

A large- N tensor model with four supercharges

Davide Lettera and Alessandro Vichi. “A large- N tensor model with four supercharges”. In: (Dec. 2020). arXiv: [2012.11600](https://arxiv.org/abs/2012.11600) [hep-th]

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Sections

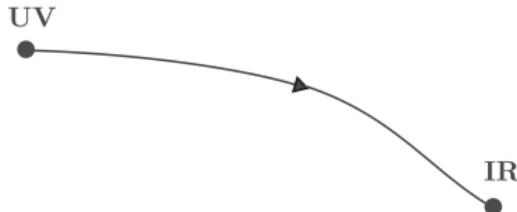
1. Introduction
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3. Supersymmetric tensor model

Introduction

Motivations

Why Tensor Field Theories are interesting?

- The melonic limit is easier to deal with than others: such as the planar limit of matrix models,
- Look for hopefully non trivial CFT,



- Connection with the SYK model:

Edward Witten. "An SYK-Like Model Without Disorder". In: *J. Phys. A* 52.47 (2019), p. 474002. doi: [10.1088/1751-8121/ab3752](https://doi.org/10.1088/1751-8121/ab3752).
arXiv: [1610.09758 \[hep-th\]](https://arxiv.org/abs/1610.09758)

I.R.Klebanov G.Tarnopolsky. "Uncolored Random Tensors, Melon Diagrams, and SYK Models". In: (2017),

- Supersymmetry simplify computations and leads to a real spectrum.

Tensor Field Theories (TFT)

Tensor Field Theory

TFT are QFT built in terms of fields that are tensors of rank- r : $\phi_{a_1 a_2 \dots a_r}$

We focus on the case: $r = 3$.

Igor R. Klebanov, Fedor Popov, and Grigory Tarnopolsky. "TASI Lectures on Large N Tensor Models". In: PoS TASI2017 (2018), p. 004. doi: 10.22323/1.305.0004. arXiv: 1808.09434 [hep-th]

Razvan Gurau. "Notes on Tensor Models and Tensor Field Theories". In: (July 2019). arXiv: 1907.03531 [hep-th]

$$\mathcal{G} : \Phi_{abc} \rightarrow \Phi'_{abc} = M_1^{aa'} M_2^{bb'} M_3^{cc'} \Phi_{a'b'c'} \quad \mathcal{G} = O(N)^3 \\ (M_1 \in O_1(N), M_2 \in O_2(N), M_3 \in O_3(N))$$

We describe the theory in terms of an $O(N)^3$ -symmetryc Lagrangian \mathcal{L}

$$\mathcal{G} : \mathcal{L}[\Phi_{abc}] \rightarrow \mathcal{L}'[\Phi'_{abc}] = \mathcal{L}[\Phi_{abc}]$$

Invariants

- There is only one possible quadratic term: $\Phi_{abc}\Phi_{abc}$.
- There are 3 quartic invariants:

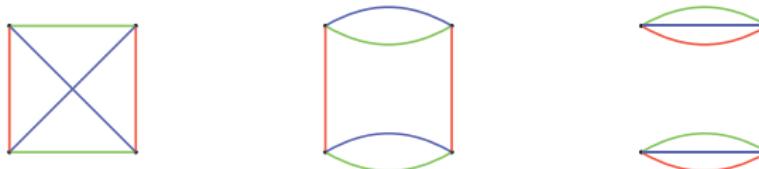


Figure 1: Each vertex represents a field and each colored lines represents two contracted indeces.

$$\mathcal{O}_t = \Phi_{a_1b_1c_1}\Phi_{a_1b_2c_2}\Phi_{a_2b_1c_2}\Phi_{a_2b_2c_1}$$

$$\mathcal{O}_p = \frac{1}{3} (\Phi_{a_1b_1c_1}\Phi_{a_2b_1c_1}\Phi_{a_1b_2c_2}\Phi_{a_2b_2c_2} + \text{perm.})$$

$$\mathcal{O}_{dt} = \Phi_{a_1b_1c_1}\Phi_{a_1b_1c_1}\Phi_{a_2b_2c_2}\Phi_{a_2b_2c_2}$$

Large-N limit

We focus on a model with only \mathcal{O}_t in \mathcal{L} . And we study the large-N limit

- Vector model: 1 index, snail diagrams dominate, keep fixed gN .
- Matrix models: 2 indices, planar diagrams dominate, keep fixed gN^2 .
- 3 indices (or more), melonic diagrams dominate, melonic dominance keeping fixed g^2N^3

Melonic diagrams are a subset of planar diagrams and can be summed exactly!

