

# Models and Groups, İstanbul 4

26-28 march 2015

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| Time                     | Thursday         | Friday    | Saturday  |
|--------------------------|------------------|-----------|-----------|
| 9.45-11.00               | Melleray         | Melleray  | Melleray  |
| 11.15-12.30              | Benli            | Benli     | Guirardel |
| <b>Lunch</b>             |                  |           |           |
| 15.00-16.15              | Sklinos          | Sela      | Ünlü      |
| 16.30-17.45              | Iwanow           | Atalan    | Ziegler   |
| <b>Contributed talks</b> |                  |           |           |
| 18.00-18.25              | Bradley-Williams | Ibarlucía |           |
| 18.30-18.55              |                  |           |           |

## Talks

### Ferihe Atalan

*Curve Complex, Mapping Class Group of a Nonorientable Surface and their Automorphisms*

**Abstract.** In this talk, I will first give the definitions of some basic concepts such as curves on a surface, the complex of curves, the mapping class group and  $Y$ -homeomorphisms. For a nonorientable surface of genus  $g$  with  $n$  punctures  $N$ , let  $\text{Mod}(N)$  and  $C(N)$  denote the mapping class group and the curve complex of the nonorientable surface  $N$ , respectively. Then, I will give an outline of the proof the fact that the automorphism group of the curve complex,  $\text{Aut}(C(N))$ , (joint with Mustafa Korkmaz) and also the automorphism group of the mapping class group (joint with Blazej Szepietowski) of  $N$ ,  $\text{Aut}(\text{Mod}(N))$ , are both isomorphic to the mapping class group of the surface  $N$ . If time permits, I will also talk about injective homomorphisms of mapping class groups of nonorientable surfaces.

### Vincent Guirardel

*Embedding of hyperbolic groups and extension of automorphisms*

**Abstract.** Given a torsion-free hyperbolic group  $G$  and a subgroup  $H$ , we study the group of automorphisms of  $H$  that extend to  $G$ . For instance, we prove that if sufficiently many automorphisms of  $H$  extend to  $G$ , we prove that  $H$  has to be embedded in a very particular way in  $G$ . In particular, if  $H$  is a free group, and if all automorphisms extend, then  $H$  has to be a free factor in  $G$ . I will try to emphasize some similarities and relations to problems in model theory. This is a joint work with Gilbert Levitt.

**Aleksander Iwanow**

*Metric groups in continuous logic*

**Abstract.** A **metric structure** is a complete metric space  $(M, d)$  with  $d$  bounded by 1, along with a family of uniformly continuous operations  $F_k$  on  $M$  and a family of predicates  $R_l$ , i.e. uniformly continuous maps from appropriate  $M^{k_l}$  to  $[0, 1]$ .

Continuous model theory provides metric structures with continuous formulas and continuous theories. Moreover a large part of classical model theory can be extended to the level of continuous logic. This gives a logical approach to some mathematical objects which were not available for logic before.

*How strong is this approach in the case of topological groups and their actions on topological spaces?* In particular is it possible to axiomatise familiar classes of groups and actions? In my talk I am going to consider several properties which are traditional in geometric group theory (for example involving actions on Hilbert spaces or real trees). I also study some model theoretic properties in the case of locally compact groups.

**Zlil Sela**

*Envelopes of definable sets and some of their applications*

**Abstract.** Although definable sets over a free or a hyperbolic group are understood and classified, they are not easy to work with when tackling natural model theoretic problems. For these purposes it is often much easier to study not the definable sets themselves but rather their envelopes.

An envelope of a definable set is a Diophantine set that contains the definable set, such that all the "generic points" of the Diophantine envelope are contained in the definable set.

We intend to explain the structure of the envelope, emphasizing its geometric structure, and briefly discussing some of the applications of this concept, that include the stability of free and hyperbolic groups, and the (geometric) elimination of imaginaries in these theories.

**Rizos Sklinos**

*Fields definable in the first order theory of the free group*

**Abstract.** Given a first order structure, one would like to know what groups are definable in it or even whether there is an infinite field definable in it. These questions played fundamental role in the development of model theory, and especially of its major subdiscipline, geometric stability theory.

