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VII FRENCH-KAZAKH COLLOQUIUM ON MODEL THEORY

ON JONSSON SPECTRA AND THEIR EXISTENTIALLY CLOSED MODELS

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Outline

- Introduction to Jonsson theories
- Jonsson spectra of classes
- Holographicness in Jonsson theories

References

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Jonsson theory

Definition 1 ([1])

A theory T is Jonsson if:
1) theory T has infinite models;
2) theory T is inductive;
3) theory T admits the joint embedding property (JEP);
4) theory T admits the amalgam property (AP).

Examples of Jonsson theories:

1) Group Theory,
2) Theory of Abelian groups,
3) Theory of fields of fixed characteristics,
4) Theory of Boolean algebras,
5) Theory of polygons over a fixed monoid,
6) Theory of modules over a fixed ring,
7) Theory of linear order.

Homogeneous-universal model

Definition 2 ([2])

Let $\kappa \geq \omega$. Model M of theory T is called

- *κ-universal for T, if each model T with the power strictly less κ isomorphically embedded in M;*
- *κ-homogeneous for T, if for any two models A and A_1 of theory T, which are submodels of M with the power strictly less then κ and for isomorphism $f : A \to A_1$ for each extension B of model A, wich is a submodel of M and is model of T with the power strictly less then κ there is exist the extension B_1 of model A_1, which is a submodel of M and an isomorphism $g : B \to B_1$ which extends f.*

Semantic model

Definition 3 ([2])

Model C of Jonsson theory T is called semantic model, if it is ω^{+}-homogeneous-universal.

Definition 4 ([2])

The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^{*}, i.e. $T^{*} = Th(C)$.

Existence of semantic model

Fact 1 ([2])

Each Jonsson theory T has k^+-homogeneous-universal model of power 2^k. Conversely, if a theory T is inductive and has infinite model and ω^+-homogeneous-universal model then the theory T is a Jonsson theory.

Fact 2 ([2])

Let T is a Jonsson theory. Two k-homogeneous-universal models M and M_1 of T are elementary equivalent.

Perfectness

Definition 5 ([2])

Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^*.

Theorem 1 ([2])

Let T is a Jonsson theory. Then the following conditions are equivalent:

1. *theory T is perfect;*
2. *theory T^* is a model companion of theory T.*

Let E_T be a class of all existentially closed models of theory T.

Theorem 2 ([2])

If T is a perfect Jonsson theory then $E_T = ModT^*$.

Cosemanticness

Let T_1 and T_2 be Jonsson theories, T_1^* and T_2^* be their centres, respectively.

Definition 6 ([2])

T_1 and T_2 are said to be cosemantic Jonsson theories (denoted by $T_1 \bowtie T_2$), if $T_1^* = T_2^*$.

Let σ be some signature, $L \supset L_0 \supset L_0^{\forall\exists} \supset Ax(T) \Rightarrow E_T \neq \emptyset$.

Let $K \subseteq Mod(T)$, $T^0(K) = Th_{\forall\exists}(K)$.

1. $K \subseteq E_T$, then

Theorem 3

$T^0(K)$ is a Jonsson theory.

The spectrum of the class K is the following set of Jonsson theories $JSp(K)$:

$$JSp(K) = \{\Delta \subseteq L_0 \mid \Delta \text{ is a Jonsson theory and } A \models \Delta \text{ for any } A \in K\}.$$

2. K is a variety (quasivariety).

Semantic Jonsson varieties and quasivarieties

Let K be a variety (quasivariety) in the usual sense as in [3]. We construct a set $\forall\exists(K)$, where $\forall\exists(K)$ is a set of Jonsson theories and obtained as follows:

$$\forall\exists(K) = \{ Th(K) \cup \varphi \,|\, \varphi \text{ is an } \forall\exists\text{-sentence and} \qquad (*)$$

$$\varphi \cup Th(K) \text{ is consistent}\}.$$

In other words, the set $\forall\exists(K) = \{T_1, T_2, ...\}$ is a list of all Jonsson theories that satisfy Condition (*). Then C_i is a semantic model of T_i from this list. Let us consider the following set:

$$JK = \{C_i \,|\, C_i \text{ is a semantic model of } T_i,\ T_i \in \forall\exists(K)\}.$$

Semantic Jonsson varieties and quasivarieties

Definition 7

The class JK is a semantic Jonsson variety (quasivariety), if the theory $Th_{\forall\exists}(JK)$ is Jonsson.