The model

We have studied a **supersymmetric** and **melonic** model:

$$S[\Phi, \bar{\Phi}] = \int d^d x \int d\theta^2 d\bar{\theta}^2 (\bar{\Phi}_{abc} \Phi_{abc}) + \int d^d y d\theta^2 W[\Phi] + \int d^d \bar{y} d\bar{\theta}^2 W[\bar{\Phi}],$$
$$W[\Phi] = \frac{1}{4} g \Phi_{abc} \Phi_{ade} \Phi_{fbe} \Phi_{fdc},$$

- Φ and $\bar{\Phi}$ are chiral and anti-chiral fields.
- $W[\Phi]$ is the chiral superpotential.
- The total amount of supercharges is 4.

Results

- Scaling dimension of Φ in the IR and large- N limit:
$$\Delta_\Phi = \Delta_{\bar{\Phi}} = \frac{d-1}{4}.$$
- Spectrum of bilinear operators.
- Stress-energy tensor, $O(N)$ conserved currents.
- Large spin sector.
- Anomalous dimension of the Tetrahedral interaction.
- Results are: real, above unitarity bounds, matches with other known results.

Key equations: general

It is always possible to write:

- For the 2 point function $G(x_1, x_2)$ (SD equation):

$$G^{-1}(x_1, x_2) = G_0^{-1}(x_1, x_2) - \Sigma(x_1, x_2).$$

- For the 3 point function $v(x_1, x_2, x_3)$ (BS equation):

$$\int dx dx' \mathcal{K}(x_1, x_2, x, x') v(x, x', x_3) = v(x_1, x_2, x_3).$$

Where Σ is the self-energy and \mathcal{K} is the kernel.

Not always possible to solve them!

Key equations: melonic

When melonic dominance occurs

- For the 2 pt function, relevant diagrams look like a melon.

$$\begin{array}{c} \text{---} \circ \text{---} = \text{---} + \bullet \text{---} + \bullet \bullet \text{---} + \\ \bullet = \text{---} \circ \text{---} \end{array}$$

- For the three point functions, relevant diagrams look like a **ladder**. The kernel operator adds a rung to the ladder.



Supersymmetry

Supersymmetry (SUSY)

SUSY

Supersymmetry: evade the Coleman-Mandula theorem

In the SUSY algebra: fermionic generators Q^I (supercharges) are allowed.

$$Q^I |Fermion\rangle = |Boson\rangle, \quad Q^I |Boson\rangle = |Fermion\rangle.$$

Being fermions, Q^I obey to an anti-commutation relation

$$\{Q_\alpha^I, \bar{Q}_{\dot{\beta}}^J\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu \delta^{IJ} \quad (\text{in 4d})$$

Supermultiplets

Due to the anti-commuting nature of fermionic generators, each supermultiplet contains a finite number of states.

the more supercharges \rightarrow the bigger are supermultiplets

Super space-time and superfields

For each supercharge in the algebra Q we introduce an associated Grassman variable θ

$$(x^\mu) \rightarrow (x^\mu, \theta_i), \quad G(x, \theta, \bar{\theta}, \omega) = e^{ixP + i\theta Q + i\bar{\theta}\bar{Q} + \frac{1}{2}\omega M}$$

Superfields

Superfields are nothing but fields in superspace: $\Phi(x, \theta^I, \bar{\theta}^J)$

We can always expand in Grassman variables and few terms survive

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) = & f(x) + \theta\psi(x)\bar{\theta}\bar{\chi} + \theta\theta m(x)\bar{\theta}\bar{\theta}n(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) + \\ & + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\rho(x) + \theta\theta\bar{\theta}\bar{\theta}d(x), \end{aligned}$$

This equation contains too many degrees of freedom.

