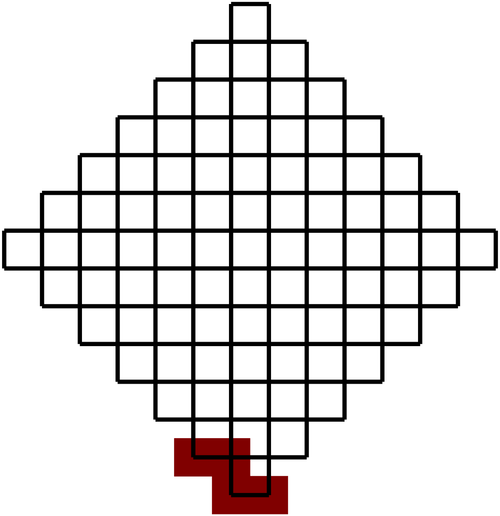

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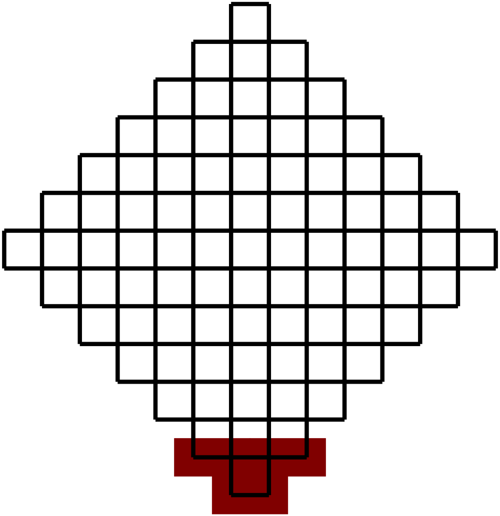

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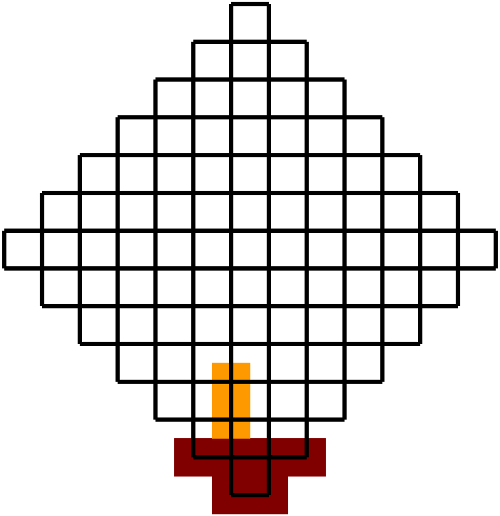

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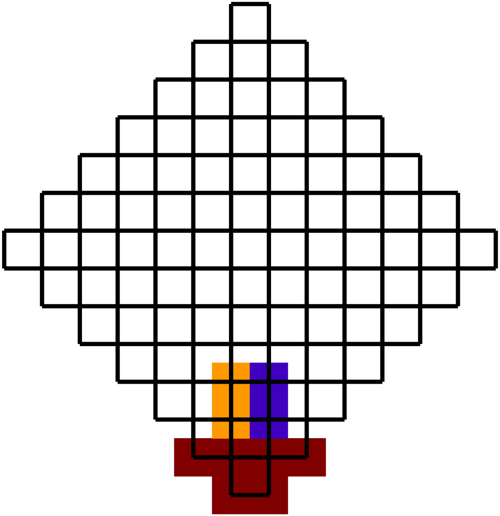

