Correlations and field theory inside the arctic circle

[or Arctic quenches]

Jean-Marie Stéphan[1]

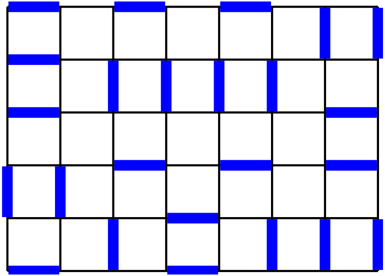
[1]Max-Planck-Institut für Physik Complexer Systeme (Dresden)

Firenze 2015

in collaboration with N. Allegra, J. Dubail, M. Haque and J. Viti.

Outline

Classical Dimers in 2d



- dimers with hardcore constraint.
- Exactly solvable: free fermions.
- $Z = \det(\ldots)$ [Kasteleyn, Fisher (1963)]
- Critical system

Long distance limit: Dirac field or free gaussian compact field

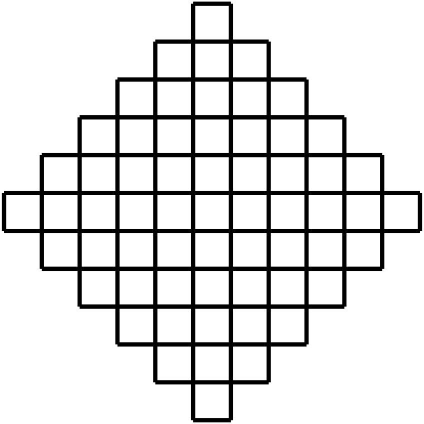
$$S = \frac{g}{4\pi}\int dxdy\,(\nabla\varphi)^2 \quad , \quad \varphi = \varphi + 2\pi$$

$$C_{dd}(\mathbf{r},\mathbf{r}') = \left|\mathbf{r}-\mathbf{r}'\right|^{-1/g} \quad , \quad C_{mm}(\mathbf{r},\mathbf{r}') = \left|\mathbf{r}-\mathbf{r}'\right|^{-g}$$

$\longrightarrow$

Aztec diamond, and the arctic circle

[Jokusch, Propp and Shor, 1995]



Aztec diamond, and the arctic circle

[Jokusch, Propp and Shor, 1995]

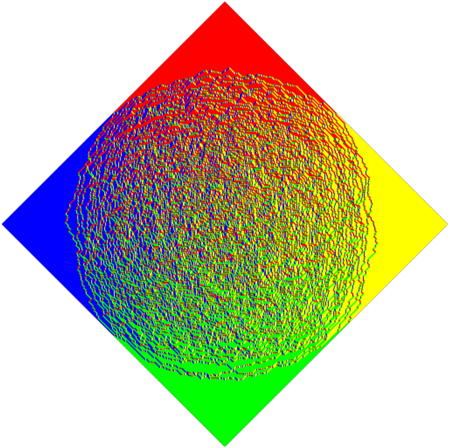
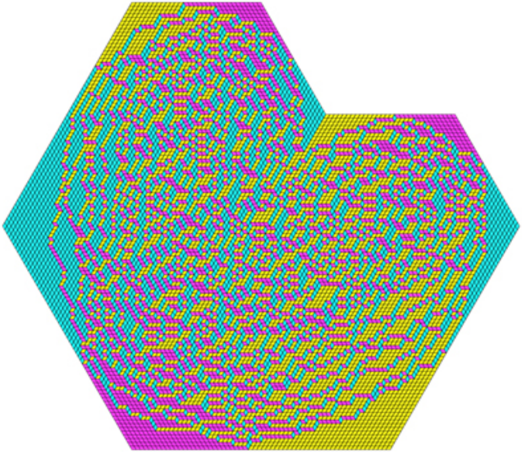