Stable structures in the sense of model theory enjoy the presence of an abstract independence relation, also known as "forking independence". The algebraic independence in the field of complex numbers is an example of this, relevant also for this talk. This independence is intimately related to the abovementioned definability questions.

In a major work, non abelian free groups have been shown to be stable structures by Zlil Sela. Moreover, forking independence in free groups has a "field-like" behavior. Surprisingly, it is conjectured that, despite this behavior, an infinite field is not definable in a non abelian free group. In this talk I will present some progress in joint work with Ayala Byron towards a solution to this conjecture.

## Özgün Ünlü

*Free group actions on highly connected manifolds*

**Abstract.** In this talk we will discuss the problem of finding group theoretic conditions that characterizes the finite groups which can act freely on a given compact highly connected manifold without boundary. The study of this problem breaks up into two aspects: (1) Find group theoretic restrictions on finite groups that can act freely on the given manifold. (2) Construct explicit free actions of finite groups on the given manifold. I will give a quick overview of the first aspect of this topic. Then I will discuss some recently employed methods of constructing such actions.

## Martin Ziegler

*Model theory of right-angled buildings (Joint work with A. Baudisch and A. Martin-Pizarro)*

**Abstract.** A *right-angled Coxeter group*  $W(\Gamma)$  is generated by the vertices of a finite graph  $\Gamma$  with defining relations  $\gamma^2 = 1$  for all  $\gamma \in \Gamma$  and  $\gamma\delta = \delta\gamma$  if there is no edge between  $\gamma$  and  $\delta$ . For each right-angled Coxeter group there is a unique countable Tits building  $B(\Gamma)$  with infinite residues. Using a suitable language, we study the first order theory of  $B(\Gamma)$ . It has a nice axiomatization, is omega-stable, equational and has trivial forking. The theory is  $n$ -ample, if  $\Gamma$  contains two vertices of distance  $n$ , and not  $n$ -ample, if  $|\Gamma| \leq n$ .

This construction generalises the free 2-pseudospace defined by Baudisch-Pillay (2000) and the higher-dimensional free pseudospaces defined by Tent(2014) and Baudisch-Pizarro-Ziegler(2014).

# Tutorials

**Mustafa Gökhan Benli**

*Space of finitely generated groups*

**Abstract.** Let  $G$  be a group and  $S$  an ordered subset of size  $k$  which generates  $G$ . The pair  $(G, S)$  is called a *marked group*. The space  $\mathcal{M}_k$  of marked groups (up to a natural equivalence) can be given a metrizable compact topology in which two pairs are close if the corresponding Cayley Graphs are isomorphic in a big radius around the identity. Elementary observations show that local properties of a given group are independent of the generating set  $S$ . This allows one to ask several questions in this space, such as: What does the neighbourhood of a given group look like? Which groups are isolated in this space? Which groups belong to the condensation part of this space? What is the Cantor-Bendixson rank of this space? What group properties are open/closed in this space? What are some interesting compact subsets of this space? etc. My tutorial will be about this space and its properties together with open problems related to it.

Also, I want to talk about neighbourhoods and generic properties of some groups of automorphisms of regular rooted trees. These are groups which are interesting because they have unusual properties related to various notions such as growth and amenability. Specifically I want to talk about neighbourhoods of contracting self-similar groups and groups of intermediate growth constructed by R. Grigorchuk.

## References

- [BCGS14] Bieri, Robert and Cornulier, Yves and Guyot, Luc and Strebel, Ralph. Infinite presentability of groups and condensation. *J. Inst. Math. Jussieu*, 13(4):811–848, 2014.
- [BGdlH13] Benli, Mustafa Gökhan and Grigorchuk, Rostislav and de la Harpe, Pierre. Amenable groups without finitely presented amenable covers. *Bull. Math. Sci.*, 3(1):73–131, 2013.
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- [CG05] Champetier, Christophe and Guirardel, Vincent. Limit groups as limits of free groups. *Israel J. Math.*, 146:1–75, 2005.
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- [Cor11] Cornulier, Yves. On the Cantor-Bendixson rank of metabelian groups. *Ann. Inst. Fourier (Grenoble)*, 61(2):593–618, 2011.
- [dCGP07] de Cornulier, Yves and Guyot, Luc and Pitsch, Wolfgang. On the isolated points in the space of groups. *J. Algebra*, 307(1):254–277, 2007.