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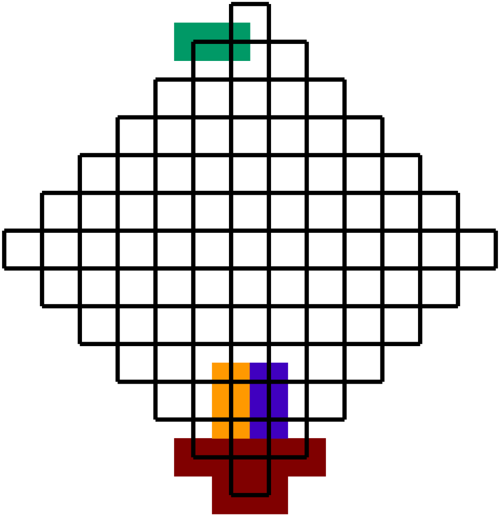

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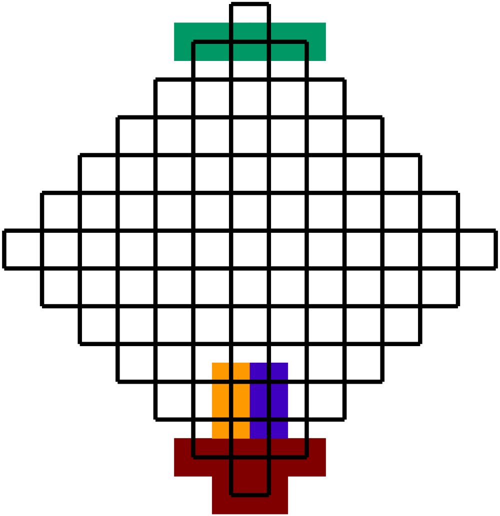

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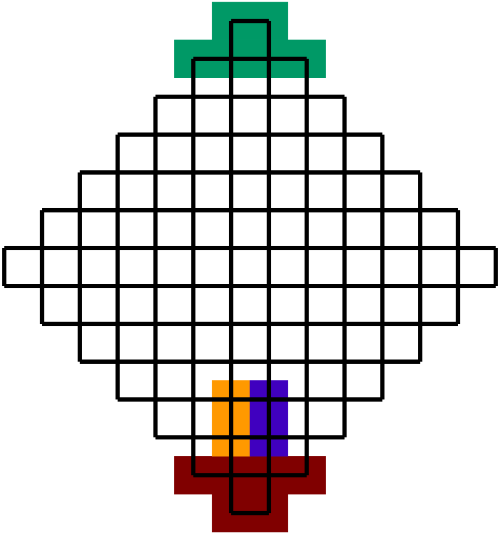

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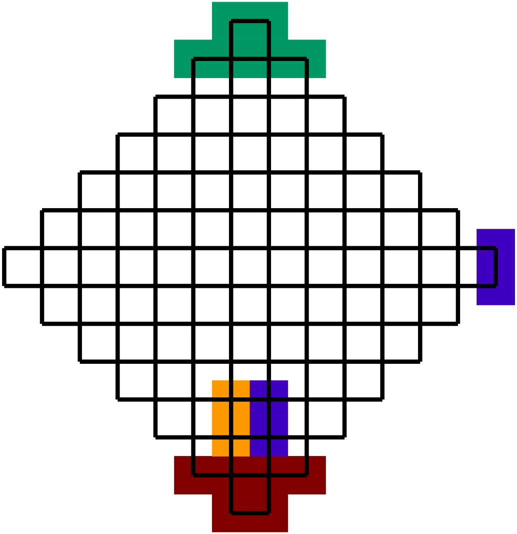

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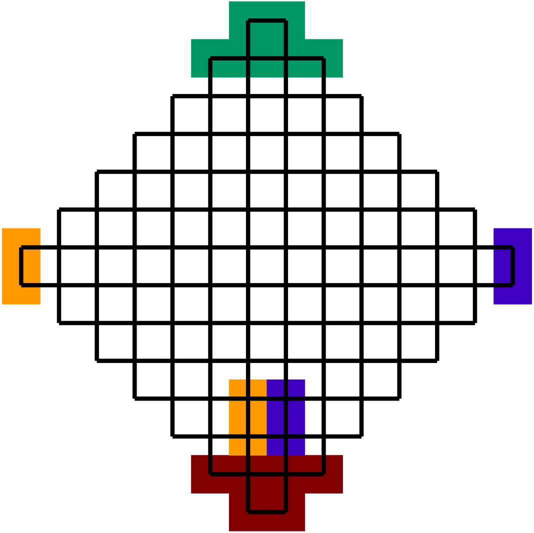