Let us denote the theory $Th_{\forall\exists}(JK)$ by $(JK)^0$. The theory $(JK)^0$ is called a Kaiser hull of the class JK.

Definition 8

The Jonsson spectrum $JSp(JK)$ of the class JK is the following set:

$$JSp(JK) = \{(JN)^0 \mid N \textit{ is a subvariety (subquasivariety) of } K\}.$$

Then $JSp(JK)_{/\bowtie}$ denotes the factor set of the Jonsson spectrum of Jonsson variety (quasivariety) of the class K by the relation $\bowtie$.

Inner and outer world of the existentially closed model of the Jonsson theory

Definition 9 ([4])

Let T be an arbitrary Jonsson theory.
$IW_T(A) = \{A' \in E_T | \ f$ is isomorphism, $f : A' \to A, \ A \in E_T\}$ is called the inner world of the model A for T.

Definition 10 ([4])

Let T be an arbitrary Jonsson theory.
$OW_T(A) = \{B \in E_T : $ there exist $A' \cong A, \ A' \subseteq B\}$ is called the outer world of the model A for T.

Definition 11 ([4])

The world of the existentially closed model A is the following set

$$W_T(A) = IW_T(A) \cup OW_T(A).$$

Inner and outer world of the existentially closed model of the Jonsson theory

Note that the above definitions connect two different existentially closed models in the case of a convex theory. In this manner, the following fact is true.

Fact 3 ([4])

Let T be a perfect strong convex Jonsson theory. Then for any models $A, B \in E_T$ the following is true:

1) $OW_T(A) \cap OW_T(B) \neq \emptyset$,
2) $IW_T(A) \cap IW_T(B) \neq \emptyset$.

The λ-comparison of two existentially closed models

Definition 12 ([4])

Let T be a Jonsson theory. Let $\omega \leq \lambda \leq \mu$, A and B be existentially closed models of the theory T. $|A| = |B| = |\mu|$. Models A and B are called λ-comparable *if for any existentially closed submodel A' of a model A, such that $|A'| \leq \lambda$, it is true that A' is an existentially closed submodel of B, and for any existentially closed submodel B' of a model B, such that $|B'| \leq \lambda$, it is true that B' is an existentially closed submodel of A.*

The λ-comparison of two existentially closed models

It is clear that the above definition defines an equivalence relation on the set of all existentially closed models of the considered Jonsson theory. Therefore, the following spectral definition of the number of model classes makes sense.

Let T be a Jonsson theory, $\omega \leq \lambda \leq \mu$. $N(E_T^{\lambda,\mu})$ is the number of classes of existentially closed models of Jonsson theory of cardinality μ concerning the λ-comparison relation.

Let $IW_T^{\lambda}(A)$ be the set of all models from $IW_T(A)$, whose cardinality does not exceed λ.

Theorem 4 ([4])

Let T be a $\exists$–complete Jonsson theory, and for some $\omega \leq \lambda \leq \mu$ $N(E_T^{\lambda,\mu}) = 1$ holds. Then the theory T^* is model complete.

The λ-comparison of two existentially closed models

Due to the fact that the semantic model of some Jonsson theory, which specifies the cosemanticness of these two theories, is existentially closed, it does not yet follow that the Cartesian product of two existentially closed submodels of the semantic model will be an existentially closed submodel of this semantic model.

Definition 13 ([4])

Let X be an arbitrary definable subset of the semantic model C of the Jonsson theory T and its closure $cl(X) = M$ in some pregeometry given on the power set of C Then the Jonsson theory $Th_{\forall\exists}(M)$ (denoted by $Fr(X)$) is called a fragment in the theory T.

