Abstracts of the 7th franco-kazakh Colloquium in Model Theory

14 – 18 November 2022

Tuna Altinel: Permutation groups of finite Morley rank
Since the beginning of the fine analysis of groups of finite Morley rank, permutation groups have proven to be an effective tool. The classification of groups of rank 3 containing elements of order 2 by Hrushovski was the most striking example. The work of Gropp and Nesin followed it. Since the classification of simple groups of finite Morley rank of *even type* has been completed, permutation groups of finite Morley rank arouse a much stronger and systematic interest through the notion of *generic transitivity* . Following the fundamental article of Borovik and Cherlin, important advances towards well-defined goals were obtained by various mathematicians: Berkman, Borovik, Deloro, Wiscons. In my talk, after introducing the notion, I will talk about a conjecture of Popov on highly generically transitive groups. Then, I will expose some results in collaboration with Joshua Wiscons and make the connection with the action of finite permutation groups on groups of finite Morley rank.

Yerzhan Baissalov & Jamalbek Tussupov: Formulas over minimal structures II
We present some purely syntactic results on the structure of formulas over a minimal structure. Let M be such a structure.
Definition 1. Let $\varphi(x;\bar{y})$ be a formula over M with free variables $\bar{y} = (y_1, y_2, \dots, y_n)$ and x. We say the variable x is *tame* if the set $\varphi(M;\bar{b})$ is finite for any $\bar{b} \in M$. The variable x is *wild* if the set $\varphi(M;\bar{b})$ is empty or cofinite for any $\bar{b} \in M$. A formula is called *faceted* if its every free variable is either tame or wild.
Theorem 1. The generic type over M is definable iff each formula over M is equivalent to a disjunction of faceted formulas.
Definition 2. A formula over M is called *tame* if every free variable in it is tame. A minimal structure with a definable generic type is *symmetric* if it does not contain a definable partial order with infinite chains [1].
Theorem 2. The minimal structure M is symmetric iff each formula over M is equivalent to a Boolean combination of tame formulas.

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Itaï Ben Yaacov: On codimension two valuations and globally valued fields
Let us consider the following problem :
Fix a field K, and consider two families of indeterminates X and Y. Consider two irreducible polynomials P and Q in $K[X,Y]$, that are homogeneous in each of X and Y separately.
For each n, find a polynomial R_n in $K[X_1, \dots, X_n, Y_1, \dots, Y_n]$ that vanishes on the common roots of $P(X_i, Y_j)$ and $Q(X_i, Y_j)$ for every i, j between 1 and n, such that the degree of R_n does not increase too quickly with n.
This was given a very simple answer by K. Adiprasito in the case where $P = X_1Y_1 + X_2Y_2$ and $Q = X_3Y_4 + X_4Y_4$, but only recently did we answer the full question.
In the talk I will describe the construction, and, if there is enough time, explain where it contributes to the "Globally Valued Field" programme.

Adrien Deloro: Quasi-Frobenius pairs of finite Morley rank
A pair of groups ($C < G$) is called quasi-Frobenius if (1) C has trivial intersections with its distinct G-conjugates and (2) C has finite index in its normaliser in G.
This generalises the classical notion of a Frobenius pair, in order to also capture natural configurations from geometric algebra, such as the geometry of $\mathrm{PGL}(2, C)$ or $\mathrm{SO}(3, R)$. The talk studies these pairs in the model-theoretic context of finite-dimensional theories, which generalise both finite Morley rank and o-minimal worlds. I shall survey recent work by a number of people.

Paolo Marimon: Universally measure zero non-forking formulas in simple ω-categorical Hrushovski constructions
A recent article of Chernikov, Hrushovski, Kruckman, Krupinski, Moconja, Pillay and Ramsey gives the first examples of simple structures with formulas which do not fork over the emptyset but are universally measure zero. In this talk we give the first known simple omega-categorical examples. These happen to be various omega-categorical Hrushovski constructions. Using a probabilistic independence theorem from Jahel and Tsankov, we show how simple omega-categorical structures where the forking ideal and the universally measure zero ideal coincide must satisfy a stronger version of the independence theorem. It is then easy to build Hrushovski constructions for which this strong independence theorem fails.