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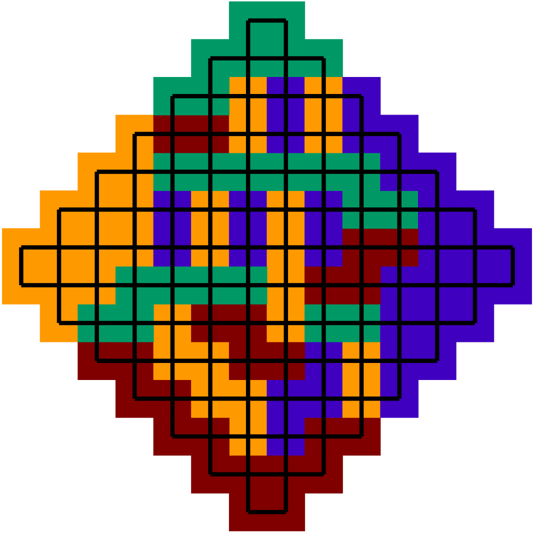

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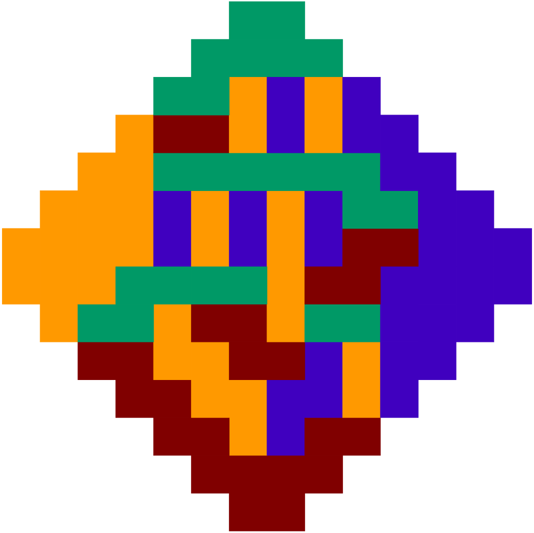
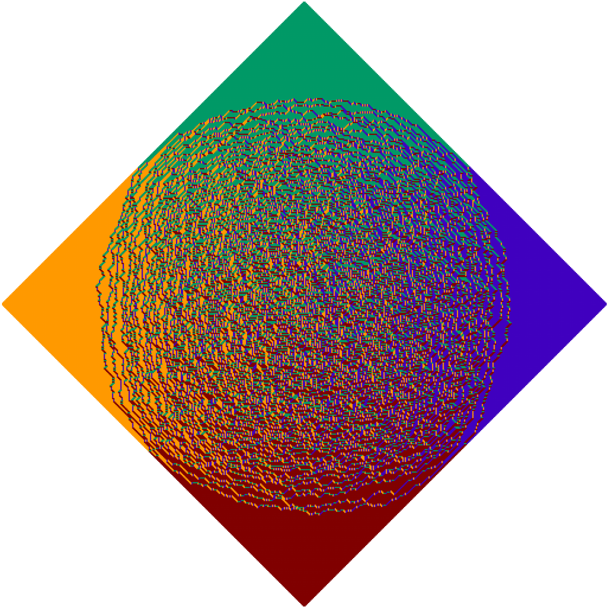
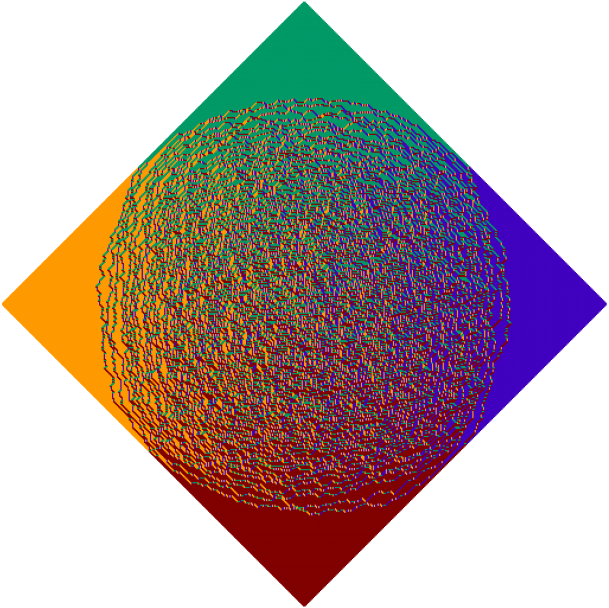
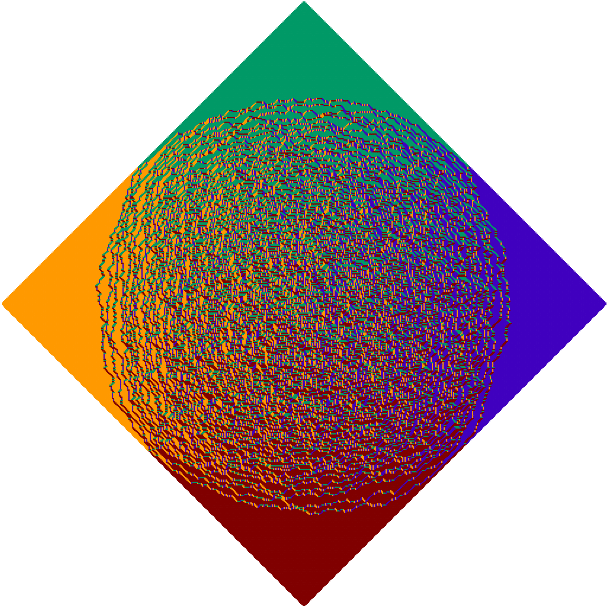




