Models and Groups İstanbul 2

27-29 march 2014

Time	Thursday	Friday	Saturday
9.45-11.00	Wagner	Wagner	Wagner
11.15-12.30	Sklinos	Sklinos	Sklinos
Lunch			
15.00-16.15	Korkmaz	Perin	Deloro
16.30-17.45	Poizat	Ersoy	Borovik
Contributed talks			
18.00-18.20	Dobrowolski	Byron	Druart
18.30-19.00	Hofmann	Jagiella	

Talks

Alexandre Borovik
Black box algebra and black box model theory

Abstract. Some natural problems in computational algebra and cryptography require analysis of finite algebraic structures (for example, groups, rings, fields, projective planes, Lie algebras) up to probabilistic polynomial time interpretability/bi-interpretability and open a fascinating new chapter of model theory. The talk will outline a few basic concepts of this new theory.

This is a joint work with Şükrü Yalçınkaya.

Adrien Deloro
Representations of finite Morley rank

Abstract. We know quite a lot about "abstract" groups of finite Morley rank. As suggested by Borovik and Cherlin, it may be time to apply the knowledge gained in the classification project and convert the methods into results on "concrete" groups, ie. permutation groups. The talk will not consider the general setting of group actions of finite Morley rank but the more specific aspect of representations, that is of groups acting on abelian groups - definably in some ranked universe. We shall review known facts and basic questions.

Kıvanç Ersoy
Locally finite groups with certain conditions centralizers

Abstract. A group G is called locally finite if every finitely generated subgroup of G is finite. Brauer-Fowler Theorem indicates that there are at most finitely many finite simple groups with a given centralizer of an involution. After Brauer-Fowler, centralizers played an important role in the proof of the classification of finite simple groups, hence it is natural to think that structures of centralizers in finite and locally finite groups give a lot of information about the structure of the group. In this talk we will give a survey of structural results obtained via imposing conditions on centralizers and fixed points of automorphisms in locally finite groups. Moreover, we will prove some new results, one of which is the following:

Theorem 1 (E.-Gupta, [1]) *Let G be an infinite simple locally finite group with an automorphism α such that $C_G(\alpha)$ has finite rank. Then, G is isomorphic to one of the following groups:*

1. $G \cong PSL(l+1,k)$ *or* $PSU(l+1,k)$ *for some infinite locally finite field k of characteristic $q \neq p$ and $p > l$*
2. *G has type $B_l(k), C_l(k)$ or ${}^2B_2$ (that is $l = 2$) over an infinite locally finite field k of characteristic $q \neq p$ (and $q = 2$ in the case of ${}^2B_2(k)$) and $p > 2l - 1$.*
3. $G \cong D_l(k)$ *or* ${}^2D_l(k)$ *or* ${}^3D_4(k)$ *for some infinite locally finite field k of characteristic $q \neq p$ and $p > 2l - 3$*
4. $G \cong E_6(k)$ *or* ${}^2E_6(k)$ *over an infinite locally finite field of characteristic $q \neq p$, and $p > 11$.*
5. $G \cong E_7(k), F_4(k)$ *or* ${}^2F_4(k)$ *over an infinite locally finite field of characteristic $q \neq p$, and $p > 17$.*
6. $G \cong E_8(k)$ *over an infinite locally finite field of characteristic $q \neq p$, and $p > 29$.*
7. $G \cong G_2(k)$ *or* ${}^2G_2(k)$ *over an infinite locally finite field of characteristic $q \neq p$, and $p > 5$.*

References

[1] K. Ersoy, C.K. Gupta “Locally finite groups with centralizers of finite rank”, in preparation.

Mustafa Korkmaz
Commutator lengths in mapping class groups

Abstract. For an element x in the commutator subgroup of a group, the commutator length of x is defined to be the minimal number of commutators needed to express x as a product of commutators. The mapping class group of a closed oriented surface is the group of isotopy classes of orientation–preserving diffeomorphisms of the surface. The commutator lengths of elements in the mapping class group is of interest in low dimensional topology.

In this talk I will discuss some of the known results on commutator lengths, stable commutator lengths in free groups and in mapping class groups. A closedly related notion bounded cohomology groups will also be discussed.

Chloé Perin

Forking in the free group

Abstract. Sela showed that the theory of the non abelian free groups is stable. In a joint work with Sklinos, we give some characterization of the forking independence relation between elements of the free group F over a set of parameters A in terms of the Grushko and cyclic JSJ decomposition of F relative to A. The cyclic JSJ decomposition of F relative to A is a geometric group theory tool that encodes all the splittings of F as an amalgamated product (or HNN extension) over cyclic subgroups in which A lies in one of the factors.

