Models and Groups, İstanbul 5

8-10 october 2015

Thursday		Friday		Saturday	
		9.00-10.15	Macpherson		
9.45-11.00	Macpherson	10.35-11.50	Kuzucuoğlu	9.45-11.00	Macpherson
11.15-12.30	Kuzucuoğlu	12.00-13.15	Güçlükan İlhan	11.15-12.30	Kuzucuoğlu
Lunch					
15.00-16.15	Yalçınkaya	15.00-16.15	Levitt	15.00-16.15	Minasyan
16.30-17.45	Uyanık	16.30-17.45	Deloro	16.30-17.45	Poizat
Contributed talks					
18.00-18.25	Göral	18.00-18.25	Turbo		
18.30-18.55		18.30-18.55			

Talks

Adrien Deloro
Representations of $\mathrm{SL}_2(\mathbb{K})$ *as* $\mathrm{SL}_2(\mathbb{K})$*-modules*

Abstract. The algebraic representations of the group $\mathrm{SL}_2(\mathbb{K})$ of 2×2 matrices with determinant 1 are well-known: there is, for each integer positive integer d, the $(d+1)$-dimensional $\mathbb{K}$-vector space $\mathbb{K}[X^d, X^{d-1}Y, \ldots, XY^{d-1}, Y^d]$ of homogeneous polynomials in two variables of degree d.

But what happens if one wants to study $\mathrm{SL}_2(\mathbb{K})$-modules more in general? Can one identify the algebraic representations among abstract $\mathrm{SL}_2(\mathbb{K})$-modules? We shall discuss some positive algebraic results and some geometric limitations.

Aslı Güçlükan İlhan
Equivariant homotopy diagrams

Abstract. In this talk, we explain what we mean by an equivariant homotopy diagram and how we can use them to obtain a gluing data for constructing topological spaces with group actions. Our main goal is to construct some examples of finite G-CW-complexes homotopy equivalent to a product of spheres with isotropy groups in a given family of subgroups of G, where G is a finite group. For this, one needs to construct the correct gluing data so that the construction has the required properties. We discuss how the combinatorial nature of our construction can be used to understand the homotopy type of the resulting space. This is a joint work with Özgün Ünlü.

Gilbert Levitt
Some finiteness properties of groups.

Abstract. I will discuss finiteness properties of a group G related to the following topics: groups containing G with a given finite index, conjugacy classes of finite subgroups, decompositions of G as a free product with amalgamation. This is based on joint work with Vincent Guirardel.

Ashot Minasyan
Introduction to conjugacy separability.

Abstract. A group G is said to be residually finite if it can be approximated by finite groups, i.e., if for any distinct elements x,y in G there is a homomorphism from G to a finite group such that the images of x and y are distinct. Similarly, a group is conjugacy separable if for any non-conjugate elements x and y there is a homomorphism to a finite group such that the images of these elements are not conjugate.

Residual finiteness and conjugacy separability are classical "residual" properties studied in Combinatorial Group Theory. They can be viewed as algebraic analogues of the solvability of the word problem and the conjugacy problem in the group respectively. For many groups residual finiteness can be shown quite easily (e.g., all finitely generated linear groups are residually finite). Conjugacy separability, on the other hand, is much harder to prove, and until recently very few classes of conjugacy separable group were known.

During the talk I will present the classical proof that free groups are conjugacy separable and will outline the argument for showing that right angled Artin groups are conjugacy separable. (Recall the right angled Artin groups, or graph groups, or partially commutative groups, are groups possessing finite presentations where the only defining relators are commutators of the generators. Free groups and free abelian groups are basic examples of right angled Artin groups, however the class of such groups is much larger and has a very rich subgroup structure.) I will then discuss some applications of this result.

Bruno Poizat
Free abelian groups in positive and negative logics.

Abstract. Le titre est explicite.

Çağlar Uyanık
Dynamics of free group automorphisms and a subgroup alternative for $Out(F_N)$.

Abstract. The study of outer automorphism group of a free group $Out(F_N)$ is closely related to the study of Mapping Class Group of a surface. We will discuss various free group analogs of pseudo-Anosov homeomorphisms of hyperbolic surfaces. We will focus mostly on dynamics of their actions on the space of currents and deduce several structural results about subgroups of $Out(F_N)$. Part of this talk is based on joint work with Martin Lustig and Matt Clay.

Şükrü Yalçınkaya
Black box methods to identify groups of Lie type

Abstract. I will focus on the recognition algorithms for the black box groups of Lie type of odd characteristics. A *black box group* is a black box (or an oracle, or a device, or an algorithm) operating with binary strings of uniform length which encrypt (not necessarily in a unique way) elements of some finite group. Group operations, taking inverses and deciding whether two strings represent the same group elements are done by the black box. In this context, a natural task is to find a probabilistic algorithm which determines the isomorphism type of a group within given arbitrarily small probability of error. More desirable algorithms, *constructive recognition algorithms*, are the ones producing an isomorphism between a black box copy of a finite group and its natural copy.

Our approach for recognising black box groups of Lie type is based on the procedures that separates the recognition of the type of the group from the underlying field structure. First, I will explain the details of our approach and then, for black box groups encrypting $\mathrm{PGL}_2(\mathbb{F})$ and $\mathrm{PGL}_3(\mathbb{F})$ over a field $\mathbb{F}$ of odd characteristic, I will present algorithms which construct a black box field isomorphic to the underlying field and produce isomorphism between the given black box group and the corresponding Lie type group on this black box field. Although it is not needed for the isomorphisms, the solution of an old problem, that is, construction of a unipotent element in black box groups of Lie type is also presented.