Arctic circle theorem [Jockusch, Propp and Shor 1998]



Inhomogeneous CFT [Allegra, Dubail, JMS, Viti 2016]



Interactions: conjecture for the arctic curves [Colomo & Pronko 2009]

Fermi gas in a harmonic potential

$$H = \int dx\, \psi^\dagger(x) \left[-\frac{1}{2}\partial_x^2 - N + \frac{x^2}{2} \right] \psi(x)$$

Exactly solvable. Single particle states $\psi_k^\dagger = \int_{\mathbb{R}} u_k(x) c^\dagger(x)\, dx$ given in terms of Hermite polynomials. Single particle energies $\epsilon_k = (k + 1/2 - N)$, $k \in \mathbb{N}$

Density profile (Wigner Semicircle law)

$$\rho(x) = \frac{k_F(x)}{\pi} \langle c^\dagger(x) c(x) \rangle \sim \sqrt{L^2 - x^2} \qquad L = \sqrt{2N} \gg 1$$

Inhomogeneous. Field theory description of correlations?

Curved CFT approach

[Dubail, JMS, Viti & Calabrese 2017] Imaginary time propagator at short distances (up to some phases)

$$\langle c^\dagger(x+\delta x, y+\delta y) c(x,y)\rangle \sim \frac{1}{2\pi}\left[\frac{1}{\delta x + i v(x)\delta y} - \frac{1}{\delta x - i v(x)\delta y}\right]$$