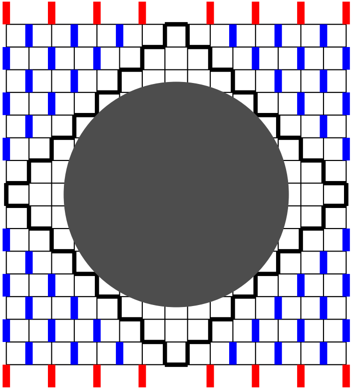
Image at http://tuvalu.santafe.edu/∼moore/gallery.html

Lots of variations on this



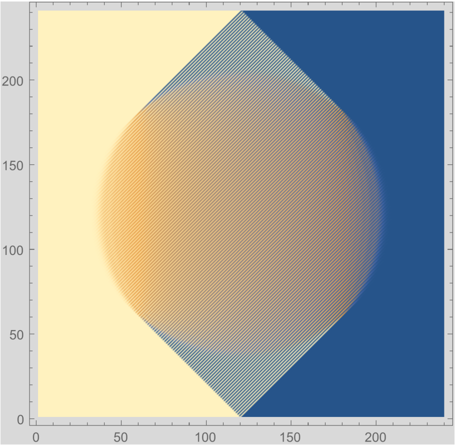
Theory for the shape [Kenyon, Okounkov, Sheffield]

Can be induced by boundary conditions



$$Z = \langle \psi_0 \,|\, T^{L_y} \,|\, \psi_0 \rangle$$

Density profile for dimers

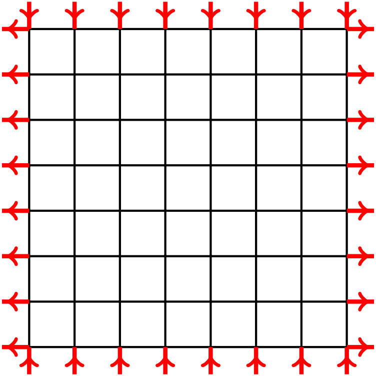


$$\rho(x,y) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{2y-1}{\sqrt{1-2x^2-2y^2}}\right)$$

[Cohn, Elkies and Propp, Duke. Math. Journ 1996]

Six vertex with domain wall boundary conditions

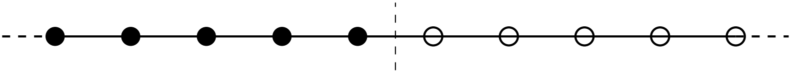
[Korepin, Izergin, Zinn-Justin, Colomo, Pronko,. . .]



Conjecture for the arctic curve [Colomo & Pronko, J. Stat. Phys 2010]

Field theory inside the circle?

A simple toy model



$$|\psi_0\rangle = \prod_{x<0} c_x^\dagger |0\rangle$$

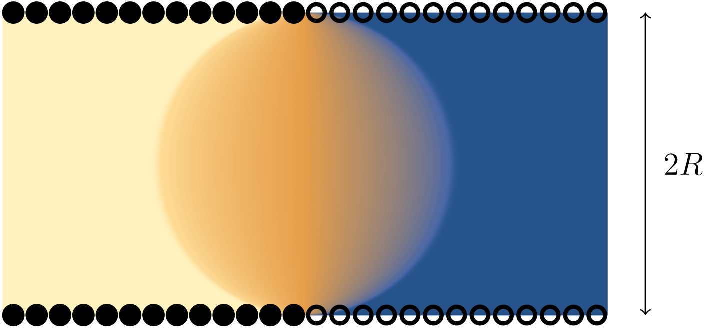
$$H = -\frac{1}{2}\sum_x \left(c_x^\dagger c_{x+1} + h.c\right)$$

Imaginary time evolution

$$Z = \left\langle \psi_0 \,\middle|\, e^{-2RH} \,\middle|\, \psi_0 \right\rangle$$

Motivations (1/3)

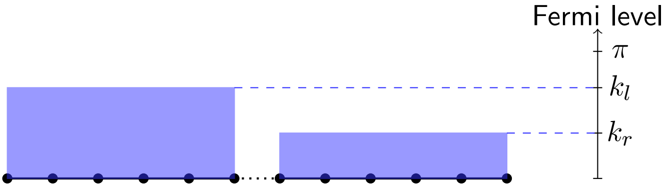
- TASEP in continuous time [Rost, Zeit. Wahrs. 1981].
- Arctic circle phenomenology.