This is a joint work with Alexandre Borovik.

Tutorials

Mahmut Kuzucuoğlu
Centralizers in infinite locally finite simple groups.

Abstract. In this series of three lectures we will survey centralizers of elements and centralizers of finite subgroups in simple locally finite groups.

One of the methods to produce new locally finite simple groups from the finite simple groups or from the finite symmetric groups is to use the direct limit method. There are two classes of well known groups obtained as a direct limit of finite symmetric groups:

(1) the homogenous symmetric groups obtained as a direct limit of finite symmetric groups one is embedded into the next one by diagonal embedding.

(2) Hall universal group which is obtained as a direct limit of finite symmetric groups one is embedded into the next one by regular embedding.

Kroshko-Sushchansky studied the first type of groups and they give a complete characterization of such groups using Steinitz numbers. We will discuss the centralizers of finite subgroups in these groups and extend their results to homogenous finitary symmetric groups. We also show that, for any given cardinality κ there are uncountably many pairwise non-isomorphic simple locally finite groups of cardinality κ. We introduce their characterization and the structure of centralizers of finite subgroups see [1].

One can attach a locally finite tree to homogenous symmetric groups. Then there is a natural metric space structure and topology attached to these trees. By using topological properties one can say some structural information on automorphisms of such groups.

P. Hall in 1959 [2], presented the locally finite simple group U defined as a countable $\aleph_0$-homogenous group. The structure of centralizers of finite subgroups in Hall universal group has been studied by many authors and the structure of centralizers of elements has been described by Hartley using the methods of existentially closed structure. Here we give a complete description of the structure of centralizers of arbitrary finite subgroups in Hall universal groups using only group theoretical construction of such groups.

Namely we prove the following:

Theorem 1 *(Kegel-Kuzucuoğlu) Let U be the Hall universal group and F a finite subgroup of U. Then the centralizer $C_U(F)$ is isomorphic to $Z(F)U$. In particular the centralizer of every finite subgroup contains an isomorphic copy of U.*

References

[1] Ü. B. Güven, O. H. Kegel, M. Kuzucuoğlu; *Centralizers of subgroups in direct limits of symmetric groups with strictly diagonal embedding*, Comm. Algebra., **43**(**6**), 1–15, (2015).

[2] P. Hall, Some constructions for locally finite groups. J. London Math. Soc. 34 (1959) 305-319.

Dugald Macpherson

Model theory of pseudofinite and profinite groups.

Abstract. A structure is *pseudofinite* if it is elementarily equivalent to an ultraproduct of finite structures. I will discuss this notion, and then focus on pseudofinite groups. John Wilson [4] showed that a pseudofinite group is simple (group-theoretically) if and only if it is a group of Lie type (possibly twisted) over a pseudofinite field. Using work of Hrushovski on the non-standard Frobenius, it follows that any pseudofinite simple group has supersimple finite rank theory. Furthermore, by work of Ryten, such a group is *measurable*, and any family of finite simple groups of fixed Lie rank forms an 'asymptotic class' (notions introduced in [3] and [1]). In the first two lectures, I will discuss this material, mentioning also results from [1] on pseudofinite groups with supersimple theory of rank at most 2, and on permutation groups.

The third talk will focus on recent work with Tent on profinite groups, viewed as 2-sorted structures (G, I) where I is an index set coding the open subgroups of G. Such a structure has NIP theory (that is, its theory does not have the independence property) if and only if G has a finite index normal subgroup N which is a direct product of finitely many compact p-adic analytic groups (for different primes p). I will discuss this, and connections to pseudofinite groups.

Basic background on notions from generalised stability theory (theories which are stable, simple, NIP, etc.) will be given.

References

[1] R. Elwes, 'Asymptotic classes of finite structures', J. Symb. Logic 72 (2007), 418–438.

[2] R. Elwes, E. Jaligot, H.D. Macpherson, M. Ryten, 'Groups in supersimple and pseudofinite theories', Proc. London Math Soc. (3) 103 (2011), 1049–1082.

[3] H.D. Macpherson, C. Steinhorn, 'One-dimensional asymptotic classes of finite structures', Trans. Amer. Math. Soc. 360 (2008), 411–448.

[4] J.S. Wilson, 'On pseudofinite simple groups', J. London Math. Soc. 51 (1995), 471–490.

Contributed Talks

Haydar Göral
Definable Groups in Mann Pairs.

Abstract. In this talk, we study algebraically closed fields expanded by two unary predicates denoting an algebraically closed subfield and a multiplicative subgroup. This will be a proper extension of algebraically closed fields with a group satisfying the Mann property, and also pairs of algebraically closed fields. We define the uniform version of the Mann property introduced by L. van den Dries and A. Günaydin. We first characterise the forking independence in the triple. Then this allows us to characterise definable groups in the triple via algebraic groups.

Turbo Ho
Describing Groups.

Abstract. Recall that the index set of a computable structure is the set of indices for its computable copies. The calculation of the complexity of index set usually involves finding an optimal Scott sentence (a sentence in $L_{\omega_1,\omega}$ that describes the structure up to isomorphism.) J. Knight and et al. determined the complexity of index sets of various structures. In this talk, we focus on finding the complexity of index sets of various groups, generalizing methods that was previously used by J. Knight and et al. We found computable Scott sentences for various different groups or class of groups, including nilpotent groups, polycyclic groups, and the lamplighter group. In some of these cases, we also showed that the sentence we had are optimal.