It is easy to see that if $M \in E_T$, then $Fr(X)$ is always Jonsson theory. In this research, we do not consider the content of the set X.

The λ-comparison of two existentially closed models

Let $Fr(C)$ denote the set of all fragments in the theory T.

Definition 14 ([4])

A Jonsson theory T is called ***totally model-consistent*** *if any of its two fragments $Fr(X_1)$, $Fr(X_2) \in Fr(C)$ are mutually model-consistent, where X_1, X_2 are some definable subsets of C.*

The λ-comparison of two existentially closed models

In what follows, we will work within the framework of a fixed totally model-consistent Jonsson theory T. It is clear that in this case, the question arises: is the cosemanticness class of Jonsson theories perfect if its center is model-consistent with regard to Jonsson spectrum of a subclass of the class of existentially closed models of theory T?

Theorem 5 ([4])

Let $K \subseteq E_T$. If the fixed center $[T]^*$ of some cosemanticness class $[T] \in JSp(K)$ of the Jonsson spectrum $JSp(K)$ is totally model-consistent, then its semantic model is saturated.

Semantic Jonsson varieties and quasivarieties

Definition 15

Let $(JK)^0 = Th_{\forall\exists}(JK)$ *and* $(JK)^* = Th(JK)$. *If* $(JK)^0 = (JK)^*$ *then the class* K *is called perfect.*

Semantic Jonsson varieties and quasivarieties

It is clear that $JSp(JK) \subseteq JSp(\mathcal{A})$ for any model $\mathcal{A} \in K$, where K is a variety (quasivariety).

Definition 16

Let K_1 and K_2 be quasivarieties. The class JK_1 is JSp(K)-cosemantic to class JK_2 ($JK_1 \underset{JSp(K)}{\bowtie} JK_2$) if

$$JSp(JK_1)/_{\bowtie} = JSp(JK_2)/_{\bowtie}$$

.

Semantic Jonsson varieties and quasivarieties

Let *cl* be the closure operator of some pregeometry defined on all subsets of semantic model of considered Jonsson theory.

Definition 17 ([5])

Let K be a quasivariety, $\mathcal{A} \in K$, $[T] \in JSp(\mathcal{A})/_{\bowtie}$. The center of Jonsson class $[T]$ is an elementary theory $[T]^*$ of its semantic model $\mathcal{C}_{[T]}$, i. e. $[T]^* = Th(\mathcal{C}_{[T]})$ and $[T]^* = Th(\mathcal{C}_{\Delta})$ for every $\Delta \in [T]$.

Semantic Jonsson varieties and quasivarieties

We denote by $E_{[T]} = \bigcup\limits_{\Delta \in [T]} E_\Delta$ the class of all existentially closed models of the class $[T] \in JSp(\mathcal{A})/_{\bowtie}$. Note that $\bigcap\limits_{\Delta \in [T]} E_\Delta \neq \varnothing$, since at least for every $\Delta \in [T]$ we have $\mathcal{C}_{[T]} \in E_\Delta$.

Let $\mathcal{A}, \mathcal{B} \in K$, where K is a variety (quasivariety).

Semantic Jonsson varieties and quasivarieties

Definition 18

A model $\mathcal{A}$ is called $JSp(K)$*-cosemantic to a model $\mathcal{B}$* ($\mathcal{A} \underset{JSp(K)}{\bowtie} \mathcal{B}$) *if* $JSp(\mathcal{A})/_{\bowtie} = JSp(\mathcal{B})/_{\bowtie}$. *Accordingly, a model $\mathcal{A}$ is said to be* $JSp(K)$*-cosemantic to a model $\mathcal{B}$ regarding Γ, where $\Gamma \subseteq L$ and we denote it by* $\mathcal{A} \overset{\Gamma}{\underset{JSp(K)}{\bowtie}} \mathcal{B}$, *if* $JSp_{\Gamma}(\mathcal{A})/_{\bowtie} = JSp_{\Gamma}(\mathcal{B})/_{\bowtie}$.