This coincides with the propagator for the following action

$$\mathcal{S} = \frac{1}{2\pi}\int dz d\bar{z}\, e^{\sigma(x,y)}\left[\psi_{\rm R}^\dagger \overleftrightarrow{\partial}_{\bar z}\psi_{\rm R} + \psi_{\rm L}^\dagger \overleftrightarrow{\partial}_{z}\psi_{\rm L}\right],$$

provided $e^{\sigma(x,y)}\delta z(x,y) = \delta x + i v(x)\delta y$. Solution:

$$z(x,y) = \int^x \frac{du}{v(u)} + iy = \arcsin\frac{x}{L} + iy \qquad e^\sigma = v = \sqrt{L^2 - x^2}$$

Entanglement entropy

Replica trick [Holzhey, Larsen & Wilczek 1994; Calabrese & Cardy 2004].
Twist fields [Cardy, Castro-Alvaredo & Doyon 2008] with dimension $\Delta_n = \frac{c}{12}\left(n - \frac{1}{n}\right)$.

$$S_n(x) \sim \frac{1}{1-n} \ln \epsilon^{\Delta_n} \left\langle \mathcal{T}_n(x, y=0) \right\rangle .$$

Map to the upper half plane through $g(z) = e^{i(z+\pi/2)}$.

$$\langle \mathcal{T}_n(z, \bar{z}) \rangle = \left(e^{\sigma(z,\bar{z})} \left| \frac{dg(z)}{dz} \right|^{-1} \operatorname{Im} g(z) \right)^{-\Delta_n}$$

Careful that the UV cutoff $\epsilon \to \epsilon(x) = \epsilon_0 / k_F(x)$ now depends on position.

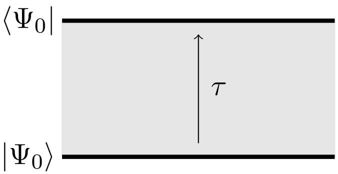
Entanglement entropy

Bipartition $(-\infty, x] \cup [x, \infty)$

$$S_n(x) = \frac{n+1}{12\,n} \ln \left[L^2 \left(1 - (x/L)^2\right)^{3/2} \right]$$

Quantum quenches

$$\mathcal{L}(\tau) = \langle \Psi_0 | e^{-H\tau} | \Psi_0 \rangle$$



[Calabrese & Cardy PRL 2006]

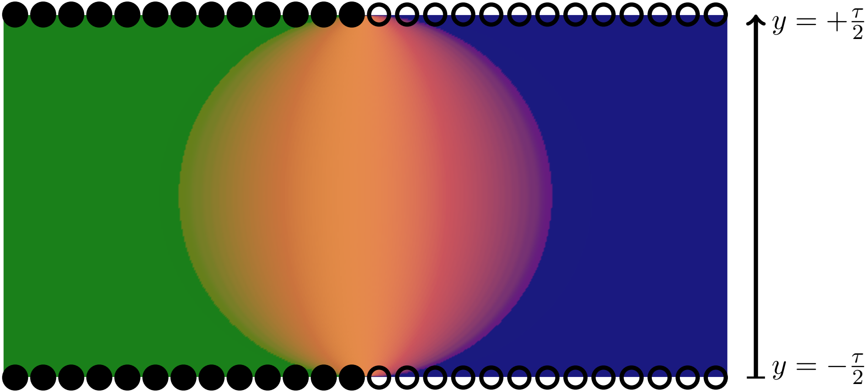
[Cardy PRL 2015]

Strategy

- Replace it by $\tau \in \mathbb{R}$
- Compute using stat. mech.
- "Remember" that $\tau = it$

This is the global quench approach. Similar treatment for local quenches.