Chiral and anti-chiral multiplets

Covariant Derivatives

Covariant derivatives are defined requiring that they anti-commute with supercharges

$$D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu,$$

$$\bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} + i\theta_\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu.$$

Chiral and anti-chiral fields

Chiral fields and anti-chiral fields can be defined requiring:

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \quad D_\alpha\bar{\Phi} = 0.$$

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) - \theta\theta F(y), \quad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$\bar{\Phi}(\bar{y}, \bar{\theta}) = \bar{\phi}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) - \bar{\theta}\bar{\theta}\bar{F}(\bar{y}), \quad y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

3d superspace

The super algebra in 3 dimension takes the form

$$\{Q_\alpha^I, Q_\beta^J\} = 2i\gamma_{\alpha\beta}^\mu P_\mu,$$

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$\mathcal{N} = 2 \rightarrow Q = Q_1 + iQ_2$ and $\bar{Q} = Q_1 - iQ_2$ brings the algebra in the same form of $\mathcal{N} = 1$ in 4d.

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma_{\alpha\beta}^\mu P_\mu + 2i\epsilon_{\alpha\beta}Z$$

Where the central charge Z is the momentum in the reduced direction (say P_3).

Super conformal-multiplets

The recipe to build super-conformal multiplets is

- Defining a **super-primary state** $|\mathcal{O}\rangle$ such that $K_\mu|\mathcal{O}\rangle = S_\alpha|\mathcal{O}\rangle = 0$ and with scaling dimension $D|\mathcal{O}\rangle = \Delta_{\mathcal{O}}|\mathcal{O}\rangle$
- Using the **algebra**

$$[D, Q_\alpha] = -\frac{1}{2}Q_\alpha, \quad [D, P_\mu] = -P_\mu,$$

we find that

$$D(P_\mu|\mathcal{O}\rangle) = (\Delta_{\mathcal{O}} + 1)(P_\mu|\mathcal{O}\rangle),$$
$$D(Q_\alpha|\mathcal{O}\rangle) = \left(\Delta_{\mathcal{O}} + \frac{1}{2}\right)(Q_\alpha|\mathcal{O}\rangle).$$

Notice

Since $Q^2 \sim P$ each descendant is also a super-descendant, but the contrary is not true.

Supersymmetric tensor model

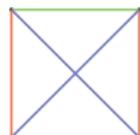
The model

We study the model with chiral (and anti-chiral) fields and $N_Q = 4$ supercharges

$$S[\Phi, \bar{\Phi}] = \int d^d x \int d\theta^2 d\bar{\theta}^2 (\bar{\Phi}_{abc} \Phi_{abc}) + \int d^d y d\theta^2 W[\Phi] + \int d^d \bar{y} d\bar{\theta}^2 W[\bar{\Phi}],$$

$$W[\Phi] = \frac{1}{4} g \Phi_{abc} \Phi_{ade} \Phi_{fbe} \Phi_{fdc}.$$

- Superpotentials W and \bar{W} are exactly marginal in $d = 3$
- In the superpotentials we include only the Tetrahedron \rightarrow melonic dominance.



DSE (1)

Let G be the 2pt function. Let G_0 be the propagator. Then

$$G = G_0 + \lambda^2 G_0 G^3 G, \quad \lambda^2 = g^2 N^3.$$

- Exploiting the analogy with the $d = 4, \mathcal{N} = 1$ case, we can use the standard form of the propagator:

$$G_0(p, \theta_1, \bar{\theta}_2) = \langle \Phi(-p, \theta_1) \bar{\Phi}(p, \bar{\theta}_2) \rangle_0 = \frac{e^{-2(\bar{\theta}_2 \gamma^\mu \theta_1) p_\mu}}{p^2}$$

- For the 2pt function G we use a conformal ansatz

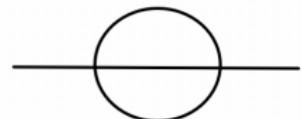
$$G(p, \theta_1, \bar{\theta}_2) = \langle \Phi(-p, \theta_1) \bar{\Phi}(p, \bar{\theta}_2) \rangle = A_2 \frac{e^{-2(\bar{\theta}_2 \gamma^\mu \theta_1) p_\mu}}{p^{2\Delta}}$$

DSE (2)

$$-\frac{e^{2(\bar{\theta}_1 \gamma^\mu \theta_2) p_\mu}}{p^2} = +2A_2^4 \lambda^2 \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{\mathcal{J}_{N=2}(p, \bar{\theta}_1, \theta_2)}{p^2 |p - k_1 - k_2|^{2\Delta} k_1^{2\Delta} k_2^{2\Delta} p^{2\Delta}},$$
$$\mathcal{J}_{N=2}(p, \bar{\theta}_1, \theta_2) = \int d^2 \theta' d^2 \bar{\theta}'' e^{2(\bar{\theta}_1 \gamma^\mu \theta' p_\mu)} e^{-2(\bar{\theta}'' \gamma^\mu \theta' p_\mu)} e^{2(\bar{\theta}'' \gamma^\mu \theta_2) p_\mu}.$$

Notice!