Bruno Poizat

The filter of supergenerics

Abstract. The talk will outline an article to be published in the Journal of Algebra. It is about a filter of large sets closed with respect to translation, which is defined in any group and which seems not to have been noticed until now.

A subset of a group is said to be *generic* if a finite number of its translates covers the entire group; it is said to be *supergeneric* if the intersection of any finite family of its translates is generic. The intersection of of two supergeneric sets is still supergeneric.

We will analyze the general elementary properties of supergeneric sets as well as the very particular uniformity properties possessed by *definable* supergeneric sets in a *stable* group.

Tutorials

Rizos Sklinos

Diophantine Geometry over Free Groups

Abstract. This tutorial concerns the first-order theories of non-Abelian free groups. The subject has been given much attention after Sela and Kharlampovich-Myasnikov independently proved that the first-order theory of non-Abelian free groups (i.e. the axioms that live in the intersection of the above mentioned first-order theories) is complete. This answers in the affirmative a long standing question that was posed around 1946 by Tarski:

Question 1 (Tarski, 1946) *Do non-Abelian free groups share the same common first-order theory?*

The main purpose of this tutorial is to analyze the notions and techniques that appear in the first steps of Sela's solution to Tarski's problem. Let us mention that the proof culminates in a series of papers [6],[7],[9], [8],[10],[11] and [12], that have not been totally absorbed by the mathematical community, despite the fact that they were available since 2001. In brief the proof splits in two parts: first Sela proves that the $\forall\exists$ first-order theories of any two non-Abelian free groups coincide, and then he proves that each first order theory eliminates quantifiers down to boolean combinations of $\forall\exists$ first-order formulas. His methods are purely geometric and a heavy use of the theory of group actions on real trees is made throughout his papers.

Our goal for this tutorial will be to give the ideas around the proof of the following intermediate result to Tarski's problem:

Theorem 1 *Let $m, n > 2$. Then $Th_{\forall\exists}(\mathbb{F}_n) = Th_{\forall\exists}(\mathbb{F}_m)$*

Note that although this theorem has been first claimed in [5], a complete proof appeared much later (in Sela's work).

The tutorial will be structured as follows: We will first define limit groups using the Bestvina-Paulin method (see [1],[4]) and record how one can describe the solution set (in a free group) of a system of equations using them. Limit groups play an important role in all steps of Sela's solution and we will see that one naturally sees them as objects of geometry rather than algebra.

We will then move to the technique of "formal solutions". This technique lies behind the main idea of the proof of Sela (and every other existing proof). The prototypical theorem being:

Theorem 2 (Merzlyakov [3]) *Let $\Sigma(\bar{x}, \bar{y})$ be a finite set of words in $\langle \bar{x}, \bar{y} \rangle$. Let $\mathbb{F}$ be a non abelian free group. Suppose $\mathbb{F} \models \forall\bar{x}\exists\bar{y}(\Sigma(\bar{x}, \bar{y}) = 1)$. Then there exists a retract $r : G_\Sigma \twoheadrightarrow \langle \bar{x} \rangle$, where $G_\Sigma := \langle \bar{x}, \bar{y} \mid \Sigma(\bar{x}, \bar{y}) \rangle$.*

We note that Merzlyakov used this theorem in order to prove that the positive first-order theories of non-Abelian free groups coincide. Let us briefly justify the term "formal solutions": the image of $\bar{y}$ under r of the previous theorem is a tuple of words in $\bar{x}$, say $\bar{w}(\bar{x})$, and it can be easily checked that $\mathbb{F} \models \forall\bar{x}\Sigma(\bar{x}, \bar{w}(\bar{x})) = 1$. Thus, the retraction can be thought of as a formal (uniform) way of assigning to each $\bar{a}$ in $\mathbb{F}$, a $\bar{b}$ in $\mathbb{F}$ (i.e. substituting $\bar{x}$ in $\bar{w}(\bar{x})$ by $\bar{a}$), that witnesses the truthfulness of Σ.

Geometry suggests some natural generalizations of the above theorem and this will lead us to the definitions of "towers" and "test sequences" on them. Our feeling is that these notions will be central to the understanding of the class of definable sets in non-Abelian free groups and thus we will try to build some intuition around them.