Quantum quench from domain wall in the XX chain



Field theory may be identified. [Allegra, Dubail, JMS & Viti 2016]

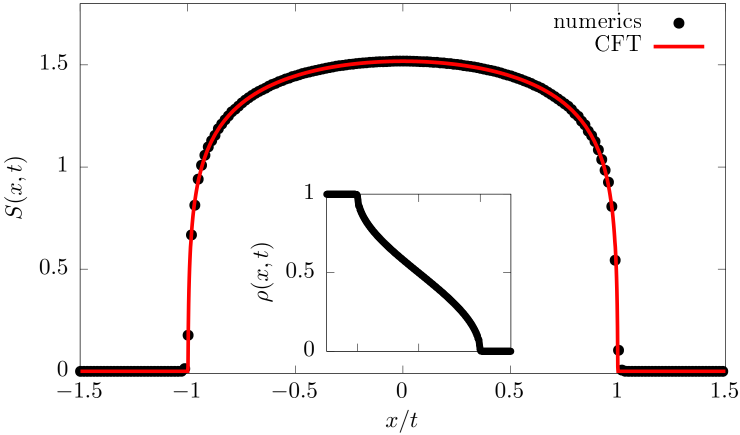
$$z(x,y) = \arccos\left(\frac{x}{\sqrt{(\tau/2)^2 - y^2}}\right) - i\,\mathrm{arctanh}\frac{2y}{\tau}$$

$$e^{\sigma(x,y)} = \sqrt{(\tau/2)^2 - x^2 - y^2}$$

Entanglement entropy [Dubail, JMS, Viti & Calabrese 2017]

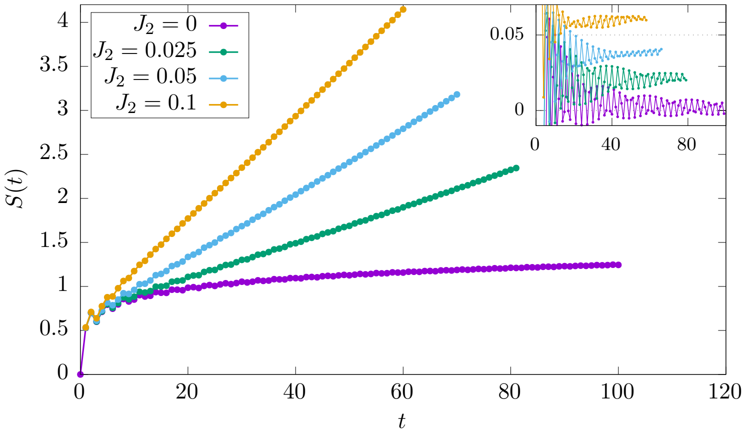
$$S_n(x,t) = \frac{n+1}{12\,n} \ln \left[t\,(1-(x/t)^2)^{3/2} \right]$$

Recovers the numerical guess made in [Eisler & Peschel 2014]



Caveat: Integrability and non Integrability

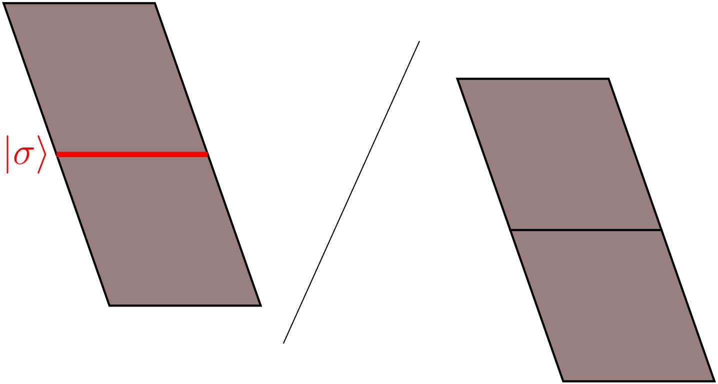
$$H = \sum_j \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y \right) + J_2 \sum_j \left(\sigma_j^x \sigma_{j+2}^x + \sigma_j^y \sigma_{j+2}^y \right)$$