- Quantum mechanics from a domain wall intial state.
  [Antal Rácz Rákos Schütz 1999 ; Antal Krapivsky Rákos 2008 ; . . .]

Motivation (2/3): Filling fraction quenches

$$H = -\sum_j \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) = \sum_k \varepsilon_k c_k^\dagger c_k$$

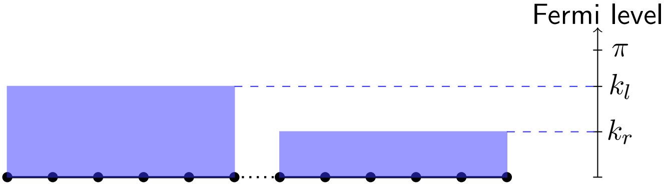


Fermions, with certain Fermi levels (k_l, k_r)

$$|\psi_0\rangle = |k_l\rangle \otimes |k_r\rangle$$

and let evolve with $H(\frac{k_l+k_r}{2})$ at time $t > 0$

Motivation (2/3): Filling fraction quenches



Limiting cases:

- $k_l = \pi$, $k_r = 0$ is the domain wall quench.
- $k_l = k_r$

  [Eisler, Karevski, Platini & Peschel, 2008] [Calabrese & Cardy, 2008]

  [JMS & Dubail, 2011]

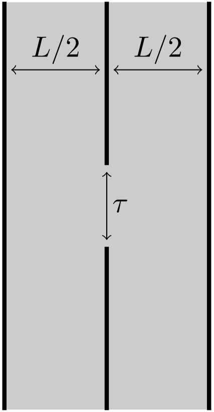
Motivation (3/3) low energy local quenches $k_l = k_r$

$$\mathcal{L}(\tau) = \left|\langle\psi_0|e^{-\tau H_{\rm tot}}|\psi_0\rangle\right|^2$$

Keep in mind

$\tau \to it$, but only at the end.

$\mathcal{F}(\tau) = -\ln \mathcal{L}(\tau)$ is a free energy! [JMS & Dubail, 2011]



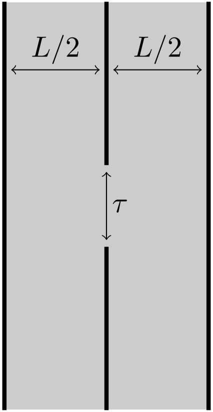
Motivation (3/3) low energy local quenches $k_l = k_r$

$$\mathcal{L}(\tau) = \left|\langle\psi_0|e^{-\tau H_{\rm tot}}|\psi_0\rangle\right|^2$$

Keep in mind

$\tau \to it$, but only at the end.

$\mathcal{F}(\tau) = -\ln \mathcal{L}(\tau)$ is a free energy! [JMS & Dubail, 2011]



Loschmidt echo

$$\mathcal{F}(\tau) = \frac{c}{4}\ln\left|\frac{L}{\pi}\sinh\left(\frac{\pi\tau}{L}\right)\right|$$

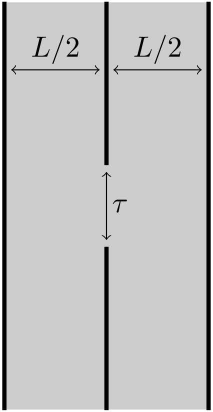
Motivation (3/3) low energy local quenches $k_l = k_r$

$$\mathcal{L}(\tau) = \left|\langle\psi_0|e^{-\tau H_{\text{tot}}}|\psi_0\rangle\right|^2$$

Keep in mind

$\tau \to it$, but only at the end.