Semantic Jonsson varieties and quasivarieties

We can define the completeness of the class $[T]$ regarding semantic Jonsson variety K (semantic Jonsson quasivariety K) as follows (Definition 19), and all four types of completeness are independent of each other and can combine. An interesting problem is the transfer of results from the Jonsson spectrum to the semantic Jonsson variety (semantic Jonsson quasivariety), when the completeness of the Jonsson theory is replaced by the following types of completeness and their combinations.
Let K be a variety (quasivariety), $[T] \in JSp(JK)$, $\Gamma \subseteq L$.

Semantic Jonsson varieties and quasivarieties

Definition 19

The class $[T]$ *is called a* Γ_i*-complete class regarding* K, *if the following* i*-conditions are true:*

i_1) $\forall \mathcal{A}, \mathcal{B} \in E_{[T]},\ \mathcal{A} \underset{JSp(K)}{\overset{\Gamma}{\bowtie}} \mathcal{B}$;

i_2) $\forall \mathcal{A}, \mathcal{B} \in E_{[T]},\ \mathcal{A} \underset{JSp(K)}{\overset{\Gamma}{\bowtie}} \mathcal{B}$ *and* $\forall \Delta \in [T],\ \Delta - \Gamma$*-complete.*

i_3) $\forall \varphi \in \Gamma,\ \forall \Delta \in [T],\ \Delta \vdash \varphi$ *or*
$\Delta \vdash \neg\varphi \Leftrightarrow \forall \mathcal{A}, \mathcal{B} \in Mod\Delta,\ \forall \Delta \in [T],\ \mathcal{A} \equiv_\Gamma \mathcal{B}$;

i_4) $\forall \varphi \in \Gamma,\ \forall \mathcal{A}, \mathcal{B} \in E_{[T]},\ \mathcal{A} \models \varphi \Leftrightarrow \mathcal{B} \models \varphi$.

Semantic Jonsson varieties and quasivarieties

It is well known that the concepts of completeness and model completeness do not coincide, but as it is shown in [2] in the case of a perfect Jonsson theory, these concepts coincide for the Jonsson theory under consideration. Therefore, in going over to the problem of the Jonsson spectrum, we must take into account that in the case of an imperfect class these concepts do not coincide.

A semantic Jonsson variety (quasivariety) JK satisfies some model-theoretic notion P if each class $[T] \in JSp(JK)$ satisfies the P property. Next, we define some particular cases of the property P for the semantic Jonsson variety (quasivariety) JK through an arbitrary class $[T] \in JSpV(JK)$:

Definition 20

The class $[T]$ is model complete if and only if $\forall \Delta \in [T]$, Δ is model complete.

Semantic Jonsson varieties and quasivarieties

A semantic Jonsson variety (quasivariety) JK is model complete if any class $[T] \in JSp(JK)$ is model complete if and only if $\forall \mathcal{A}, \mathcal{B} \in E_{[T]}$, $\forall$ monomorphism $f : \mathcal{A} \to \mathcal{B}$ is elementary if and only if $\forall \varphi \in L, \exists \psi \in \forall \cap \exists$: $[T] \vdash (\varphi \sim \psi)$ $[T] \vdash (\varphi \sim \psi) \leftrightharpoons \forall \Delta \in [T], \Delta \vdash (\varphi \sim \psi)$.

Lemma 4 ([5])

If $\Delta \in [T]$ and Δ is imperfect, then $\exists \mathcal{B} \in E_{\Delta}$, $\mathcal{B} \in E_{\Delta'}$ for some $\Delta' \in [T]$.