Integrated momenta running into a loop cancel in the exponents !



Results

Integrals over momenta can be performed.

Integrals over Grassman variables can be computed straightforwardly.

$$\Delta = \frac{d+1}{4},$$

$$A_2^4 = \frac{1}{8} \frac{(4\pi)^d}{\lambda^2} \frac{\Gamma^3(\frac{d+1}{4}) \Gamma(\frac{3-d}{4})}{\Gamma^3(\frac{d-1}{4}) \Gamma(\frac{3-d}{4})}$$

- Δ is related to the dimension of Φ : $\Delta_\Phi = \frac{d}{2} - \Delta = \frac{d-1}{4}$
- In $d = 3 - \epsilon$ we find $\gamma_\Phi = \frac{\epsilon}{4}$
- We cannot set ϵ to 0.
- The super conformal algebra fixes the dimension of chiral operators: $\Delta_\Phi = \frac{d-1}{2} R_\Phi = \frac{d-1}{4}$.

Bilinear operators

The possible bilinear operators are:

$$\Phi_{abc} \square^h \Phi_{abc},$$

$$\Phi_{abc} \square^h D^2 \Phi_{abc},$$

$$\bar{\Phi}_{abc} \square^h \Phi_{abc},$$

$$\bar{\Phi}_{abc} \square^h \bar{\Phi}_{abc}$$

$$\bar{\Phi}_{abc} \square^h \bar{D}^2 \bar{\Phi}_{abc}$$

$$\bar{\Phi}_{abc} \square^h D^2 \Phi_{abc}.$$

- In blue: chiral or anti-chiral \rightarrow fixed dimension.
- In red: are not chiral or anti-chiral: but there are no corrections at large N.
- In brown: receive radiative corrections: but it is sufficient to study $\bar{\Phi}_{abc} \square^h \Phi_{abc}$

$\bar{\Phi} D^2 \Phi$ is a super-descendants of $\bar{\Phi} \Phi$

$$D^2 (\bar{\Phi} \Phi) = \frac{1}{2} D^\alpha D_\alpha (\bar{\Phi} \Phi) = \frac{1}{2} D^\alpha (\bar{\Phi} D_\alpha \Phi) = \bar{\Phi} D^2 \Phi,$$

$$D_\alpha \bar{\Phi} = D^\alpha \bar{\Phi} = 0.$$

Scalar singlet sector (1)

We study the 3pt function of $O = \bar{\Phi}_{abc}\Phi_{abc}$ with Φ and $\bar{\Phi}$. Super conformal symmetry **constrains its form**. We choose the **ansatz**

Fedor K. Popov. "Supersymmetric tensor model at large N and small ϵ ". In: *Phys. Rev. D* 101.2 (2020), p. 026020. doi:

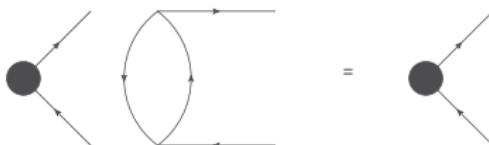
10.1103/PhysRevD.101.026020. arXiv: 1907.02440 [hep-th]

$$V(p, \theta_1, \bar{\theta}_2) = \langle O(0)\Phi(-p, \theta_1)\bar{\Phi}(p, \bar{\theta}_2) \rangle = \frac{e^{-2(\bar{\theta}_2 \gamma^\mu \theta_1)p_\mu}}{p^{2\Delta + \Delta_O}}.$$

We require that it is an **eigenfunction of the kernel**

$$(\mathcal{K}V)(p, \theta_1, \bar{\theta}_2) = g_{N=2}(\Delta_O)V(p, \theta_1, \bar{\theta}_2)$$

