NULL CONTROLLABILITY OF THE HEAT EQUATION IN UNBOUNDED DOMAINS

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Abstract

We are interested in the following null controllability problem for the heat equation in the positive half-line $\mathbb{R}_+ = (0,\infty)$: given a number $T > 0$, a function $u_0 \in L^2(\mathbb{R}_+)$ and an open set $\omega \subset \mathbb{R}_+$, find a control function f such that the solution u of

$$\begin{cases} u_t(t,x) - u_{xx}(t,x) = f(t,x)\chi_\omega(x), & (t,x) \in (0,T)\times\mathbb{R}_+, \\ u(t,0) = 0, & t \in (0,T), \\ u(0,x) = u_0(x), & x \in \mathbb{R}_+, \end{cases} \tag{1}$$

satisfies

$$u(T,\cdot) \equiv 0\,.$$

Here, χ_ω denotes the characteristic function of ω.

Micu and Zuazua [?] proved that, within the class of solutions defined by transposition, there is no smooth compactly supported initial datum u_0 that might be driven to zero in finite time if ω is a bounded interval. This lack of null controllability comes from the fact that the controlled heat equation holds in an unbounded domain, while control is localized on a bounded set, so that an unbounded region is left without control. More recently, Cabanillas, De Menezes and Zuazua [?] gave a positive result to the above problem when ω is an unbounded interval of the form (α,∞) with $\alpha > 0$.

Then, it seems natural to ask if it possible to control the heat equation with a control acting on an unbounded region ω of *finite measure*, which would be the "smallest" control region that one can hope to be efficient to obtain null controllability results. We will show that this is indeed the case discussing the results of [?] for problem (??) in weighted spaces.

References

[1] V. R. Cabanillas, S. B. De Menezes, E. Zuazua, *Null controllability in unbounded domains for the semilinear heat equation with nonlinearities involving gradient terms*, Journal of Optimization Theory and Applications, Vol. 110, No 2, p. 245-264, 2001.

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