$\mathcal{F}(\tau) = -\ln \mathcal{L}(\tau)$ is a free energy! [JMS & Dubail, 2011]



Loschmidt echo

$$\mathcal{F}(\tau) = \frac{c}{4}\ln\left|\frac{L}{\pi}\sinh\left(\frac{\pi\tau}{L}\right)\right|$$

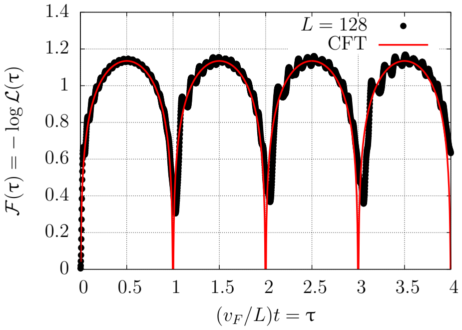
Back to real time

$$\mathcal{F}(t) = \frac{c}{4}\ln\left|\frac{L}{\pi}\sin\left(\frac{\pi t}{L}\right)\right|$$

Symmetric case $L_A = L_B = L/2$

The Loschmidt echo is periodic

$$\mathcal{F}(t) = \frac{c}{4} \ln \left| \frac{L}{\pi} \sin\left(\frac{\pi t}{L}\right) \right|$$



Non symmetric case $L_A = L/3$

Loschmidt echo

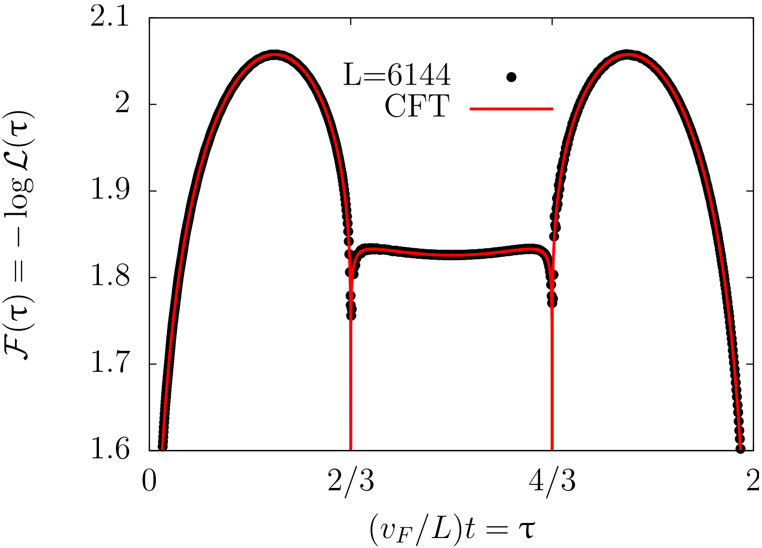
$$\mathcal{F}(a) = \frac{c}{4}\ln L + \frac{c}{24}\ln\left|\frac{a^3(a+1)^6(a+2)(2a+1)}{(a-1)^7}\right|$$

a is one of the solutions of

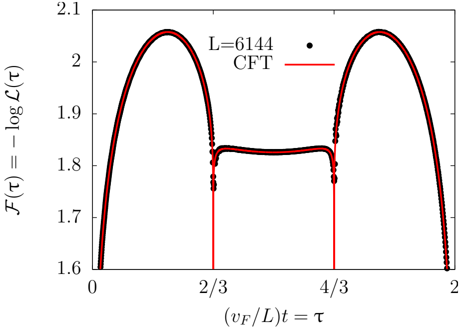
$$it = \frac{2L}{\pi}\left[\frac{1}{3}\ln\left(\frac{b-1}{b+1}\right) + \frac{2}{3}\ln\left(\frac{a-b}{a+b}\right)\right]$$

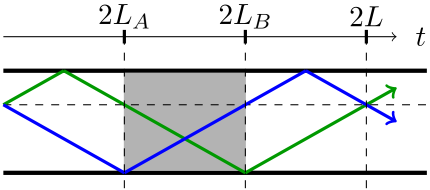
$$b^2 = a\frac{a+2}{2a+1}$$

Non symmetric case $L_A = L/3$



Non symmetric case $L_A = L/3$





Motivation (end)

Here it is not that simple, because (need both!)

