## Seminaire Hypatie :

# Towards a quantitative theory of stochastic homogenization

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***et***

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**Quand** : 31 janvier 2014

**Où** : Institut Camille Jordan, Université Lyon 1. Salle Fokko du Cloux (batiment Braconnier)

**Accès** : http://math.univ-lyon1.fr/divers/?option=acces

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**Programme** :

10:30-11:30 Felix Otto: Quantification of ergodicity by spectral gap and decay of the environment seen by the particle

11:45-12:45 Antoine Gloria: Application to stochastic homogenization: Bounds on the corrector and approximation of homogenized coefficients

Lunch

14:30-15:30 Felix Otto: Logarithmic-Sobolev inequality and annealed bounds on the elliptic Green's function

15:45-16:45 Antoine Gloria: Application to stochastic homogenization: A quantitative two-scale expansion

**Résumé** :

In this series of lectures we shall address recent developments on a quantitative theory of stochastic homogenization of linear elliptic equations on the integer lattice. This theory starts with the quantification of ergodicity in the form of a Spectral Gap estimate (SG) for the Glauber dynamics on the space of coefficients. We shall show in the first lecture how this assumption (SG) combined with quenched integrated bounds on the gradient of the (non-constant coefficients) parabolic Green's function and linear PDE theory allows one to prove the optimal decay of the semi-group associated with the non-constant coefficient operator (the so-called environment seen by the particle) in any dimension. In the second lecture we shall use this optimal decay estimate to prove new results in stochastic homogenization, namely that all the finite moments of the gradient of the corrector are bounded in any dimension, and that all the moments of the modified corrector are bounded (up to a logarithmic divergence in dimension 2). This will allow us to give a complete and sharp error analysis of the approximation of homogenized coefficients by periodization.

In the third lecture, we shall strengthen the (SG) assumption in the form of a Logarithmic-Sobolev Inequality (LSI) and show how this stronger assumption allows one to propagate moment bounds of a given order to any finite order. We shall apply this approach to the case of the gradient and mixed second gradient of the elliptic Green's function, and upgrade to any moment the annealed estimates by Delmotte and Deuschel. In the fourth and last lecture we shall make use of all these results to prove an optimal quantitative two-scale expansion for the solution of a discrete elliptic equation with random coefficients on the torus.