Local quantum quenches in critical one-dimensional systems: entanglement and light cone effects

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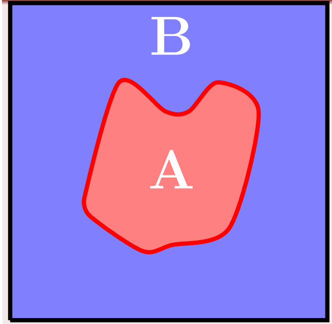
Workshop on Correlations and Entanglement in Many-body Systems Out of Equilibrium – Taiwan 2012

Outline

1. Entanglement and quantum quenches
   - Entanglement entropy in condensed matter physics
   - Quenches from (conformal) field theory
2. Local quenches in finite-size 1d systems
   - The cut and glue quench
   - Light-cone effects: Entanglement entropy and Loschmidt echo
   - Does the extrapolation length hide somewhere?
3. Entanglement and a (bipartite) fidelity
   - Orthogonality catastrophe
   - Universal scaling functions
   - Higher dimension

Entanglement entropy

Bipartition



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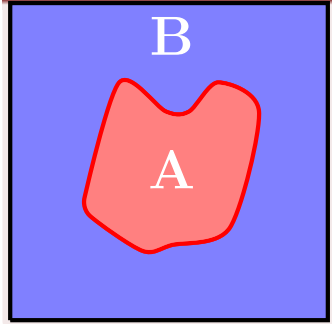

[von Neumann, 1955]

- $|\psi\rangle$ ground state of $H_{A\cup B}$
- $\rho_A = \mathrm{Tr}_B\,|\psi\rangle\langle\psi|$
- $S = -\mathrm{Tr}\,\rho_A \ln \rho_A$

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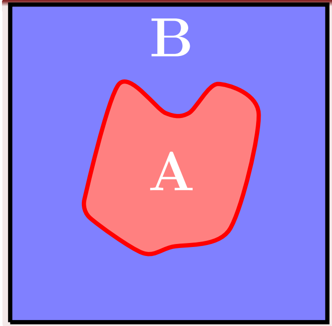

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System of two spins

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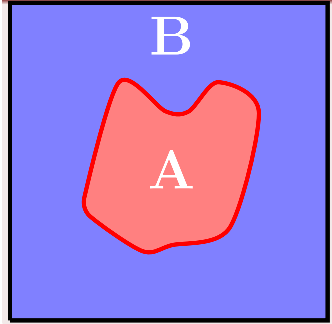

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System of two spins

- $|\psi\rangle = |\uparrow\rangle \otimes |\downarrow\rangle \quad \longrightarrow \quad S = 0$
- $|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \quad \longrightarrow \quad S = \ln 2$

Entanglement entropy

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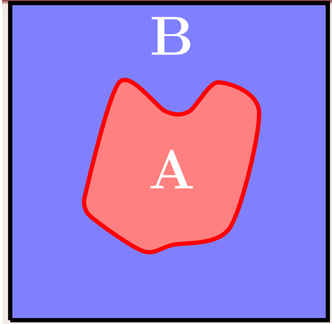

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Extended quantum system: *Schmidt decomposition*

- $|\psi\rangle = \sum_i c_i |\psi_A^i\rangle |\psi_B^i\rangle$ $\qquad \langle \psi_\Omega^i | \psi_{\Omega'}^{i'} \rangle = \delta_{ii'} \delta_{\Omega\Omega'}$
- $S = -\sum_i c_i^2 \ln c_i^2$

Entanglement entropy

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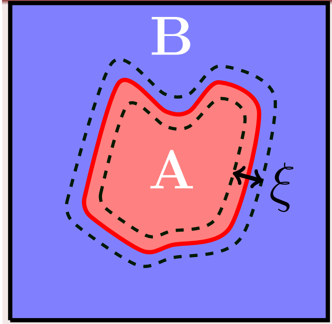

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Extended quantum system: *Boundary law*

- correlation length ξ, dimension d.
- $S(L) = aL^{d-1} + o(L^{d-1})$

Entanglement entropy (2/2)

Why studying this quantity?

- How to store efficiently quantum states in a computer?
- Tool to distinguich between subtly different phases of matter.
- Replica trick: $S = \lim_{n \to 1} \dfrac{1}{1-n} \ln \left(\mathrm{Tr}\, \rho^n \right)$

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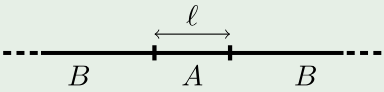
Classic results

- $1d$ critical systems: $S = \frac{c}{3} \ln \ell + \mathrm{cst} + o(1)$ [Holzhey et al, NPB 1994 — Vidal et al, PRL 2003 — Calabrese & Cardy, JSM 2004]



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- Topological order in gapped systems: $S = aL + S_{\mathrm{topo}} + o(1)$ [Kitaev & Preskill, PRL 2006 — Levin & Wen, PRL 2006]
- Entanglement spectrum [Li & Haldane, PRL 2008]

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Issues

- Difficult to compute in dimension $d > 1$
- What about experiments? [Cardy, PRL 2011]

Global quench

[Calabrese & Cardy, PRL 2006 — Cardy, Talk in Florence 2012]

Type of quench studied

- Initial (translational invariant) state $|\psi(0)\rangle$ in a gapped phase.
- Let evolve with the critical Hamiltonian H.
- Look at large distances and late times, hope for universality.
- Physical picture: entangled quasiparticles emitted everywhere.