1. Conservation of the number of particles
2. Inhomogeneous initial state

Naive low energy-field theory does not work.

Correlations inside the "circle"

Want to compute

$$\left\langle c_x^\dagger(y) c_{x'}(y') \right\rangle = \frac{\langle \psi_0 | e^{-(R+y)H} c_x^\dagger e^{-(y-y')H} c_{x'} e^{-(R-y')H} | \psi_0 \rangle}{\langle \psi_0 | e^{-2RH} | \psi_0 \rangle}$$

$$c^\dagger(k) = \sum_{x\in\mathbb{Z}} e^{-ikx} c_x^\dagger \qquad , \qquad [H, c^\dagger(k)] = \varepsilon(k) c^\dagger(k)$$

Appearance of the Hilbert transform

$$\left\langle c^{\dagger}(k,y)c(k',y')\right\rangle = \frac{\left\langle \psi_0 \left| e^{-(R-y)H} c^{\dagger}(k) e^{-(y'-y)H} c(k') e^{-(R+y)H} \right| \psi_0 \right\rangle}{\langle \psi_0 \,|\, e^{-2RH} \,|\, \psi_0 \rangle}$$

One can show that

$$\boxed{\left\langle c^{\dagger}(k,y)c(k',y')\right\rangle = \frac{e^{-iR[\tilde{\varepsilon}(k)-\tilde{\varepsilon}(k')]} e^{y\varepsilon(k)-y'\varepsilon(k')}}{2i\sin\left(\frac{k-k'}{2}+i0^+\right)}}$$

$$\tilde{\varepsilon}(k) = \text{Hilbert transform of } \varepsilon(k) = \mathrm{pv}\int_{-\pi}^{\pi} \frac{dq}{2\pi}\varepsilon(q)\cot\frac{k-q}{2}.$$

Bosonisation trick

$$c^\dagger(k) \to : e^{i\varphi(k)} :$$

$$c(k) \to : e^{-i\varphi(k)} :$$

$$c^\dagger(k)c(k) \to \partial\varphi(k)$$

Fusion of two vertex operators:

$$: e^{\alpha\varphi(k)} :: e^{\beta\varphi(k')} := e^{\alpha\beta\langle\varphi(k)\varphi(k')\rangle} : e^{\alpha\varphi(k)+\beta\varphi(k')} :$$

Need to define

- Normal order $: c_x^\dagger c_x :$
- $\langle\varphi(k)\varphi(k')\rangle = \log\sin\frac{k-k'}{2}$

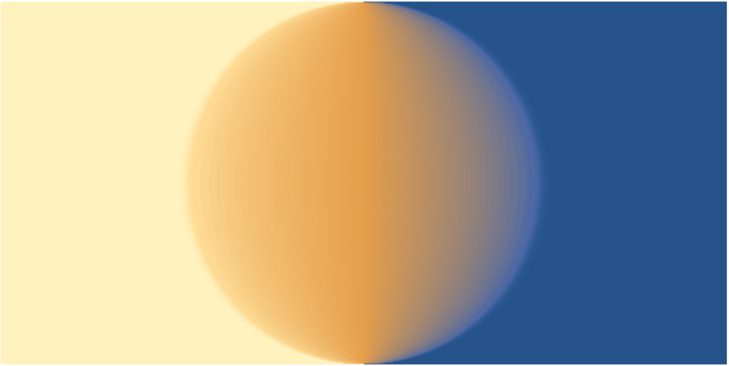
Comments

- The result is exact.
- This completely solves the problem in principle.
- Real space correlations: inverse Fourier transform+stationary phase approximation.
- Stationary points