Nurlan Markhabatov: Model theory of finite and pseudofinite graphs
We continue to study various approximations [11, 6, 9, 7] for natural classes of theories. It is planned to study the model-theoretic properties of pseudofinite graphs [2, 3, 5, 10, 12, 13, 14] using methods and constructions such as ultraproduct, Fraisse limit, Hrushovski construction, probabilistic argument.
Definition.[1] Let L be (relational) language. An infinite L-structure $\mathcal{M}$ is *pseudofinite* if for all L-sentences φ, $\mathcal{M} \models \varphi$ implies that there is a finite structure $\mathcal{M}_0$ such that $\mathcal{M}_0 \models \varphi$. The theory $T = \mathrm{Th}(\mathcal{M})$ of the pseudofinite structure $\mathcal{M}$ is called *pseudofinite.* A pseudofinite graph is an infinite graph that satisfies every first order sentence L that is true for some finite graphs.
Following [6], we denote by $\mathcal{G}_{inf}(\lambda)$, for arbitrary cardinality λ, the family of all infinite acyclic graphs consisting of λ connected components of infinite diameters.
Theorem 1. [6] Let T be the theory of an acyclic graph Γ from the class $\mathcal{G}_{inf}(\lambda)$, for finite cardinality λ, with finitely many rays. If the number of rays in Γ is even then T is pseudofinite theory.
Theorem 2. There exists a pseudofinite acyclic graph Γ from the class $\mathcal{G}_{inf}(\lambda)$, for finite cardinality λ, with an odd number of rays.

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Nazerke Mussina: Jonsson hybrids and their similarities

Definition 1. [1] Let $\mathcal{A}$ be an arbitrary model of signature σ. Let us call the Jonsson spectrum of model $\mathcal{A}$ a set:

$$JSp\,(\mathcal{A}) = \{T \mid T \text{ is the Jonsson theory in a language } \sigma \\ \text{and } \mathcal{A} \in Mod\,T\}.$$

Definition 2. (Mustafin T.) [1] We say that Jonsson theory T_1 is cosemantic to the Jonsson theory T_2 $(T_1 \bowtie T_2)$ if $C_{T_1} = C_{T_2}$, where C_{T_i} is a semantic model T_i, $i = 1, 2$.
The relation of cosemanticness on a set of theories is an equivalence relation. Then $JSp(\mathcal{A})/_{\bowtie}$ is the factor set of the Jonsson spectrum of the model $\mathcal{A}$ with respect to $\bowtie$.
Definition 3. [2] Let T_1 and T_2 be some Jonsson theories of the countable language L of the same signature σ; C_1 and C_2 are their semantic models, respectively. In the case of common signature of Jonsson theories T_1, T_2, let us call a hybrid of Jonsson theories T_1 and T_2 of the first type the following theory $Th_{\forall\exists}(C_1 \times C_2)$ if that theory is Jonsson in the language of signature σ and denote it by $H(T_1, T_2)$ and $C_1 \times C_2 \in Mod\,\sigma$.
Through $T_1 \overset{S}{\sim} T_2$ will be denote the Jonsson syntactic similarity [1] of theories T_1 and T_2. The syntactic similarities of the complete theories [1] T_1 and T_2 will be denoted $T_1 \overset{S}{\approx} T_2$.
Theorem 1. *Let K be an axiomatizable class of models of a countable language L of signature σ. Let $[T_1], [T_2], [T_3], [T_4] \in JSp(K)/_{\bowtie}$, $H_1 = H([T_1], [T_2])$ and $H_2 = H([T_3], [T_4])$ are complete for existential sentences perfect hybrids, then following conditions are equivalent:*

1. $H_1 \overset{S}{\sim} H_2$;
2. $H_1^* \overset{S}{\approx} H_2^*$.

Theorem 2. *Let K be an axiomatizable class of models of a countable language L of signature σ, $[T_1], [T_2] \in JSp(K)/_{\bowtie}$. For any perfect complete for $\exists$-sentences hybrid $H([T_1], [T_2])$ there is a Jonsson $\exists$-complete theory of the polygon T'_Π such that $H([T_1], [T_2]) \overset{S}{\sim} T'_\Pi$.*
All the concepts that are undefined here can be extracted from [1,2].

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Bruno Poizat: The Weil and Hrushovski Theorems for Groups and Symmetrons

The function $(x+y)/(1+xy)$ defines a law of group on the real segment $]-1,1[$, which is in fact isomorphic to the additive group of the reals. Over the complex numbers, it is only partially defined, and is generically associative and generically simplifiable ; therefore, according to Weil's Theorem, which has been extended by Hrushovski to the stable case, it must correspond to a generic chunk of an algebraic group, which happens to be, in this very case, the multiplicative group of the field.

The partial function $(x+y)/(1+x\bar{y})$ defines a loop chunk on the complex numbers ; by taking coordinates in the ring $C[X]/(X^2+1)$, it defines a loop chunk on $C \times C$, which is not a chunk of any algebraic loop.