Global quench

[Calabrese & Cardy, PRL 2006 — Cardy, Talk in Florence 2012]

Example of the one-point function. Use imaginary time.

$$\langle \mathcal{O}(t) \rangle = \langle \psi(0) | e^{iHt} \mathcal{O} e^{-iHt} | \psi(0) \rangle \longrightarrow \langle \psi(0) | e^{-H\tau_2} \mathcal{O} e^{-H\tau_1} | \psi(0) \rangle$$

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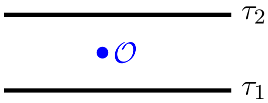

$|\psi(0)\rangle$ not too far from a boundary state $|B\rangle\rangle$

$$|\psi(0)\rangle = \exp\left(-\sum_\alpha \frac{\lambda_\alpha}{2\pi}\int_\partial \phi_\alpha(x)dx\right)|B\rangle\rangle$$

The least irrelevant operator should be the stress-tensor T.

- We have $\frac{\lambda_T}{2\pi}\int_\partial T(x)dx = \lambda_T H$
- $\langle O\rangle = \langle\langle B|e^{(-\tau_2+\lambda_T)H}\mathcal{O}e^{(-\tau_1-\lambda_T)H}|B\rangle\rangle$

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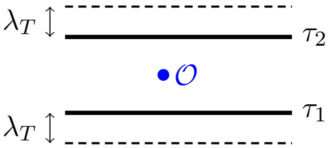

λ_T is the extrapolation length.

$$\langle \mathcal{O}(t) \rangle \sim \exp\left(-\frac{\pi \Delta}{2\lambda_T}\, t \right)$$

- Relaxation time may be modified by the other operators.
- Those are needed to find a Generalized Gibbs ensemble for the steady state.
- Does λ_T play a role in the local quench, and if so, can we measure it?

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The "cut and glue" local quench

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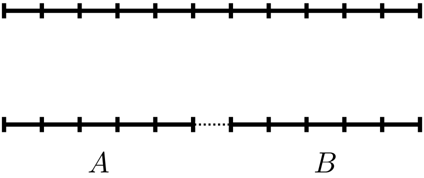

$$H_{A \cup B} = H_A + H_B + H^{int}$$

$$H_{A \otimes B} = H_A + H_B$$

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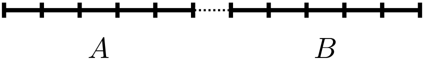

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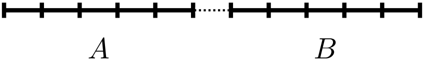

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Natural if one thinks of defects, Fermi-edge singularity, . . .

A simple example: XX chain (or itinerant fermions)

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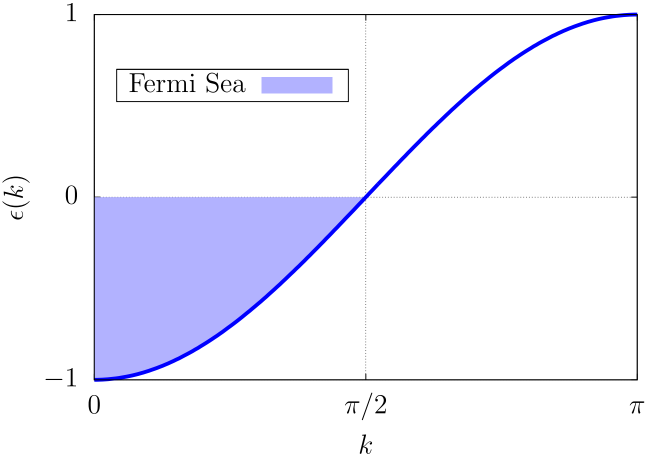

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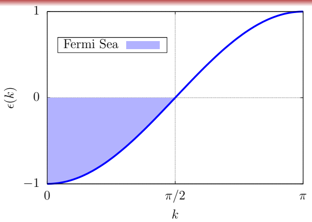

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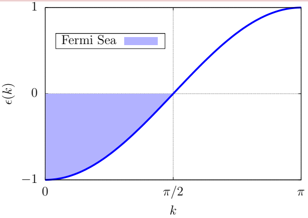
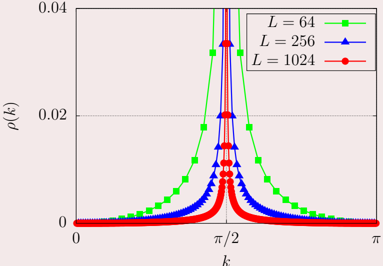


Where are the excitations localized in k space?

$$\rho(k) = \left|\langle A\cup B|d_k^\dagger d_k|A\cup B\rangle - \langle A\otimes B|d_k^\dagger d_k|A\otimes B\rangle\right|$$



- Excitations concentrate at the Fermi level $k_F = \pi/2$.
- $\Rightarrow$ Linearization $\Rightarrow$ CFT!