As noted above, Merzlyakov's theorem lies behind the main idea of all existing proofs to the Tarski's problem. Generalizing it to the case where the universal variables are bounded by a system of equations is a hard task and depends on the geometric structure of the system of equations. Unfortunately, the generalization of Merzlyakov's theorem to an arbitrary variety, that bounds the universal variables, is not possible. We have to restrict ourselves to varieties that their corresponding group has a certain structure. In particular, if a group $G_R := \langle \bar{x} \mid R(\bar{x})\rangle$ has the structure of a "tower", then the following statement (up to some tuning) is true:

Statement 1 *Let $\Sigma(\bar{x},\bar{y})$ be a finite set of words in $\langle \bar{x},\bar{y}\rangle$. Let $\mathbb{F}$ be a non abelian free group. Suppose $\mathbb{F} \models \forall\bar{x}(R(\bar{x})=1 \to \exists\bar{y}(\Sigma(\bar{x},\bar{y})=1))$. Then there exists a retract $r : G_\Sigma \twoheadrightarrow G_R$, where $G_\Sigma := \langle \bar{x},\bar{y} \mid \Sigma(\bar{x},\bar{y})\rangle$.*

Finally, the addition of inequalities to the sentences above, i.e. sentences of the form $\forall\bar{x}\exists\bar{y}(\Sigma(\bar{x},\bar{y})\wedge \Psi(\bar{x},\bar{y})\neq 1)$ require new machinery and ideas in order to be shown that their truthfulness does not depend on a particular non-Abelian free group. This machinery includes the generalization of Merzlyakov's theorem as stated above, but also requires the development of more delicate tools. We will finish this tutorial by giving the extra ideas needed for completing the proof of Theorem 1 .

Our exposition will be based on the following papers of the bibliography below: [6],[7],[8] and [2].

References

[1] M. Bestvina. Degenerations of the hyperbolic space. *Duke Math. J.*, 56:143–161, 1988.

[2] M. Bestvina. $\mathbb{R}$-trees in topology, geometry and group theory. In R.J. Daverman and R.B. Sher, editors, *Handbook of geometric topology.* North-Holland, 2001.

[3] Yu. I. Merzlyakov. Positive formulae on free groups. *Algebra i Logika*, 5:257–266, 1966.

[4] F. Paulin. Topologie de Gromov équivariante, structures hyperboliques et arbres réels. *Invent. Math.*, 94:53–80, 1988.

[5] G. Sacerdote. Elementary properties of free groups. *Trans. Amer. Math. Soc.*, 178, 1973.

[6] Z. Sela. Diophantine geometry over groups I. Makanin-Razborov diagrams. *Publ. Math. Inst. Hautes études Sci.*, 93:31–105, 2001.

[7] Z. Sela. Diophantine geometry over groups II. Completions, closures and formal solutions. *Israel J. Math.*, 134:173–254, 2003.

[8] Z. Sela. Diophantine geometry over groups IV. An iterative procedure for the validation of sentence. *Israel J. Math.*, 143:1–130, 2004.

[9] Z. Sela. Diophantine geometry over groups III. Rigid and solid solutions. *Israel J. Math.*, 147:1–73, 2005.

[10] Z. Sela. Diophantine geometry over groups V_1. Quantifier elimination I. *Israel J. Math.*, 150:1–197, 2005.

[11] Z. Sela. Diophantine geometry over groups V_2. Quantifier elimination II. *Geom. Funct. Anal.*, 16:537–706, 2006.

[12] Z. Sela. Diophantine geometry over groups VI: The elementary theory of free groups. *Geom. Funct. Anal.*, 16:707–730, 2006.

Frank Wagner
Approximate subgroups and model theory

Abstract. A subset A of a group (or a local group) G is a K-approximate subgroup, for some integer K, if $A.A$ is covered by K cosets of A. Finite approximate subgroups have recently been classified by Breuillard, Green and Tao, building on earlier work of Hrushovski as well as Goldbrings solution of Hilbert's fifth problem for local groups; they are essentially analogs of arithmetic progressions in a nilpotent group, modulo some actual subgroup. I shall try to give an overview of the proof from a model-theoretic point of view, as well as pose some questions concerning model-theoretic generalisations.

References

[1] E. Breuillard, B. Green and T. Tao. The structure of approximate groups. *Publ. Math. IHES*, 116:115–221, 2012.

[2] I. Goldbring. Hilbert's fifth problem for local groups. *Ann. of Math.*, 172:1269–1314, 2010.

[3] E Hrushovski. Stable group theory and approximate subgroups. *J. Amer. Math. Soc.*, 25:189–243, 2012.

[4] L. van den Dries. Approximate groups [after Hrushovski, and Breuillard, Green, Tao]. *Séminaire Bourbaki*, 1077, 2013.