$$x + iy\frac{d\varepsilon(k)}{k} + R\frac{d\tilde{\varepsilon}(k)}{dk} = 0$$

Density profile

$$\langle c_x^\dagger(y) c_x(y) \rangle = \frac{1}{\pi} \arccos \frac{x}{\sqrt{R^2 - y^2}}$$



$$Z = e^{-R^2/2}$$

Dirac in curved space

$$c_x^\dagger = \frac{e^{-i(\pi/4+\theta(x,y))}}{\sqrt{2\pi}}\psi^\dagger(x,y) + \frac{e^{i(\pi/4+\theta^*(x,y))}}{\sqrt{2\pi}}\overline{\psi}^\dagger(x,y)$$

where

$$\left\langle \psi^\dagger(x,y)\psi(x',y') \right\rangle = \frac{e^{-\frac{1}{2}[\sigma(x,y)+\sigma(x',y')]}}{\sin\left(\frac{z(x,y)-z(x',y')}{2}\right)}$$

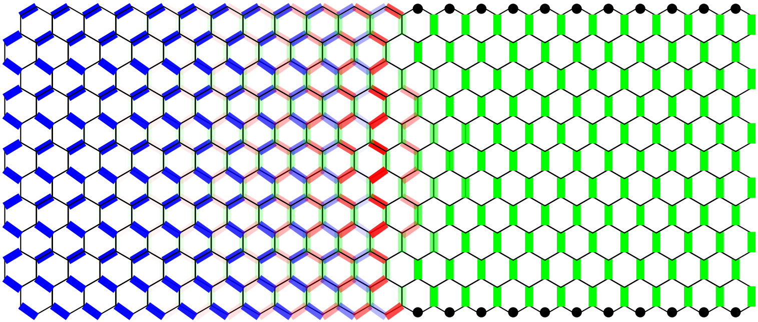
$$\begin{aligned} z(x,y) &= \arcsin\frac{x}{\sqrt{R^2-y^2}} + i\,\mathrm{arcth}\frac{y}{R} \\ \theta(x,y) &= -z(x,y)x - \sqrt{R^2-x^2-y^2} \\ \sigma(x,y) &= \log\sqrt{R^2-x^2-y^2} \end{aligned}$$

Dirac theory with a metric $ds^2 = e^{2\sigma}(dx^2+dy^2)$

Dimers on honeycomb

$$T^2 = \exp\left[-\int \frac{dk}{2\pi}\varepsilon(k)c(k)^\dagger c(k)\right]$$

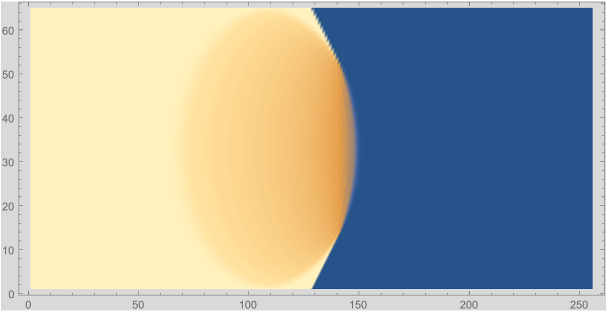
$$\varepsilon(k) = \log(1 + u^2 + 2u\cos k)$$



Dimers on honeycomb

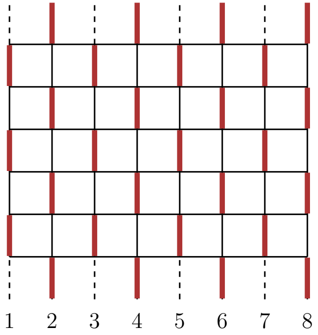
$$T^2 = \exp\left[-\int \frac{dk}{2\pi}\varepsilon(k)c(k)^\dagger c(k)\right]$$