Conformal spectrum and quasiparticles

Low energy spectrum described by CFT

$$E_n(L) = aL + b + \frac{\pi v_F}{L}\left(h_n - \frac{c}{24}\right) + \mathcal{O}(1/L^2) \qquad , \qquad h_n \in \mathbb{N}$$

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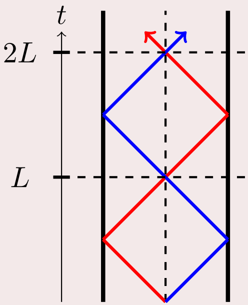

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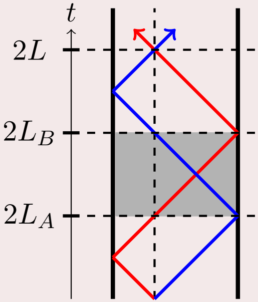

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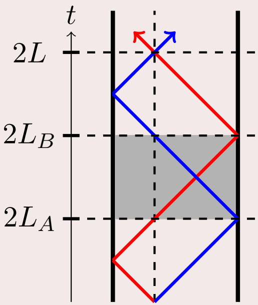

Global observables

- Entanglement entropy
  $S(t) = -\mathrm{Tr}\left[\rho(t)\ln\rho(t)\right]$, extending
  [Eisler & Peschel, J. Stat. Mech. 2007]
  [Calabrese & Cardy, J. Stat. Mech 2007],
  ...
- Loschmidt echo
  $\mathcal{L}(t) = |\langle\psi(t)|\psi(0)\rangle|^2$
  [JMS & Dubail, J. Stat. Mech. 2011]

An example: Loschmidt echo in CFT

$$\mathcal{L}(\tau) = \left|\langle A \otimes B| e^{-\tau H_{A\cup B}} |A \otimes B\rangle\right|^2$$

$\mathcal{F}(\tau) = -\ln \mathcal{L}(\tau)$ is a free energy!

Keep in mind

$\tau \to i v_F t$, but only at the end.

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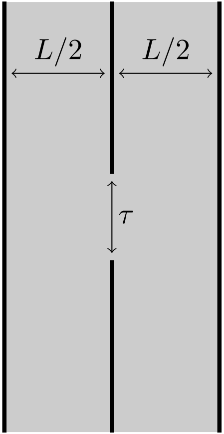

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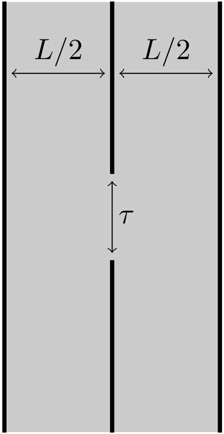
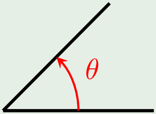


Corner singularities and the Cardy-Peschel formula



$$\Delta F = \frac{c}{24}\left(\frac{\theta}{\pi} - \frac{\pi}{\theta}\right)\ln L$$

Here we have corners with $\theta = 2\pi$, and we have to leading order

$$\mathcal{F}(\tau) = \frac{c}{4}\ln L + \text{subleading terms}$$

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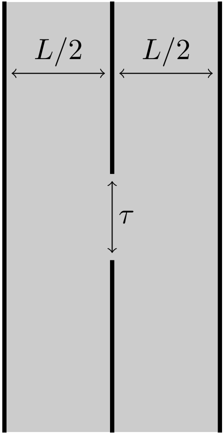

Logarithmic Loschmidt echo

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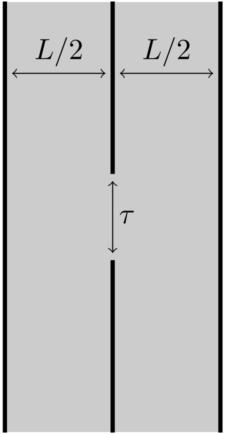

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Back to real time

$$\mathcal{F}(t) = \frac{c}{4}\ln\left|\frac{L}{\pi}\sin\left(\frac{\pi v_F t}{L}\right)\right|$$

Numerics tests, symmetric case $L_A = L_B = L/2$

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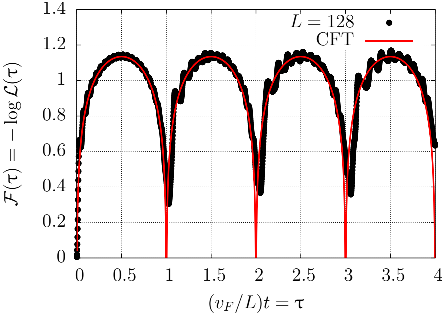

