Feuille 9 : Primitives

Exercice 9-1 Primitives usuelles à connaître

$a)$ $\displaystyle\int (2x^3-3x^2+4)\,dx$ $b)$ $\displaystyle\int \sqrt{x}\,dx$ $c)$ $\displaystyle\int \frac{3}{2x+4}\,dx$

$d)$ $\displaystyle\int \frac{2}{x^2+1}\,dx$ $e)$ $\displaystyle\int \frac{1}{\cos^2 x}\,dx$ $f)$ $\displaystyle\int \sin x\,dx$

$g)$ $\displaystyle\int e^{3x+1}\,dx$ $h)$ $\displaystyle\int \frac{1}{\sqrt{x+1}}\,dx$ $i)$ $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$

Exercice 9-2 Primitives "décalées", à reconnaître

$a)$ $\displaystyle\int (x^2-3x+4)^5(4x-6)\,dx$ $b)$ $\displaystyle\int \sin^7 x\cos x\,dx$ $c)$ $\displaystyle\int \frac{\sin x}{\cos x}\,dx$

$d)$ $\displaystyle\int \frac{9x^2-6}{x^3-2x+5}\,dx$ $e)$ $\displaystyle\int \frac{3x}{\sqrt{x^2-2}}\,dx$ $f)$ $\displaystyle\int e^{2x^2-4x}(x-1)\,dx$

$g)$ $\displaystyle\int \frac{3}{1-\sin^2 x}\,dx$ $h)$ $\displaystyle\int \frac{2x}{\sqrt{2-x^2}}\,dx$ $i)$ $\displaystyle\int \frac{x}{(x^2-4)^4}\,dx$

Exercice 9-3 Fractions rationnelles

$a)$ $\displaystyle\int \frac{-2}{2x-5}\,dx$ $b)$ $\displaystyle\int \frac{3}{x^2+4}\,dx$ $c)$ $\displaystyle\int \frac{2x-5}{2x^2+3}\,dx$

$d)$ $\displaystyle\int \frac{4}{(x-1)^5}\,dx$ $e)$ $\displaystyle\int \frac{x+2}{x^2+2x+5}\,dx$ $f)$ $\displaystyle\int \frac{2}{x(x-1)(x-2)}\,dx$

$g)$ $\displaystyle\int_3^7 \frac{x^4}{x^3-3x+2}\,dx$ $h)$ $\displaystyle\int \frac{x}{x^3-1}\,dx$ $i)$ $\displaystyle\int \frac{3x^2}{1-x^2}\,dx$

Exercice 9-4 Fonctions circulaires

$a)$ $\displaystyle\int \frac{1}{\sin x\cos x}\,dx$ $b)$ $\displaystyle\int \frac{\cos^3 x}{\sin^4 x}\,dx$ $c)$ $\displaystyle\int \sin^3 x\cos^2 x\,dx$

$d)$ $\displaystyle\int \sin^4 x\,dx$ $e)$ $\displaystyle\int \cos(2x)\sin x\,dx$ $f)$ $\displaystyle\int \frac{1}{\cos^2 x\sin^2 x}\,dx$

$g)$ $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos^2 x}{\sin x}\,dx$ $h)$ $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{-\pi}{6}} \frac{2}{\sin x}\,dx$ $i)$ $\displaystyle\int \frac{3\sin x}{1-\cos^2 x}\,dx$

Exercice 9-5 Intégration par parties

$$a)\ \int \ln x\,dx \qquad b)\ \int (3x+5)\,e^{2x}\,dx \qquad c)\ \int \sin(x)\,e^{-x}\,dx$$
$$d)\ \int (x^2-1)\ln(2x)\,dx \qquad e)\ \int \arctan x\,dx \qquad f)\ \int 3x^2\ln(x^2+1)\,dx$$
$$g)\ \int (x^2+1)\arctan x\,dx \qquad h)\ \int \arcsin x\,dx \qquad i)\ \int x\arctan x\,dx$$

Exercice 9-6 Changement de variables

$$a)\ \int \frac{dx}{\sqrt{1+e^x}} \qquad b)\ \int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} \qquad c)\ \int \frac{dx}{1+\sqrt{x}}$$
$$d)\ \int \frac{dx}{\sqrt{1+x^2}} \qquad e)\ \int \frac{dx}{2+\sin x} \qquad f)\ \int (2x+1)\sqrt{x+2}\,dx$$

Exercice 9-7 Calculez $\displaystyle\int \frac{dx}{x^2(x-1)}$.

A l'aide du changement de variable $x = \sqrt{\dfrac{t}{t+1}}$, déterminez les primitives de

$$\frac{1}{2}\int \frac{dt}{t(t+1)(\sqrt{\frac{t}{t+1}}-\frac{t}{t+1})}$$

Exercice 9-8 Pour tout entier n, on définit $(I_n)_{n\in\mathbb{N}}$ par $I_n = \displaystyle\int_0^{\frac{\pi}{2}} sin^n(x)\,dx$.

1. Calculez I_0 et I_1.
2. Montrez que $\forall n \in \mathbb{N}, \quad I_{n+2} = \dfrac{n+1}{n+2}\,I_n$
3. Montrez que pour $n = 2k$, on a : $\quad I_{2k} = \dfrac{1\cdot 3\cdot \ldots \cdot (2k-1)}{2\cdot 4\cdot \ldots \cdot 2k}\cdot\dfrac{\pi}{2}$
4. Montrez que pour $n = 2k+1$, on a : $\quad I_{2k+1} = \dfrac{2\cdot 4\cdot \ldots \cdot 2k}{3\cdot 5\cdot \ldots \cdot (2k+1)}$
5. Montrez que $(I_n)_{n\in\mathbb{N}}$ est une suite décroissante.
6. Montrez que $\quad \lim\limits_{k\to\infty} \dfrac{2}{1}\cdot\dfrac{2}{3}\cdot\dfrac{4}{3}\cdot\dfrac{4}{5}\cdot\dfrac{6}{5}\cdot\dfrac{6}{7}\cdot \ldots \cdot \dfrac{2k}{2k-1}\cdot\dfrac{2k}{2k+1} = \dfrac{\pi}{2}$ (Formule de Wallis).