Definition 21

*Two semantic Jonsson varieties (quasivarieties) JK_1, JK_2 are **existentially mutually model complete to each other** ($JK_1 \leftrightarrow JK_2$) if for any $[T]_1 \in JK_1, [T]_2 \in JK_2$ it follows that the classes $[T]_1$, $[T]_2$ are existentially mutually model complete to each other ($[T]_1 \leftrightarrow [T]_2$), i.e. $\forall \mathcal{B} \in E_{[T]_1}$, $\exists \mathcal{B}' \in E_{[T]_2} : \mathcal{B} \overset{\cong}{\to} \mathcal{B}'$ and the converse is true.*

Semantic Jonsson varieties and quasivarieties

Lemma 5

$JK_1 \leftrightarrow JK_2 \Leftrightarrow Th_{\forall}(JK_1) = Th_{\forall}(JK_2)$.

Let us consider some properties of the Jonsson spectrum at fixed completeness (a special case of Definition 19 (i_2)).
Let K be a semantic Jonsson variety (quasivariety).

$[T] \in JSp(JK)$.

$$Mod[T] = \{\mathcal{A} \in Mod\sigma_K / \mathcal{A} \models T_i, \forall T_i \in [T]\}.$$

$$Mod(JSp(\mathcal{A})) = \{\mathcal{B} \in Mod\sigma_K / \mathcal{B} \models T_j, \forall T_j \in JSp(\mathcal{A})\}.$$

$$Mod(JSp(\mathcal{A})/_{\bowtie}) = \{\mathcal{B} \in Mod\sigma_K / \mathcal{B} \models [T], \forall [T] \in JSp(\mathcal{A})/_{\bowtie}\}.$$

The following fact on elementarity is clear from above definitions: the $E_{[T]}$ is elementary class if and only if $[T]$ has a model companion.

Semantic Jonsson varieties and quasivarieties

Theorem 6 ([1])

$[T]$ has a model companion if any E_{T_i} will be an elementary class, $T_i \in [T]$.

Lemma 6

Let K be a quasivariety. $JK_1 \underset{JSp(K)}{\bowtie} JK_2 \Leftrightarrow JSp(\mathcal{A})/_{\bowtie} = JSp(\mathcal{B})/_{\bowtie}$, $\forall \mathcal{A} \in E_{K_1}, \forall \mathcal{B} \in E_{K_2}$.

Let K be a (variety) quasivariety.

Fact 7

$JSp(JK)$ is perfect, i.e. for any $[T] \in JSp(JK)$ it is true that $[T]$ is perfect iff $C_{[T]}$ is saturated.

Semantic Jonsson varieties and quasivarieties

Fact 8

Let JSp(JK) be a perfect, then for any $[T] \in JSp(JK)$ *the following conditions hold:*

1)$[T]$ *is complete iff* $\forall \mathcal{A}, \mathcal{B} \in E_{[T]}$, $\mathcal{A} \underset{JSp}{\bowtie} \mathcal{B}$.

2) $E_{[T]} = \bigcup_{i \in I} E_{T_i}$.

3) *The* $[T]$ *is* $\forall\exists$*-complete if and only if* $\forall \mathcal{A}, \mathcal{B} \in E_{[T]}$, $\mathcal{A} \overset{\forall\exists}{\underset{JSp(K)}{\bowtie}} \mathcal{B}$.

4) $A \overset{\forall\exists}{\underset{JSp(K)}{\bowtie}} \mathcal{B}$ *iff* $JSp_{\forall\exists}(\mathcal{A})/_{\bowtie}$ *iff* $JSp_{\forall\exists}(\mathcal{B})/_{\bowtie}$, *where K is a quasivariety.*

5) $JSp_{\forall\exists}(\mathcal{A})/_{\bowtie} = \{T \mid T$ *is* $\forall\exists$*-complete Jonsson theory*, $\mathcal{A} \models T\}$, *where K is a quasivariety.*

Semantic Jonsson varieties and quasivarieties

The following definition defines the inner world ($IW_{[T]}(A)$) of the model A of the class $[T]$ when $A \in E_{[T]}$.