Previous numerical result for $S(t)$ [Iglói, Szatmári & Lin, Phys. Rev. B 2009]

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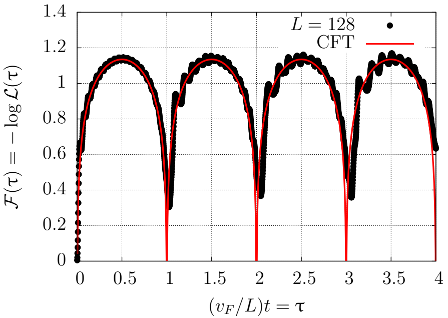

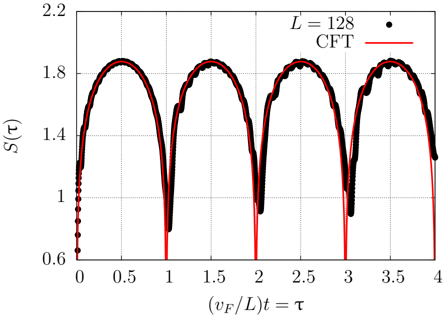

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Numerics, non symmetric case $L_A = L/3$ (I)

Loschmidt echo

$$\mathcal{F}(a) = \frac{c}{4}\ln L + \frac{c}{24}\ln\left|\frac{a^3(a+1)^6(a+2)(2a+1)}{(a-1)^7}\right|$$

a is <u>one of the solutions</u> of

$$it = \frac{2L}{\pi}\left[\frac{1}{3}\ln\left(\frac{b-1}{b+1}\right) + \frac{2}{3}\ln\left(\frac{a-b}{a+b}\right)\right]$$

$$b^2 = a\frac{a+2}{2a+1}$$

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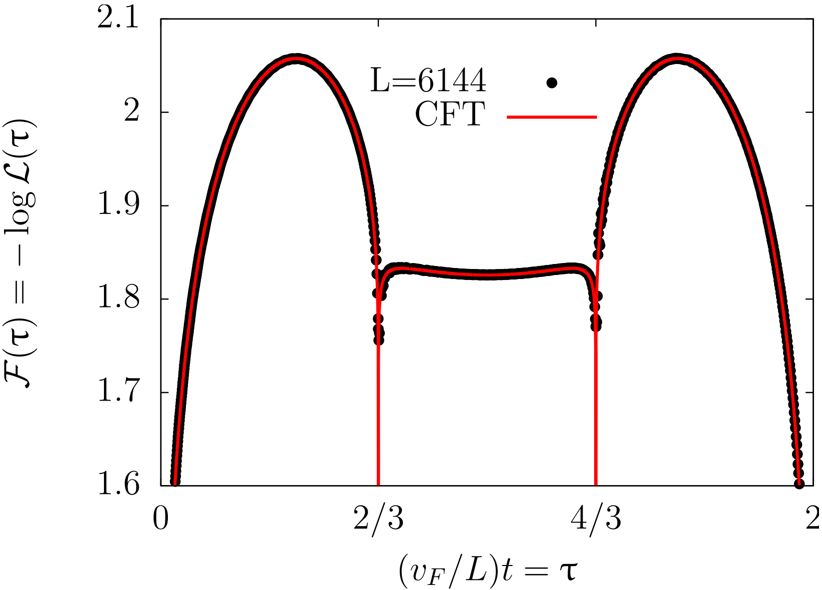

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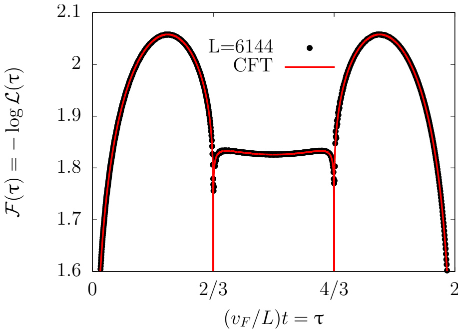
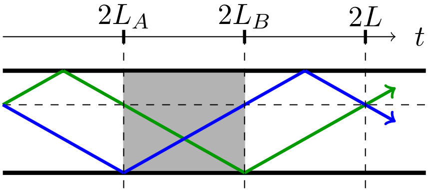




Numerics, non symmetric case $L_A = L/3$ (II)