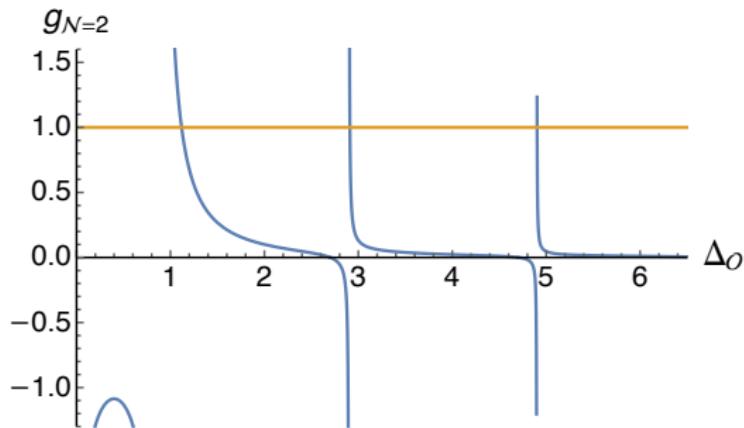
$$\mathcal{K}(y_1\theta_1, \bar{y}_2\bar{\theta}_2, y'\theta', \bar{y}''\bar{\theta}'') = 3\lambda^2 G^2(y'\theta', \bar{y}''\bar{\theta}'')G(y_1\theta_1, \bar{y}''\bar{\theta}'')G(\bar{y}_2\bar{\theta}_2, y'\theta')$$



Scalar singlet sector (2)

$$g_{\mathcal{N}=2}(\Delta_{\mathcal{O}}) = -3 \frac{\Gamma(\frac{d+1}{4})\Gamma(3\frac{d-1}{4})\Gamma(\frac{d-1}{4} - \frac{\Delta_{\mathcal{O}}}{2})\Gamma(\frac{3-d}{4} - \frac{\Delta_{\mathcal{O}}}{2})}{\Gamma(\frac{d-1}{4})\Gamma(\frac{3-d}{4})\Gamma(\frac{d+1}{4} + \frac{\Delta_{\mathcal{O}}}{2})\Gamma(3\frac{d-1}{4} - \frac{\Delta_{\mathcal{O}}}{2})} = 1.$$

The dimensions of operators are the solutions of $g_{\mathcal{N}=2}(\Delta_{\mathcal{O}}) = 1$.



$$\ln d = 3 - \epsilon$$

$$\Delta_{\bar{\Phi}\Phi} = 1 + \epsilon + O(\epsilon^2)$$

$$\Delta_{\bar{\Phi}D^2\Phi} = 2 + \epsilon + O(\epsilon^2)$$

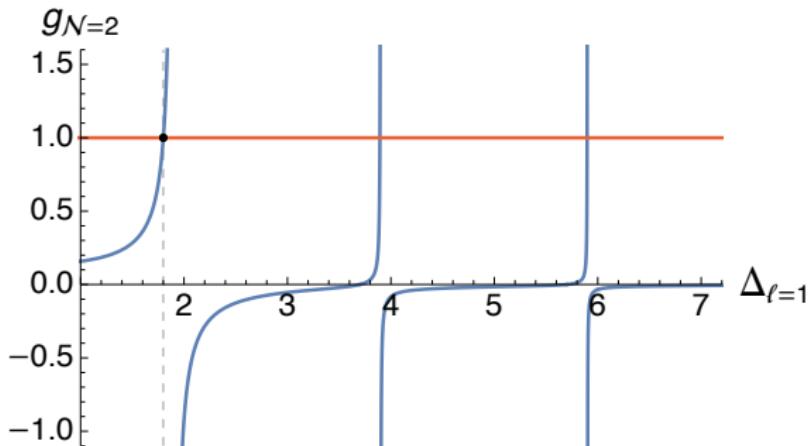
Figure 2: Plot in $d = 2.8$.

Spinning singlet sector

Spinning bilinear operators: $\mathcal{O}^{(\ell)} = \bar{\Phi}_{abc} \partial_{\mu_1} \dots \partial_{\mu_\ell} \square^h \Phi_{abc}$.

$$V_{\mu_1 \dots \mu_\ell}^\ell(p, \theta_1, \bar{\theta}_2) = \langle \mathcal{O}^{(\ell)}(0) \Phi(-p, \theta_1) \bar{\Phi}(p, \bar{\theta}_2) \rangle = \frac{e^{-2(\bar{\theta}_2 \gamma^\mu \theta_1) p_\mu}}{p^{2\Delta + \Delta_\ell + \ell}} p_{\mu_1} \dots p_{\mu_\ell}.$$

$$g_{N=2}(\Delta_\ell, \ell) = -(-1)^\ell 3 \frac{\Gamma\left(\frac{3(d-1)}{4}\right) \Gamma\left(\frac{d+1}{4}\right) \Gamma\left(\frac{d-1}{4} + \frac{\ell - \Delta_\ell}{2}\right) \Gamma\left(\frac{3-d}{4} + \frac{\Delta_\ell + \ell}{2}\right)}{\Gamma\left(\frac{3-d}{4}\right) \Gamma\left(\frac{d-1}{4}\right) \Gamma\left(\frac{3(d-1)}{4} + \frac{\ell - \Delta_\ell}{2}\right) \Gamma\left(\frac{d+1}{4} + \frac{\Delta_\ell + \ell}{2}\right)}.$$