2. The book of the CFT

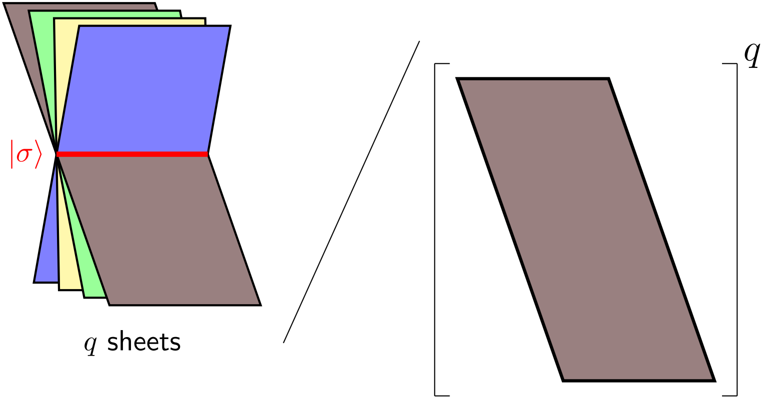
Ground state wave function

$$|\psi(\sigma)|^2 \;=\; \lim_{\tau\to\infty} \frac{\langle s|e^{-\tau H}\,|\sigma\rangle\,\langle\sigma|\,e^{-\tau H}|s\rangle}{\langle s|e^{-2\tau H}|s\rangle} \;=\; \text{picture below}$$



Ground state wave function

$$|\psi(\sigma)|^{2q} \;=\; \text{picture below}$$



Free compact field ($c = 1$)

$$\mathcal{S} = \frac{1}{8\pi K}\int_0^L dx \int_{-\infty}^{\infty} d\tau (\nabla\varphi)^2 + \sum_p \underbrace{\lambda_p \cos(p\,\varphi)}_{\text{irrelevant if } p^2 > 2\kappa}$$

Can show: $K \to K/q$ for the q binding.
[JMS, Misguich & Pasquier 2011]

If q becomes too large irrelevant operators might become relevant, and trigger a phase transition.

Initial motivation in the XXZ spin chain [JMS, Furukawa, Misguich & Pasquier 2009] [JMS, Misguich & Pasquier 2011]

$$Z_q = \left(\sum_{\sigma} \psi(\sigma)^{2q} \right)^{1/q}$$

Here $p_{\min} = 2$, $K = f(\Delta)$, $q_c = 4K$ at half filling

$$Z_q = e^{-a_q L} \mathcal{Z}_q (1 + o(1))$$

with

$$\mathcal{Z}_q = \begin{cases} K^{1/2 - 1/2q} q^{1/2q} & , \quad q < q_c \\ 2^{1/q} \sqrt{K} & , \quad q > q_c \end{cases}$$

$q = \infty$, exact lattice computation [Brockmann & JMS 2017]

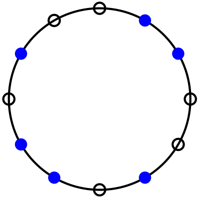
$$Z_\infty = \sqrt{K} \exp \left(-L \left[\log 2 - \int_{\mathbb{R}} \frac{\sinh(\frac{\pi}{\gamma} - 1)k \sinh k}{2 \sinh \frac{\pi k}{\gamma} \cosh 2k} \frac{dk}{k} \right] \right) (1 + o(1))$$

A simple example: the XX chain at half filling

Wave function has Jastrow-type form (spins $\leftrightarrow$ particles)

$$\psi(x_1,\ldots,x_n) = \prod_{1\leq i<j\leq n} (z_i - z_j)^\alpha$$

considered in [Cirac & Sierra 2009]



$\alpha = 1$ for the XX chain. "Binding procedure" just changes α.
Note $\alpha = 2$ is gives the ground state of the Haldane-Sastry chain

RG analysis [Narayan & Shastry 1998] of the classical particle system also compatible with the previous slide.

The point

Even if some correlations gap out for $n > n_c$, in the book picture the spins are still connected through the bulk, which is critical.