Definition 22

Let K be a variety (quasivariety) and let $[T] \in JSp(JK)$. $IW_{[T]}(A) = \{A' \in E_{[T]} | \ f$ is isomorphism, $f : A' \to A, \ A \in E_{[T]}\}$ is called the inner world of the model A for $[T]$.

The following definition defines the outer world ($OW_{[T]}(A)$) of the model A of the class $[T]$ when $A \in E_{[T]}$.

Definition 23

Let K be a variety (quasivariety). $[T] \in JSp(JK)$. $OW_{[T]}(A) = \{B \in E_{[T]}$: there exist $A' \cong A, \ A' \subseteq B\}$ is called the outer world of the model A for $[T]$.

Semantic Jonsson varieties and quasivarieties

Definition 24

The world of the existentially closed model A is the following set

$$W_{[T]}(A) = IW_{[T]}(A) \cup OW_{[T]}(A).$$

Note that the above definitions can connect two different existentially closed models in the case of a convex theory. As the following theorem is true.

Fact 9

Let T be the perfect, strong convex Jonsson theory. Then for any models $A, B \in E_T$ the following is true:
1) $OW_T(A) \cap OW_T(B) \neq \emptyset$,
2) $IW_T(A) \cap IW_T(B) \neq \emptyset$.

Semantic Jonsson varieties and quasivarieties

The following question from [6] it well-known. We can formulate is as follows:

"For what varieties V of algebras with AP is it the case that every algebra A which is algebraically closed (in the sense of groups) is existentially closed? We would like to obtain a complete characterization of such varieties."

We found the sufficient conditions to above question in the frame of the study of Jonsson varieties.

Semantic Jonsson varieties and quasivarieties

Let K be a variety.

Definition 25 ([5])

The class $[T] \in JSp(K)/_{\bowtie}$ *is called* *elementarily closed*, *if* $\forall [T]' \in JSp(K)/_{\bowtie}$: $[T]' \neq [T] \Rightarrow E_{[T]} \cap E_{[T]'} = \emptyset$.

Definition 26

The class $[T] \in JSp(K)/_{\bowtie}$ *is called* *companion-convex*, *if the theory* $\nabla = Th_{\forall\exists}(\bigcap_{\Delta \in [T]} E_{\Delta})$ *is a Jonsson theory and has a model companion.*

Semantic Jonsson varieties and quasivarieties

Theorem 7

Let K be a variety, $JSp(JK)$ be a Jonsson spectrum of the semantic Jonsson variety JK, then if K is a perfect class ($(JK)^0 = (JK)^*$), for any class $[T] \in JSp(JK)$ such that $[T]$ satisfies Definitions 25 and 26, its semantic model, which is algebraically closed, also belongs to $E_{[T]}$.

Abelian groups

Let $[T] \in JSp(JK)$. JK is a semantic Jonsson variety of abelian groups, then $C_{[T]} \in E_{[T]}$ its semantic model of the center $[T]^*$, and $C_{[T]}$ be a divisible group and its standard Shmelev group is representable in the form $\oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)} \oplus \mathbb{Q}^{(\beta)}$, where $\alpha_p, \beta \in \omega^+$, $2^\omega = |C_{[T]}|$.

Definition 27 ([5])

The pair $(\alpha_p, \beta)^A_{C_{[T]}}$ is called the Jonsson invariant of the abelian group A if the standard group of the Shmelian group A is representable as $\oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)} \oplus \mathbb{Q}^{(\beta)}$, where $C_{[T]}$ is semantic model of $[T] \in JSpV(A)/_{\bowtie}$.

Abelian groups

Let K be a variety of abelian groups, JK be its class of semantic Jonsson variety. We define the following set $\{(\alpha_p, \beta)^K_{C_{[T]}} : [T] \in JSp(JK)/_{\bowtie}$, for all prime $p\}$ as the Jonsson invariant of the factor-set of the semantic Jonsson variety $JSpV(JK)/_{\bowtie}$ and denote it by $JInv(JSp(JK)/_{\bowtie})$.