A loop is a set equipped with a (total) binary operation $x * y$ such that for any a and b there is a unique x such that $x * a = b$, and a unique y such that $a * y = b$. It is a K-loop (or Bruck's loop), if (i) it possesses a two-sided identity, (ii) it satisfies a weak form of associativity called Bol's equation, implying that the left-inverse equals the right inverse, (iii) the inverse is an automorphism, i.e. $x^{-1} * y^{-1} = (x * y)^{-1}$ (an associative K-loop is a commutative group).

The example above satisfies generically these conditions ; in it, a generic point has two different square roots.

A special kind of loop, which I suggest to call symmetrons, plays a role in the theory of groups of finite Morley rank without involutions: they are the dei ex machina of the proof by Frécon of the inexistence of simple groups of rank three without involutions.

A symmetric space is a binary operation $s(x,y)$ such that (i) $s(x,x) = x$, (ii) $s(s(x,y),y) = x$, (iii) $s(s(x,y),z) = s(s(x,z),s(y,z))$; when we call "symmetry of center a " the unary operation $s(x,a)$, the meaning of the axioms is transparent : each symmetry fixes its center, is involutive, and is an automorphism of the structure. For any group G , the operation $yx^{-1}y$ is a symmetric space.

A symmetron is a symmetric space which is a loop: for any x and y there is a unique z, called the middle point $m(x,y)$ of x and y, such that $s(x,z) = y$. The symmetric space of a group G is a symmetron when G is uniquely 2-divisible, as it is the case with groups of finite Morley rank without involutions.

Many properties of groups of finite Morley rank can be extended to symmetrons of finite Morley rank, and an open question is whether Weil-Hrushovski Theorem is valid for them. In fact, any symmetron is bi-interpretable with a uniquely 2-divisible K-loop (for a group G, the K-loop is $y^{1/2}x^{-1}y^{1/2}$), and the counter-example above does not provide a symmetron chunk.

Another question is whether symmetrons of finite Morley rank can be characterized by "algebraic" conditions in Borovik's style ; a partial answer has been given by Zamour in his recently defended doctoral thesis.

Indira Tungushbayeva: Properties of companions of AP-theories

We consider the model-theoretic properties of the theory of differentially closed fields (zero and positive characteristics) in the framework of the study of Jonsson AP-theories. In addition, the forcing companions of the Jonsson AP-theories in the enriched signature in the more general situation are considered.

The concept of AP-theory was introduced in [1].

Definition. [1] A theory T is called to be an AP-theory if in T the amalgam property implies joint embedding property.

The examples are the theories of differential fields of zero and positive characteristic, such as DF_0, DCF_0, DPF_p, DCF_p. In addition, concerning this theories, the following results are obtained:

Theorem. DF_0, DCF_0, DPF_p, DCF_p are perfect Jonsson AP-theories. Meanwhile, DF_p is not a Jonsson theory.

The case of positive characteristic in differential fields gave rise to the study of the connection between a Jonsson AP-theory and a non-Jonsson theory. We consider the theories Δ_1, Δ_2, Δ_3 that satisfy the following conditions:
1) Δ_1 is an inductive theory that is not a Jonsson theory, but has a model companion which is the theory Δ_3,
2) Δ_2 is a hereditary Jonsson AP-theory that has a model companion, which is also Δ_3.
We have the following extensions of the theories Δ_1, Δ_2, Δ_3 in the enrichment of L by adding new symbols c and P.

$$\overline{\Delta_1} = \Delta_1 \cup \Delta_1^f \cup \{P, \subseteq\},$$

where $\{P, \subseteq\}$ is an infinite list of $\exists$-sentences and interpretation of P is an existentially closed submodel in a model of Δ_1,

$$\overline{\Delta_2} = \Delta_2 \cup \Delta_2^f \cup Th_{\forall\exists}(C_2, c),$$

where C_2 is a semantic model of Jonsson theory Δ_2.
The following results are obtained:
Theorem. [2] $\overline{\Delta_1}^f = \overline{\Delta_2}^f = \Delta_1^f$.
Let $\overline{\Delta_3} = \overline{\Delta_1} \cup \overline{\Delta_2} \cup P(c)$. Then
Theorem. [2] Let $\overline{\Delta_3} = \overline{\Delta_1} \cup \overline{\Delta_2} \cup P(c)$. Then 1) $\overline{\Delta_3}$ is consistent; 2) $(\overline{\Delta_3})^f = \Delta_1^f = \Delta_2^f$.

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Frank Wagner: Soluble dimensional groups
I shall generalize the results about soluble groups of finite Morley rank to the (fine finite-) dimensional context.
Definition. A theory T is (fine finite-) dimensional if there is a dimension function dim from the collection of all parameter-interpretable sets to $\{-\infty\} \cup \omega$ such that

- $\dim X = -\infty$ iff $X = \emptyset$ and $\dim X = 0$ iff X is finite.
- $\dim(X \cup Y) = \max\{\dim X, \dim Y\}$.
- If $f : X \to Y$ is an interpretable function with $\dim f^{-1}(y) = k$ for all $y \in Y$, then $\dim X = k + \dim Y$.
- If $f : X \to Y$ is a interpretable function, then $\{y \in Y : \dim f^{-1}(y) = k\}$ is definable for all $k \in \omega$.