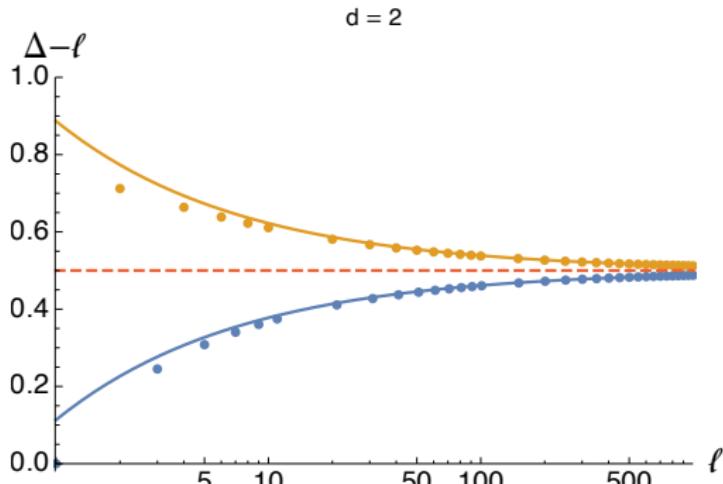
Large spin

- $\ln d = 3 - \epsilon$

$$\Delta_\ell = 2\Delta_\Phi + \ell + \frac{3(-1)^\ell}{4\ell+2}\epsilon + O(\epsilon^2)$$

- For any d : $\Delta_\ell = 2\Delta_\Phi + \ell + \gamma$ and we can expand in small γ

$$\gamma \simeq \frac{1}{e} \frac{6(-1)^\ell \Gamma\left(\frac{3(d-1)}{4}\right) \Gamma\left(\frac{d+1}{4}\right)}{\Gamma\left(\frac{3-d}{4}\right) \Gamma\left(\frac{d-1}{4}\right) \Gamma\left(\frac{d-1}{2}\right)} \frac{1}{\ell^{2\Delta_\Phi}} + \dots, \quad e = 1, +3, -3.$$



Non singlet sector

The possible irreps are 10:

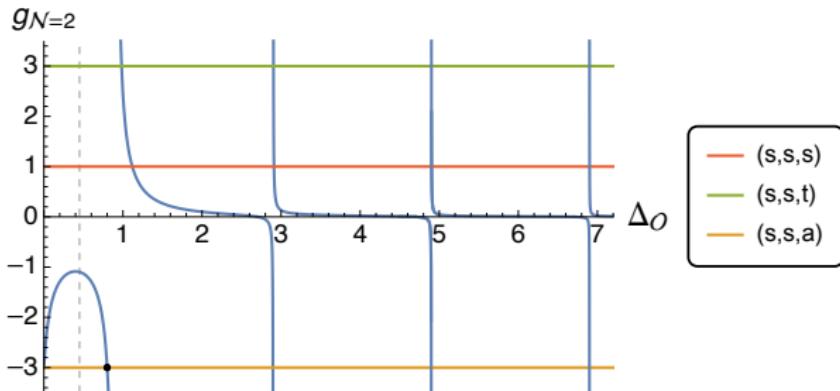
$$(v, v, v) \otimes (v, v, v) \sim (s, s, s) \oplus (s, s, t) \oplus (s, s, a) \oplus (s, t, t) \oplus \dots$$

We checked which irrp gets corrections in the large N

$$(s, s, s) : \quad g_{N=2}(\Delta_O) = 1$$

$$(s, s, a) : \quad g_{N=2}(\Delta_O) = -3$$

$$(s, s, t) : \quad g_{N=2}(\Delta_O) = 3$$



Perturbative check

In perturbation theory we find

$$\gamma_\Phi = \frac{\epsilon}{4}.$$

$$\left(\frac{\partial \beta}{\partial \lambda} \right)_{ij} = \begin{pmatrix} 2\epsilon & 0 & 0 \\ 2\epsilon^{\frac{1}{2}} \lambda_p & 0 & 0 \\ 2\epsilon^{\frac{1}{2}} \lambda_{dt} & 0 & 0 \end{pmatrix}.$$

