Maths IV, Analyse (Printemps 2011) - Fiche 6.

Exercice 3.1.

f: R^p → R

1. g,k | f(a+h) = f(a) +df(a).h + ½ d²f(a).h² + o(||h||²)

2. $g(x,y) = g(2,2) + h\partial g\partial x(2,2) + k\partial g\partial y(2,2) + ½ (h^2\partial^2 g\partial x^2(2,2) + hk\partial g\partial x\partial y(2,2) + k^2\partial^2 g\partial y^2(2,2))$

   + o(||(h,k)||²)
   étudier RT – S².

   $R = \partial^2 g\partial x^2(2,2)$ $S = \partial^2 g\partial x\partial y(2,2)$ $T = \partial^2 g\partial y^2(2,2)$

   excosx-y

   R =

(th)d2fa.h

Symétrie de la matrice Hessienne dépend du théorème de Schwarz.

g: R² → R

(x,y) → excos(x-y)

a=(2,2) € D_g

g est de classe C∞ sur son domaine..

$\partial g\partial x = ex\ cosx\text{-}y+sinx\text{-}ycos2x\text{-}y$

$\partial g\partial y = \text{-}ex\ sinx\text{-}ycos2x\text{-}y$

$\partial^2 g\partial x^2 = ex\{\text{-}sinx\text{-}ycosx\text{-}ycos2x\text{-}y+2cosx\text{-}ysinx\text{-}ycosx\text{-}y+sinx\text{-}ycos4x\text{-}y\}$

$\partial 2g\partial x22,2= e21+1=2e2$

$\partial^2 g\partial y^2 = \text{-}ex\{\text{-}cos3x\text{-}y\text{-}2cosx\text{-}ysin^2x\text{-}ycos4x\text{-}y\}$

$\partial^2 g\partial y^2\ (2,2) = e2$

$\partial 2g\partial x\partial y= \text{-}exsinx\text{-}y+excosx\text{-}ycos2x\text{-}y+2cosx\text{-}ysinx\text{-}yexsinx\text{-}ycos4x\text{-}y$

En (2,2), $\partial 2g\partial x\partial y=e2$

f((2,2)+(h,k)) = f(2,2) + (∂g∂x∂g∂y) hk+ 12 (h,k)$\partial^2 g \partial x^2$ $\partial^2 g \partial x \partial y \partial^2 g \partial x \partial y \partial^2 g \partial y^2$ hk

f(2+h,2+k)) = e2 + (e20) hk+ 12 2e2h^2-2e2hk+$e^2 k^2$ +o(||(h,k)||²)

f(2+hu
v
a+h

,2+ku
v
a+h

) = e2+ e2h+e2h2-e2hk+e2k22 + o(||(h,k)||²)

h = u – 2

- ⇨ f(u,v) = $e^2 + e^2(u-2) + e^2(u-2)^2 - e^2(u-2)(v-2)$ +e2v-22 + o(||(h,k)||²)

k = u +2

Exercice 3.2.

f :	R	→	R	$C^2(R)$
F :	R^2	→	R	

(x,y) → fx-f(y)x-y si $x \neq y$

f'(x) si x = y

On fixe (a,a)

∂F∂x, ∂F∂y en (a,a) ?

∂f∂x (a,a) = limh→0Fa+h,a- F(a,a)h=

limh→0fa+h-f(a)h- f'(a)h =

limh→0fa+h-fa-hf'(a) h^2 = (*)

f(a+h) – f(a) = f’(a)h + f’’ch22!

c entre a et a+h.

Or f est de classe C^2 sur R. Donc, f ''(a+h) tend vers f''(a) quand h tend vers 0. En particulier, f''(cu
v
a+h
) → f''(a).

(*) = limh→0f''ch²2h²=f''a2