Renormalization Theory, I

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Lyon, September 2008

Quantum Field Theory

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Remark/statement/suggestion: quantum field theory has a soul which is renormalization.

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Many approaches currently in competition: string theory, loop gravity, noncommutative geometry...

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NCQFT is a step towards understanding what is fundamental in QFT.

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- In constructive field theory (cluster expansions, Mayer expansions, Brydges-Kennedy-Abdesselam-R. forest formulas...)
- In NCQFT they are also there but everything is different in a subtle way.

Quantum Field Theories as weighted species

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- Constructive field theory adresses the second problem. Species of Graphs $\rightarrow$ Species of Trees.

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Multiscale constructive theory tries to combine the two previous steps in a consistent way, but...

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This phenomenon always occur (either at the "infrared" or at the "ultraviolet" end of the renormalization group) in field theory on ordinary four dimensional space time (except possibly for extremely special models). This is somewhat frustrating.

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- Z is the normalization, so that this measure is a probability measure;
- $D\phi$ is a formal product $\prod_{x \in \mathbb{R}^d} d\phi(x)$ of Lebesgue measures at each point of $\mathbb{R}^4$.

The ordinary ϕ^4_4 propagator

An infinite product of Lebesgue measures is ill-defined. So it is better to define first the Gaussian part of the measure

$$d\mu(\phi) = \frac{1}{Z_0} e^{-(m^2/2)\int \phi^2 - (a/2)\int (\partial_\mu \phi \partial^\mu \phi)} D\phi. \tag{2.2}$$

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where we recognize the heat kernel.

Feynman Rules

The full interacting measure may now be written as the multiplication of the Gaussian measure $d\mu(\phi)$ by the interaction factor:

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Expanding the exponential as a formal power series in the coupling constant λ we get perturbative field theory:

$$S_N(z_1, ..., z_N) = \frac{1}{Z}\sum_{n=0}^{\infty} \frac{(-\lambda)^n}{n!} \int \Big[\int \frac{\phi^4(x)dx}{4!}\Big]^n \phi(z_1)...\phi(z_N) d\mu(\phi) \tag{2.6}$$

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By Wick theorem, S_N is a sum over "Wick contractions schemes", i.e. ways of pairing together $4n + N$ fields into $2n + N/2$ pairs. There are exactly $(4n + N - 1)(4n + N - 3)...5.3.1 = (4n + N)!!$ such contraction schemes.

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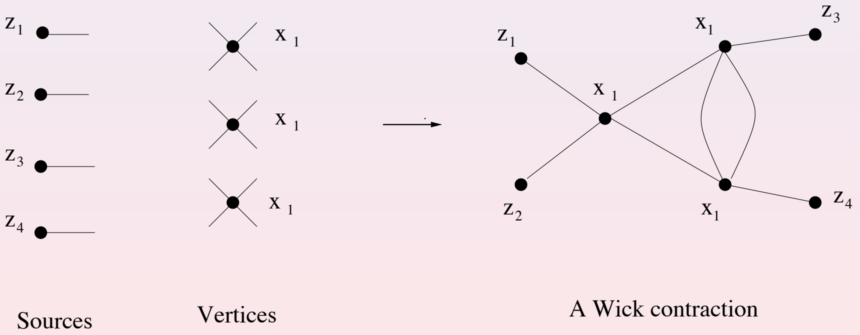

Figure: A contraction scheme

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Feynman amplitudes are functions (in fact distributions) of the external positions $z_1, ..., z_N$. They may diverge either because of integration over all of $\mathbb{R}^4$ or because of the singularity in the propagator $C(x, y)$ at $x = y$.

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The first two elements are quite universal. The third depends on the details of the model.

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At fixed scale attribution, some subgraphs play an essential role. They are the connected subgraphs whose internal lines all have higher scale index than all the external lines of the subgraph. Let's call them the "high" subgraphs. They form a forest for the inclusion relation.

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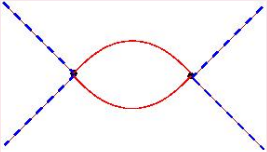

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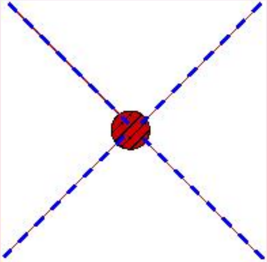

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In four dimension by the previous estimates of a single scale propagator C, power counting delivers a factor M^{2i} per line and M^{-4i} per vertex integration $\int d^4x$. There are $n-1$ "internal" integrations to perform to compare a high connected subgraph to a local vertex. For a connected ϕ^4 graph, the net factor is $2l(G) - 4(n(G)-1) = 4 - N(G)$ (because $4n = 2l + N$). When this factor is strictly negative, the sum is geometrically convergent, otherwise it diverges.

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For instance the previous graph diverges (logarithmically) because there are two line factors M^{2i} and a single internal integration M^{-4i}.

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Such models are called (perturbatively) renormalizable. But...

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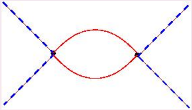

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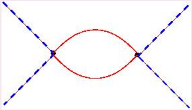

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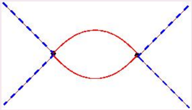

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- 'tHooft and Veltmann succeeded to show that these theories are still renormalisable. They used a new technical tool calleddimensional renormalization.

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- *Happy end*, all the people in red in this page got the Nobel prize...