Theorem. Let G be a connected group acting faithfully on an infinite abelian group A.

1. If G is abelian, $C_A(G)$ is trivial and A has minimal dimension possible, then A is G-minimal and there is a definable field K such that $A \cong K^+$ and $M \hookrightarrow K^\times$.
2. If G is nilpotent, then either the action is nilpotent, or a field is naturally interpretable in some section.
3. If $C_A(G)$ is trivial, A is G-minimal and G has an infinite normal abelian subgroup M, then there is a definable field K over which A is definably a vector space of finite linear dimension, such that the action of G is K-linear and the action of M scalar (so M is central in G).

Corollary. A connected soluble non-nilpotent dimensional group interprets naturally a field.

Aibat Yeshkeyev: On Jonsson spectra and their existentially closed models
The concept of the spectrum of existentially closed structures and classes of semantic Jonsson varieties (quasivarieties) is defined. Let σ be a signature, $L \supset L_0 \supset L_0^{\forall\exists} \supset Ax(T) \;\Rightarrow E_T \neq \emptyset$. Let $K \subseteq Mod(T)$, $T^0(K) = Th_{\forall\exists}(K)$. Here we consider two cases. In the first one, $K \subseteq E_T$. Then the spectrum of K is defined as follows.
Definition. The spectrum of the class K is the following set of Jonsson theories $JSp(K)$:

$$JSp(K) = \{\Delta \subseteq L_0 \mid \Delta \text{ is a Jonsson theory and } A \models \Delta \text{ for any } A \in K\}.$$

In the second case, let K be a variety (quasivariety) in the usual sense as in [1]. For a given variety (quasivariety) K we define a Jonsson semantic variety (quasivariety) JK.
Definition. The Jonsson spectrum $JSp(JK)$ of the class JK is the following set:

$$JSp(JK) = \{(JN)^0 \mid N \text{ is a subvariety (subquasivariety) of } K\}.$$

A sufficient condition has been found to solve Forrest's question [2] in the language of the inner and outer worlds for a class of fixed signature structures:
Theorem. Let K be a variety, $JSp(JK)$ be a Jonsson spectrum of the semantic Jonsson variety JK, then if K is a perfect class ($(JK)^0 = (JK)^*$), for any class $[T] \in JSp(JK)$ such that $[T]$ is elementarily closed and companion-convex, its semantic model, which is algebraically closed, also belongs to $E_{[T]}$.
The solution for the question of the existence of holographic structures posed in the paper [3] is given:
Theorem. If T is a perfect, $\forall\exists$ complete, Jonsson theory, then $Hol_T \neq \varnothing$, i.e. such a theory has a holographic model.
In the language of the inner and outer worlds, the following result is obtained with respect to λ-similarity of existentially closed structures:
Theorem. [4] Let T be a $\exists$–complete Jonsson theory, and for some $\omega \leq \lambda \leq \mu \;\; N(E_T^{\lambda,\mu}) = 1$ holds. Then the theory T^* is model complete.

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Kabylda Zhetpissov: Tolendi Garifovich Mustafin and the Soviet-French Colloquium on Model Theory
In my short report, I want to talk about Tolendi Garifovich Mustafin (1942-1994), a famous Kazakh scientist, about his role in the development of model theory in Kazakhstan.
Early realizing the importance of scientific cooperation in such a fundamental science as model theory, he was one of the first to break through Soviet isolationist approach and organize the first scientific forum in 1990 in Karaganda (Kazakhstan), which went down in history as "Soviet-French Colloquium on Model Theory".
Fortunately, French scientists from the universities of Lyon-1 and Paris-7 also acted as an active side of the cooperation, which continues to this day.
Here are all the colloquia that took place as part of this collaboration (the history is reflected in the variation of their names):

1. Soviet-French Colloquium on Model Theory, 1990, Karaganda (Kazakhstan)
2. French-Ex-Soviet Colloquium on Model Theory, 1992, Marseille (France)
3. Kazakh-French Colloquium on Model Theory, 1994, Almaty (Kazakhstan)
4. French-Turan Colloquium on Model Theory, 1997, Marseille (France)
5. Kazakh-French Colloquium on Model Theory, 2000, Karkaraly (Kazakhstan)
6. French-Kazakh Conference on Model Theory and Algebra, 2005, Astana (Kazakhstan)

The report will demonstrate various archival printed and photographic materials.