- The two null eigenvalues corresponds to two exactly marginal operators O_p e O_{dt}
- The eigenvalue 2ϵ means that an anomalous dimension for O_t is generated.

$$\Delta_t = 2 + \epsilon + O(\epsilon^2) = \Delta_{\bar{\Phi} D^2 \Phi}$$

J.A. Gracey et al. "a-function for $N = 2$ supersymmetric gauge theories in three dimensions". In: *Phys. Rev. D* 95.2 (2017), p. 025005. doi:

10.1103/PhysRevD.95.025005. arXiv: 1609.06458 [hep-th]

Remarks

We need to interpret the anomalous dimension of \mathcal{O}_t

Being chiral we did not expect $\gamma_{\mathcal{O}_t} \neq 0$!

In the IR, a multiplet recombination must happen \rightarrow and \mathcal{O}_t became a super descendant of $\bar{\Phi}\Phi$: $D^2(\bar{\Phi}\Phi) \propto \mathcal{O}_t$.

Clay Cordova, Thomas T. Dumitrescu, and Kenneth Intriligator. "Multiplets of Superconformal Symmetry in Diverse Dimensions". In: *JHEP* 03 (2019), p. 163. doi: 10.1007/JHEP03(2019)163. arXiv: 1612.00809 [hep-th]

$U(1)$ explicit symmetry breaking

The free theory has an additional $U(1)$ symmetry: rotating Φ and $\bar{\Phi}$ (but not θ and $\bar{\theta}$). The superpotential $W[\Phi]$ in $d < 3$ is relevant and, in the IR breaks explicitly $U(1)$. We expect

$$D^2(\bar{\Phi}\Phi) = B$$

B being an operator that breaks $U(1)$. We guess $B = \mathcal{O}_t$ because it has the correct scaling dimension and R-charge!

Conclusions

Conclusions

- Spectrum of bilinear operators: **real and above unitarity bound (also in d=2)**

Chi-Ming Chang, Sean Colin-Ellerin, and Mukund Rangamani. "Supersymmetric Landau-Ginzburg Tensor Models". In: *JHEP* 11 (2019), p. 007. doi: 10.1007/JHEP11(2019)007. arXiv: 1906.02163 [hep-th]

- The structure of superspace with $N_Q = 4$ helps us: **non-renormalization theorems, chiral-antichiral fields**
- We observed a **multiplet recombination**
- Results in the **large spin sector**

Future directions

- Investigate the **large spin sector with analytical bootstrap**
- Compute the $\frac{1}{N}$ corrections

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-  Clay Cordova, Thomas T. Dumitrescu, and Kenneth Intriligator. "Multiplets of Superconformal Symmetry in Diverse Dimensions". In: *JHEP* 03 (2019), p. 163. DOI: [10.1007/JHEP03\(2019\)163](https://doi.org/10.1007/JHEP03(2019)163). arXiv: [1612.00809 \[hep-th\]](https://arxiv.org/abs/1612.00809).

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-  J.A. Gracey et al. "a-function for $N = 2$ supersymmetric gauge theories in three dimensions". In: *Phys. Rev. D* 95.2 (2017), p. 025005. doi: [10.1103/PhysRevD.95.025005](https://doi.org/10.1103/PhysRevD.95.025005). arXiv: [1609.06458](https://arxiv.org/abs/1609.06458) [hep-th].
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-  Igor R. Klebanov, Fedor Popov, and Grigory Tarnopolsky. "TASI Lectures on Large N Tensor Models". In: *PoS TASI2017* (2018), p. 004. doi: [10.22323/1.305.0004](https://doi.org/10.22323/1.305.0004). arXiv: [1808.09434](https://arxiv.org/abs/1808.09434) [hep-th].

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-  Edward Witten. "An SYK-Like Model Without Disorder". In: *J. Phys. A* 52.47 (2019), p. 474002. doi: [10.1088/1751-8121/ab3752](https://doi.org/10.1088/1751-8121/ab3752). arXiv: [1610.09758](https://arxiv.org/abs/1610.09758) [hep-th].