Contributed Talks

Ayala Byron
Homogeneity in torsion-free hyperbolic groups - a geometric point of view

Abstract. Why do we like (torsion-free) hyperbolic groups? They are finitely presented, so they come with a JSJ-decomposition, which allows us to see all their tower structures. We'll explain these notions and see how those tower structures let us tell which group is homogeneous. This is work in progress with Chloé Perin.

Jan Dobrowolski
Locally finite profinite rings

Abstract. The aim of this talk is to present some results from [2].

First, I will discuss some connections between certain conjectures concerning small profinite (in the sense of Newelski) groups and the ones concerning small profinite rings. These connections are among the motivations for our interest in profinite rings.

Next, I will focus on two theorems from [2]. First of them states that the Jacobson radical of a locally finite profinite rings is nil of finite nilexponent. This is a generalization of a result from [1], which gives the same conclusion for small profinite rings. Our second theorem is a complete classification of semisimple locally finite profinite rings. It applies, in particular, to the context of small compact G-rings, yielding also a classification of semisimple small compact G-rings.

References

[1] K. Krupiński, F. Wagner, *Small profinite groups and rings*, ***Journal of Algebra*** (306), 494-506, 2006.

[2] J. Dobrowolski, K. Krupiński, *Locally finite profinite rings*, ***Journal of Algebra***, DOI: 10.1016/j.jalgebra.2013.11.020

Benjamin Druart
Definable subgroups in $\mathrm{SL}_2(\mathbb{Q}_p)$

Abstract. A Cartan subgroup of an abstract group G is a maximal nilpotent subgroup H such that every normal subgroup of finite index in H has finite index in its normalizer in G. This is a notion rooted in the theory of algebraic groups. In this talk, we will describe all Cartan subgroup in $\mathrm{SL}_2(\mathbb{Q}_p)$ and we wil make connections with notion of genericity. This description allows us to discuss some algebraic and model theoretic properties of definable subgroups in $\mathrm{SL}_2(\mathbb{Q}_p)$.

Daniel Hoffmann

Iterative derivations and group schemes

Abstract. Let k be a field of characteristic $p > 0$. Recall that a Hasse-Schmidt derivation is a k-algebra homomorphism

$$\mathbb{D} : A \to A[\![X]\!],$$
$$\mathbb{D}(a) = \sum_{i=0} D_i(a)X^i.$$

It is additively iterative if $D_i \circ D_j = \binom{i+j}{i} D_{i+j}$. Such a composition law is induced by an algebraic group, in this case by the additive group. Another possibility is the multiplicative group. Both are nice for a model-theoretic approach, because they are strongly integrable in the sense of [1]. Thus one can consider a geometric axiomatisation of existentially closed Hasse-Schmidt fields with one additively/multiplicatively iterative derivation as was done in [2] for the additive iterativity condition.

It is natural to ask: are there more (not only one-dimensional) such nice algebraic groups? I will develop terminology of G-iterative derivations for higher dimensional algebraic groups and group schemes, then show examples of unipotent commutative groups, that are strongly integrable.

References

[1] Hideyuki Matsumura, *Integrable derivations*, ***Nagoya Math. J.***, Volume 87 (1982), 227-245.

[2] Piotr Kowalski, *Geometric axioms for existentially closed Hasse fields*, ***Annals of Pure and Applied Logic***, vol.135 (2005), no.1-3, pp.286-302.

Grzegorz Jagiella

Definable topological dynamics and real Lie groups

Abstract. Methods of topological dynamics have been introduced to model theory by Newelski in [3] and since then saw further development in that field by other authors. Given (over a model M) a definable group G, we consider the category of definable flows. This category has a universal object $S_G(M)$, the space of types in G over M. Its Ellis semigroup is $S_{G,ext}(M)$, the space of external types. It can be considered as a model-theoretic equivalent to βG, the large compactification of G.

In the talk I will describe the results from [2] that give description of definable topological dynamics of a large class of groups interpretable in an o-minimal expansion of the field of reals along with their universal covers interpreted in a certain two-sorted structure. The results provide a wide range of counterexamples to a question by Newelski whether the Ellis group of the universal definable G-flow is isomorphic to G/G^{00} and generalize methods from [1] that provided a particular counterexample.

References

[1] J. Gismatullin, D. Penazzi, A. Pillay, *Some model theory of $SL(2, R)$*, preprint

[2] G. Jagiella, *Definable topological dynamics and real Lie groups*, preprint

[3] L. Newelski, *Topological dynamics of definable group actions*, J. Symbolic Logic Volume 74, Issue 1 (2009), pp 50-72