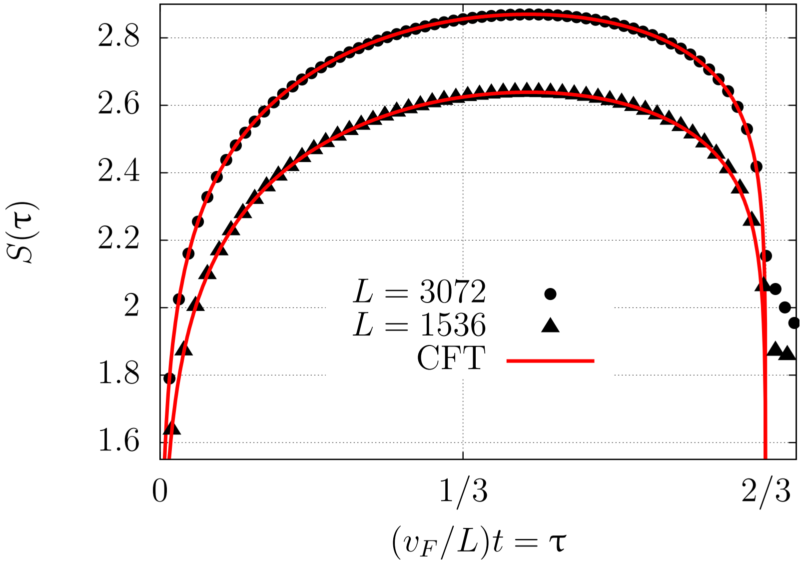
Entanglement entropy

$$f_\epsilon(t) = \frac{\left\{4(1-3\epsilon)\left[17-\cos(3\pi t)\right]\cos\left(\frac{3\pi t}{2}\right)-24i\sqrt{2}(1-\epsilon)\sqrt{1+16\epsilon-\cos(3\pi t)}\sin\left(\frac{3\pi t}{2}\right)\right\}^{1/3}}{(4+2\epsilon)\sin\left(\frac{3\pi t}{2}\right)}$$

With $f_\epsilon = g_\epsilon + ih_\epsilon$ and the derivative $f'_\epsilon = g'_\epsilon + ih'_\epsilon$, we have

$$S(t) = \frac{c}{3}\ln L + \frac{c}{12}\ln\left\{\frac{\left[h_\epsilon(t)\right]^2}{\left[\frac{3\pi}{2}\csc^2\left(\frac{3\pi t}{2}\right) + g'_\epsilon(t)\right]^2 - 3\left[h'_\epsilon(t)\right]^2}\right\} + \text{cst.}$$

Numerics, non symmetric case $L_A = L/3$ (II)



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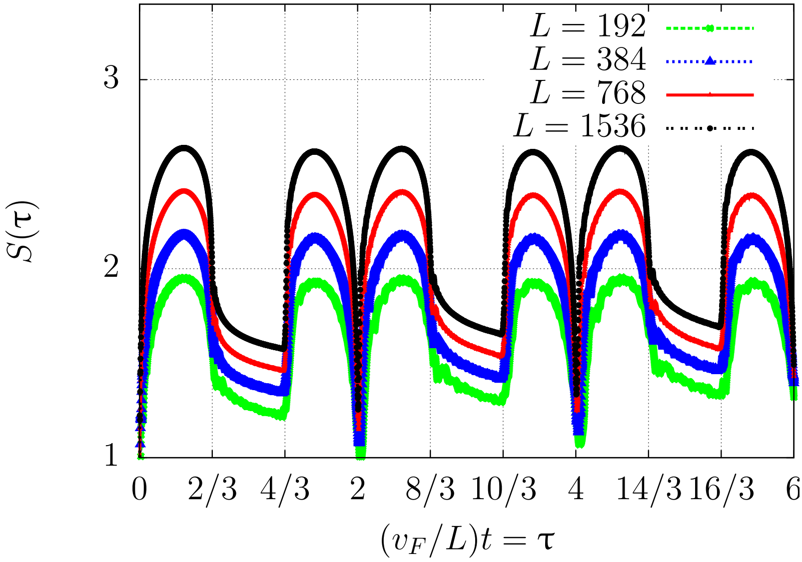

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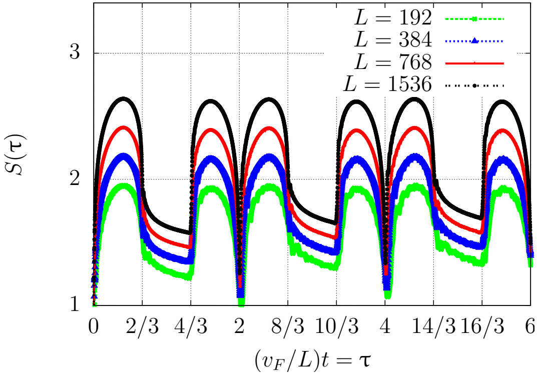

In the plateau region, CFT prediction ($S(t) = \text{cst}$) breaks down.
Observed before in [Eisler, Karevski, Platini & Peschel, J. Stat. Mech 2008]

Corrections to scaling [Dubail & JMS, in preparation]

- Lattice-effects captured by the leading irrelevant operators.
- One is the stress-energy tensor $T(z)$. It is always there.
- Perturbed (boundary) CFT:

$$S \to S_{CFT} + \frac{\lambda_T}{2\pi} \int_\partial T(z) dz$$

λ_T is the extrapolation length. Crucial role in

(a) Global quench [Calabrese & Cardy, Phys. Rev. Lett 2006]

(b) Entanglement spectrum/Edge spectrum correspondence [Qi, Katsura & Ludwig, Phys. Rev. Lett 2012] [Dubail, Read & Rezayi, arXiv 2012]

- Here, we have an exotic correction:

$$\mathcal{F}, S = \frac{c}{\cdot} \ln L + f(\tau) + \lambda_T \times g(\tau) \frac{\ln L}{L} + \text{other subleading terms}$$

(General consequence of a 2π corner singularity + finite-size)

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- Example : symmetric Loschmidt echo

$$g(\tau) = \frac{c}{4} \frac{1}{\tanh(\pi\tau/L)}$$

Problem: $g(\tau = it)$ is pure imaginary!