Bozonization $S_j^z \sim \partial_x \varphi + (-1)^j \cos\varphi$
$S_j^+ \sim e^{i\tilde{\varphi}} \left((-1)^j \cos\varphi + cst\right)$

For $q > q_c$, $\varphi = \varphi_L + \varphi_R$ locks to Dirichlet (Néel) at the boundary, but the dual field $\tilde{\varphi} = \varphi_L - \varphi_R$ still fluctuates!
This point was also made in [Kumano, Misguich & Oshikawa, unpub. 2014]

Correlation functions

$q < 4$

$$\langle S^z_j S^z_{j+r} \rangle \sim \frac{(-1)^r}{r^{2/q}} - \frac{1}{2q\pi^2 r^2}$$

$$\langle S^x_j S^x_{j+r} \rangle \sim \frac{\mathrm{cst}}{r^{q/2}}$$

$q > 4$

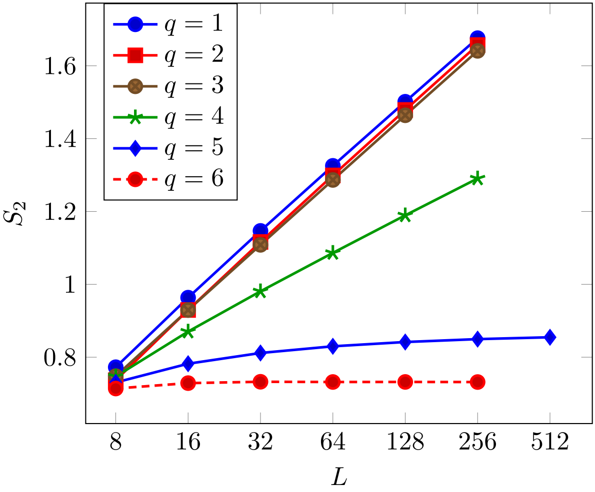
$$\langle S^z_j S^z_{j+r} \rangle \sim \mathrm{cst}$$

$$\langle S^x_j S^x_{j+r} \rangle \sim \frac{\mathrm{cst}}{r^{q/2}}$$

What about entropy?

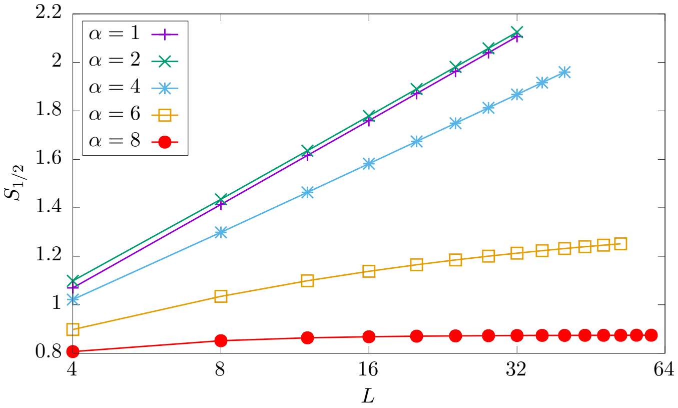
Back to entanglement scaling

S_2 can be efficiently computed in Monte Carlo



Back to entanglement scaling

$S_{1/2}$ is less easy, smaller system sizes



Approximability by MPS

Usual logic is that if $S_{n<1}$ saturates, then the wave function can be approximated by an MPS with finite bond dimension.

But all correlations decay exponentially for an MPS with finite bond dimension.

[Hastings 2007] showed that if S_1 saturates for the ground state of a local Hamiltonian, then approximability follows. So the states we consider are not ground states of local Hamiltonian for $\alpha > 4$.

Summary

1. Inhomogeneous systems.

Field theory: $dzd\bar{z}$, $x+iy \to e^{\sigma(x,y)}dzd\bar{z}$, $z=\tilde{x}+iy$. Easy to work out the extra contributions for free fermions. Interacting more complicated, but logic is the same.

Careful about quantum quenches, where integrability plays a role. Quenches of domain wall type are neither local nor global, and in this case $S(t)$ is a chaos detector ($\log t$ vs t).

2. It is possible to cook up critical states in 1d with $S \sim \mathrm{const}$.

Thank you!