**Julien Melleray**

*Automorphism groups of metric structures*

**Abstract.** Following the talks by D. Evans during the previous meeting of this series (no familiarity with the contents of these talks will be assumed), we will present some of the basics of the theory of Polish groups, as well as examples of such groups and motivations for their study. For a model theorist, the most natural examples are probably the automorphism groups of countable structures, and any nonarchimedean Polish group is of this form. There are many other Polish groups, though; we will explain why these groups may be considered as automorphism groups of *metric structures*, and one of the objectives of my talks will be to compare and contrast the theory of automorphism groups of countable (classical) structure vs. the theory of automorphism groups of separable (metric) structures.

Topics I hope to cover, time permitting: basics of descriptive set theory as it pertains to the theory of Polish groups (Baire category notions and techniques); how to view Polish groups as automorphism groups; metric Fraïssé theory; some topological dynamics, including extreme amenability and its link to the Ramsey property (and a continuous analogue of it) and universal minimal flows of Roelcke-precompact Polish groups; properties of the space of actions of a countable group on a Polish metric structure, and what it can be used for.

## Contributed Talks

**David Bradley-Williams**

*Reducts of Trees (via Jordan groups)*

**Abstract.** In this talk, my aim was to discuss a recent result appearing in my thesis [1]. It is a classification, up to interdefinability, of the reducts of certain tree-like structures. To be more precise, a *semilinear ordering* is a partially ordered set  $(\Omega, <)$  such that every pair of elements is bounded below, and for any  $a$  in  $\Omega$ , the set of lower bounds of  $a$  is linearly ordered by  $<$ . For the purpose of this talk, the set  $\Omega$  will be countably infinite. In a semilinear ordering  $(\Omega, <)$ , for any  $x, z$  in  $\Omega$ , there is a natural notion of the interval between  $x$  and  $z$ , denoted by  $[x, z] \subseteq \Omega$ . In  $(\Omega, <)$  we define the *betweenness relation*  $B(x, y, z)$  which holds whenever  $y \in [x, z]$ . A semilinear ordering  $S = (\Omega, <)$  for which any partial  $<$ -isomorphism  $p : A \rightarrow B$  between 2-element subsets  $A, B \subset \Omega$  extends to an automorphism  $g \in \text{Aut}(S)$  is called a *relatively 2-transitive* semilinear ordering. The countably infinite, relatively 2-transitive semilinear orderings are all  $\omega$ -categorical and were classified by Droste in [3], there are countably infinitely many up to isomorphism. My result is that if  $S = (\Omega, <)$  is a countably infinite, relatively 2-transitive semilinear order which is not a linear order then  $(\Omega, B)$  is the unique non-trivial, proper reduct of  $S$  up to interdefinability. The proof (which we did not discuss) uses results on infinite primitive Jordan groups with primitive Jordan sets by Adeleke and Neumann. This work was inspired by the joint work [2] with Bodirsky, Pinsker and Pongrácz in which we use Ramsey theoretic methods to study the reducts of one such semilinear order  $S_2$  in more detail, which also yields the classification of reducts of  $S_2$  up to interdefinability.

## References

- [1] D. Bradley-Williams, *Jordan groups and homogeneous structures*, PhD Thesis, Leeds.
- [2] M. Bodirsky, D. Bradley-Williams, M. Pinsker and A. Pongrácz, *The universal binary branching tree*, preprint, arXiv:1409.2170 .
- [3] M. Droste, *Structure of partially ordered sets with transitive automorphism groups*, Mem. Amer. Math. Soc. **334** (1985).

**Tomás Ibarlucía**

*$\aleph_0$ -categorical models and Roelcke precompact groups*

**Abstract.** We will discuss how the model theory of metric structures meets the theory of Banach representations of dynamical systems. The latter has been developed by Glasner and Megrelishvili in recent years to classify the actions of topological groups.

The main connection with model theory comes from the study of Roelcke precompact Polish groups, which Ben Yaacov and Tsankov have shown to be precisely the automorphism groups of  $\aleph_0$ -categorical metric structures. A rich parallelism can be traced between the topological classification and several model-theoretic notions, with applications to both sides.