Theorem 8 ([5])

Let K_1 and K_2 be an arbitrary classes of Jonsson variety of abelian groups, then the following conditions are equivalent:

1) $JK_1 \underset{JSp(K)}{\bowtie} JK_2$;

2) $JInv(JSp(JK_1)/_{\bowtie}) = JInv(JSp(JK_2)/_{\bowtie})$.

Holographicness in Jonsson theories

Definition 28 ([7])

The structure $\mathfrak{M}$ of the predicate signature σ of finite height is called holographic if there exists a finite set $S \subseteq \mathfrak{M}$ such that for any set $A \subseteq \mathfrak{M}$ of power no more than $||\sigma||$ there exists $\varphi \in Aut\mathfrak{M}$ with the property $\varphi(A) \subseteq S$.

The set S is called the set of prototypes for the structure $\mathfrak{M}$, where $||\sigma||$ denotes the height of the signature - the maximum number of arguments of symbols of the signature σ.

$Aut\mathfrak{M}$ is the group of all automorphisms of the structure $\mathfrak{M}$.

Definition 29 ([7])

A group G acts almost n-tuple transitively on a set M if, under its action, the number of orbits of n composed of pairwise distinct elements of the set M is finite.

Holographic Jonsson theory

Theorem 9 ([7])

An arbitrary structure of a finite predicate signature σ is holographic if and only if the group of all its automorphisms acts on it almost $||\sigma||$-tuple transitively.

Corollary 1 ([7])

Any countably categorical structure of a finite predicate signature is holographic.

Theorem 10 ([7])

There is a countable holographic structure that is not countably categorical.

Holographic Jonsson theory

Theorem 11 ([7])

An arbitrary linear order is holographic if and only if it is almost 2-tuple transitive.

Theorem 12

T is ω –categorical ($\forall\exists$ – complete) theory if and only if the center of this theory $T^* = Th(C_T)$ is also ω – categorical theory.

Theorem 13

Let T be κ-categorical, $\forall\exists$-complete Jonsson theory, where $\kappa \geq \omega$, then T is perfect.

Theorem 14

If T is a perfect, $\forall\exists$ complete, Jonsson theory, then $Hol_T \neq \varnothing$, i.e. such a theory has a holographic model.

Holographic Jonsson theory

Definition 30

A Jonsson theory T is called the holographic if $S_n^J(T)$ is finite, where $n = h(\sigma)$ and $S_n^J(T)$ is the set of all complete $\forall\exists$ types of n free variables.

Definition 31

A model A of a holographic Jonsson theory T is holographic if
1. *$Th_{\forall\exists}(A)$ is a Jonsson theory,*
2. *$Th_{\forall\exists}(A)$ is holographic.*

Holographic Jonsson theory

Fact 10

If the Jonsson theory T is holographic, then $S_m^J(T)$ is finite for any $m < n$, where $n = h(\sigma)$.

Fact 11

Let T be a perfect $\forall\exists$ complete Jonsson theory, then the following conditions are equivalent.
1. T is a holographic theory,
2. its center $T^* = Th(C_T)$ is a holographic theory,
3. The number of orbits of the action of the automorphism group $Aut(C_T)$ on the set $C_T^{h(\sigma)}$ is finite.

Holographic Jonsson theory

Definition 32

Let K be the class of structures of some fixed signature. Then this class is called the Jonsson-holographic class if $Th_{\forall\exists}(K)$ is a Jonsson holographic theory.

Example 12

Let $T = Th_{\forall\exists}(K)$ be the theory of a class of finite cyclic groups. It is easy to see that T is categorical in all finite powers, but not countably categorical.
Thus, taking into account that the holographicity of Abelian groups is related to the finiteness due to the results of [6], all models of this theory are holographic.

Holographic Jonsson theory

In this manner, there arises a problem of describing the Jonsson spectra of various semantic Jonsson varieties of holographic models in a given signature.

Let K be a class holographic structures of a signature σ. Then there appears a new area of researches linking with different types of Jonsson spectra for such classes in the previous above spoken manner.

Thank you for your attention!