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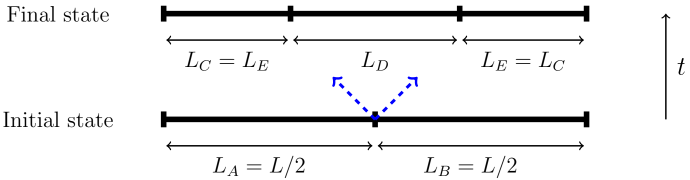
Problem: $g(\tau = it)$ is pure imaginary!
More proper regularization $\tau \to \epsilon + it$ yields:

$$\mathcal{F}_{\int_\partial T(z)dz} = c\lambda_T \times \frac{\pi\epsilon}{2} \frac{1}{1 - \cos(2\pi t/L)} \frac{\ln L}{L^2}$$

very difficult to observe numerically.

Detector idea (I)



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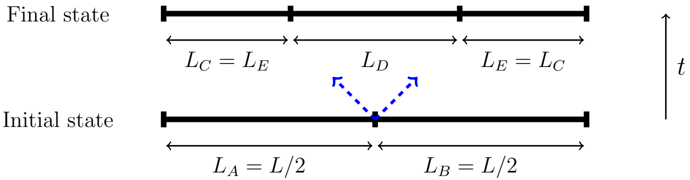
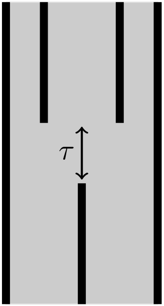


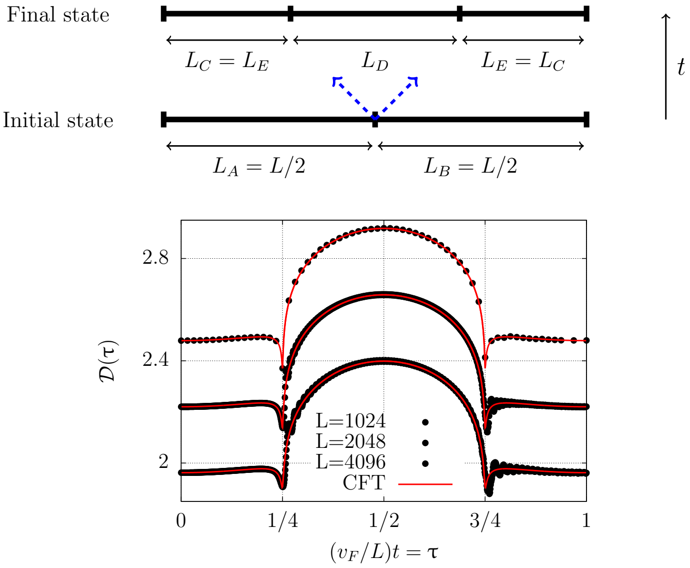


The detector

$$\mathcal{D}(t) = -\ln \left|\langle A \otimes B | e^{iHt} | C \otimes D \otimes E \rangle\right|^2$$

Similar behavior, but starts with a plateau.

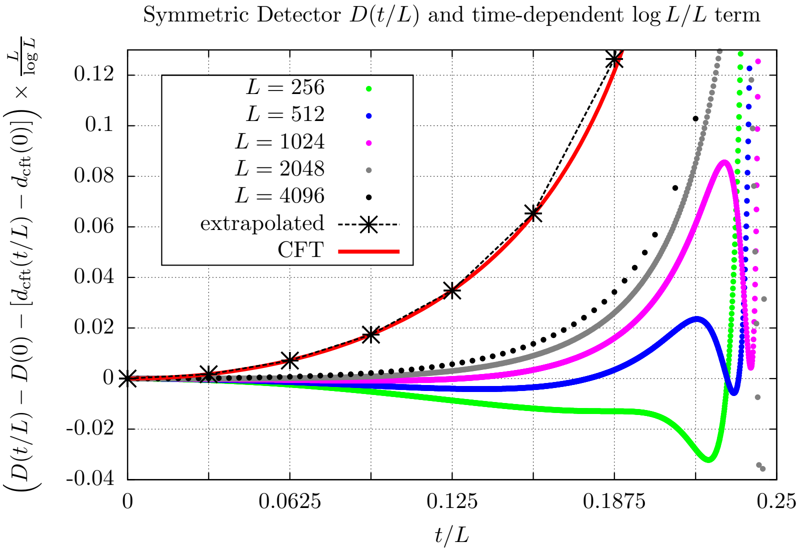
Detector idea (I)