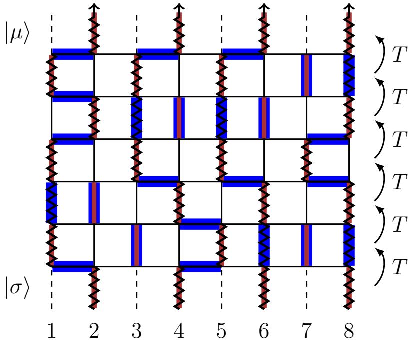
$$\varepsilon(k) = \log(1 + u^2 + 2u\cos k)$$



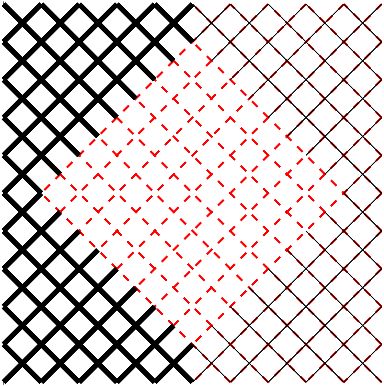
Dimers on square

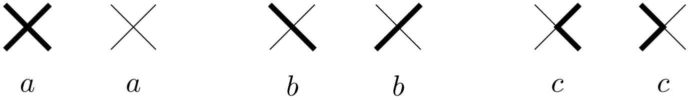
Mapping: two bands





Six vertex





Back to the toy model

Is the analytic continuation $y = it$, $R \to 0$ consistent?

Analytic continuation

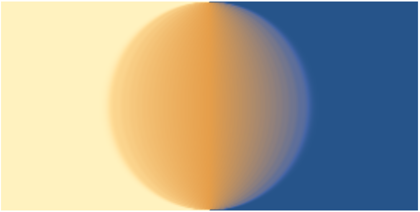
- Loschmidt echo

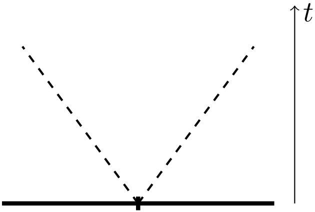
$$L(\tau) = \langle \psi_0 | e^{-\tau H} | \psi_0 \rangle = e^{\tau^2/8} \quad \longrightarrow \quad L(t) = e^{-t^2/8}$$

- Density

$$\rho(x,y,R) = \frac{1}{\pi} \arccos \frac{x}{\sqrt{R^2-y^2}} \quad \longrightarrow \quad \rho(x,t) = \frac{1}{\pi} \arccos \frac{x}{t}$$

Analytic continuation of the circle = light cone.





Analytic continuation

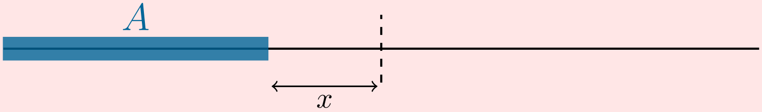
- Loschmidt echo

$$L(\tau) = \langle\psi_0|e^{-\tau H}|\psi_0\rangle = e^{\tau^2/8} \quad \longrightarrow \quad L(t) = e^{-t^2/8}$$

- Density

$$\rho(x,y,R) = \frac{1}{\pi}\arccos\frac{x}{\sqrt{R^2-y^2}} \quad \longrightarrow \quad \rho(x,t) = \frac{1}{\pi}\arccos\frac{x}{t}$$

Prediction for the entanglement entropy



$$S(x,t) = \frac{1}{6}\log\left[t\,(1-x^2/t^2)^{3/2}\right]$$

Conclusion

- Arctic circle revisited: correlations.
- Inhomogeneous system, so inhomogeneous field theory.
- Other initial states.
- Interactions?
- Logarithmic terms in six-vertex DWBC?

Thank you!