Detector idea (II)

$$\mathcal{D}(t)_{\int_\partial T} = \frac{c\lambda_T}{8} \times \frac{3 + 2\sqrt{2}\cos(\pi t)}{\sqrt{\cos(2\pi t)}} \times \frac{\ln L}{L}$$

Detector idea (II)



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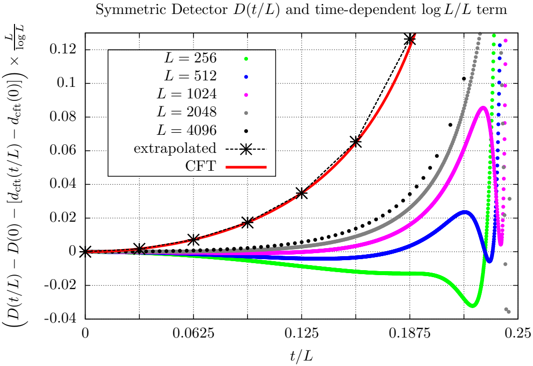

- Slow convergence because of an extra $1/L$ term.
- XX chain: results compatible with $\lambda_T = 1$
- Will prove $\lambda_T = 1$ later on.

1. Entanglement and quantum quenches
   - Entanglement entropy in condensed matter physics
   - Quenches from (conformal) field theory
2. Local quenches in finite-size 1d systems
   - The cut and glue quench
   - Light-cone effects: Entanglement entropy and Loschmidt echo
   - Does the extrapolation length hide somewhere?
3. Entanglement and a (bipartite) fidelity
   - Orthogonality catastrophe
   - Universal scaling functions
   - Higher dimension

What about $\langle A \cup B | A \otimes B \rangle$?

Bipartite fidelity

- $|\langle A \cup B | A \otimes B \rangle|^2$ [Dubail & JMS, J. Stat. Mech(L) 2011]
- Probability to observe the ground-state energy of $H_{A \cup B}$ immediately after the quench
- $\mathcal{F}_{A,B} = -\ln |\langle A \cup B | A \otimes B \rangle|^2$ is the LBF

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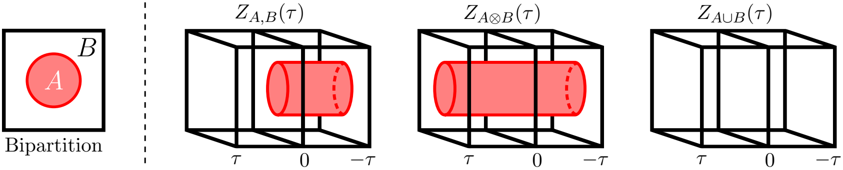

$$\mathcal{F}_{A,B} = 2f_{A,B} - f_{A\otimes B} - f_{A\cup B} \quad , \quad f_{\dots} = -\ln Z_{\dots}$$

$\mathcal{F}_{A,B} = aL^{d-1} + \mathcal{O}(L^{d-2})$ $\Longrightarrow$ Orthogonality catastrophe

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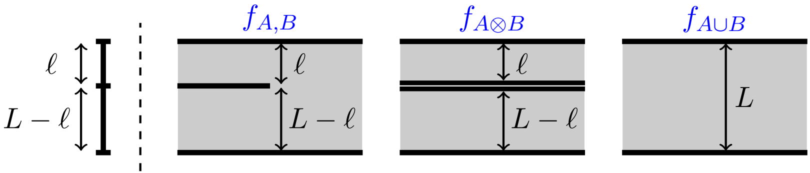

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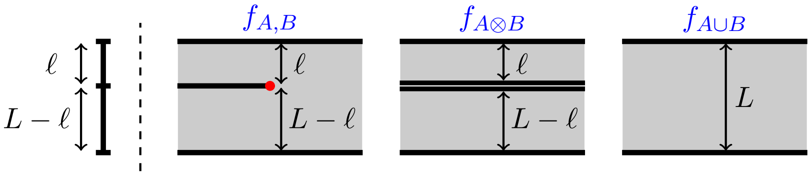
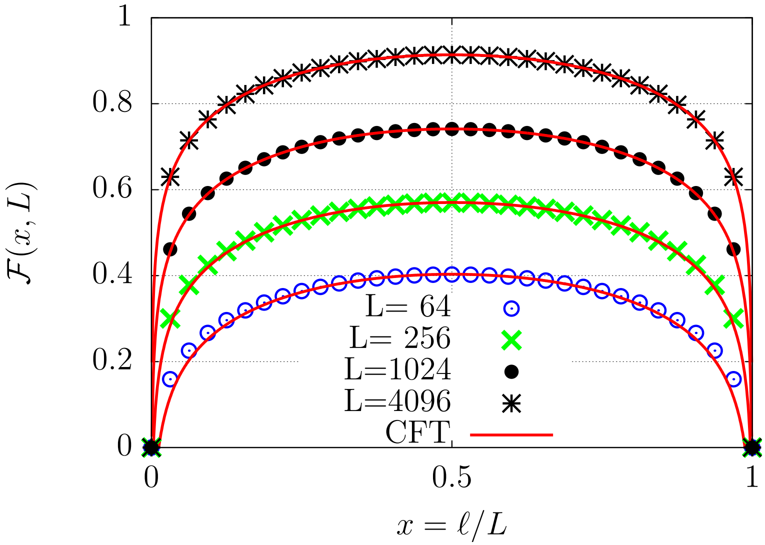


Aspect ratio $x=\ell/L$ in CFT

$$\mathcal{F}_{A,B}(x,L) = \frac{c}{8}\left[\ln L + \frac{3-3x+2x^2}{3(1-x)}\ln x + \frac{2-x+2x^2}{3x}\ln(1-x)\right] + \text{cst}$$

Some numerical checks



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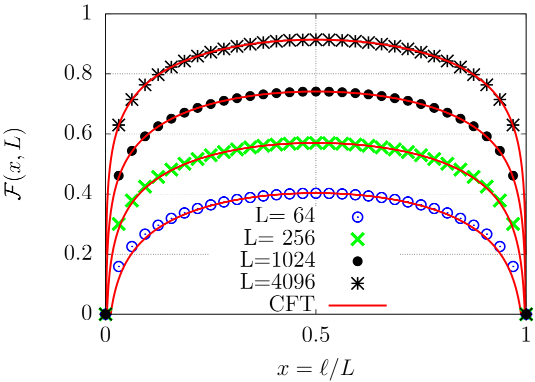
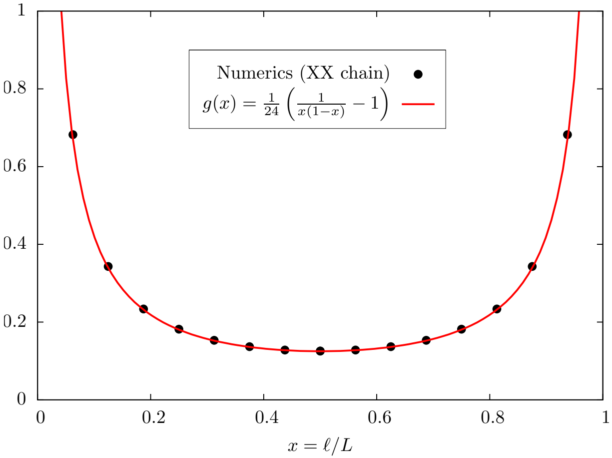


For comparison: entanglement entropy

$$S(x,L) = \frac{c}{6}\left[\,\ln L + \ln(\sin x)\right] + \text{cst}$$

Extrapolation length

$$\mathcal{F}_{\int_\partial T(z)dz} = \frac{c\lambda_T}{24}\left(\frac{1}{x(1-x)} - 1\right) \times \frac{\ln L}{L}$$



An exact result for the XX chain (cut in the middle)

Fidelity given by a Cauchy-type determinant. Asymptotics can even be computed exactly.

$$\mathcal{F}_{A,B} = \frac{1}{8}\ln L + \ln(\mathrm{cst}) + \frac{1}{8}\frac{\ln L}{L} + \mathcal{O}(1/L)$$

proves $\lambda_T = 1$! (Natural in a bosonization picture)

Remarks

- $\mathrm{cst} = \dfrac{\pi^{1/8}\exp\left(\frac{3}{8}+\frac{C}{\pi}-\frac{7}{8}\frac{\zeta(3)}{\pi^2}\right)}{A^{9/2}}\left(\dfrac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}\right)^{1/2}$
- can also be done for the ICTF, and one gets $\lambda_T = 1/2$.
- λ_T can be changed by weakening boundary links.

Higher dimension

Topological order ($2d$ quantum)

$\mathcal{F} = S_\infty$ exact for trial wave functions, which means $\mathcal{F}$ contains the topological term.

- Quantum dimer states [JMS, Misguich & Pasquier, J. Stat. Mech 2012]
- Quantum Hall [Dubail, Read & Rezayi, arXiv 2012]

Otherwise, argument as that in [Kitaev & Preskill, Phys. Rev. Lett 2006] applies, and gives

$$\mathcal{F} = aL + S_{\text{topo}}$$

Miscellaneaous

- Area law violation for free fermions $\mathcal{F} \sim L^{d-1} \ln L$
- XXZ close to criticality. In the limit $1 \ll \xi \ll L$, we get $\mathcal{F} \sim \frac{c}{8} \ln \xi$. [Weston, J. Stat. Mech 2012]
- Can include boundary changing operators in the $1d$ case.

Conclusion

- Local quantum quenches in critical $1d$ systems.
- Light-cone picture, exact results for the entanglement as well as the Loschmidt echo.
- Determination of the extrapolation length.
- Introducing a (bipartite) fidelity, very similar to the entanglement entropy.
- Analytical and exact results.

Questions

- Checks in non free-fermionic systems?
- Better understanding of the $\ln L/L$ and $\ln L/L^2$ terms.
- Apply this picture to other quantities: Emptiness formation probability,...