Models and Groups, İstanbul 6

24-26 march 2016

Thursday		Friday		Saturday	
9.45-11.00	Deloro	9.45-11.00	Deloro	9.45-11.00	Deloro
11.15-12.30	Yalçın	11.15-12.30	Kazachkov	11.15-12.30	Yalçın
Lunch					
15.00-16.15	Ersoy	15.00-16.15	Lubotzky	15.00-16.15	Casals-Ruiz
16.30-17.45	Byron	16.30-17.45	Sklinos	16.30-17.45	Hempel
Contributed talks					
18.00-18.25		18.00-18.50	Brück		
18.30-18.55					

Talks

Ayala Byron
Definable fields in the free group

Abstract. In the early 2000s Sela proved that all non-abelian free groups share a common first-order theory. Together with R. Sklinos, we use tools developed in his work to show that no infinite field is definable in this theory. In this talk we will survey the line of proof for a formal solution theorem for a simple sort of definable sets, i.e., that have a structure of a hyperbolic tower, and use it to characterize definable sets that do not carry a definable structure of an abelian group.

Montserrat Casals-Ruiz
Limit groups of partially commutative groups

Abstract. Basic questions on the elementary theory of free groups were formulated by Tarski around 1945 and since then have been the motivation for a large body of research in Group Theory. A first step towards Tarski problems is understanding finitely generated models of the universal theory of a free group, or in other terms, the study of limit groups.

In the first part of the talk, I will introduce the class of limit groups and explore different characterizations that connect Group Theory, Algebra, Geometry, and Model Theory.

In the second part of the talk, I will turn my attention to the algebraic structure of limit groups over free groups and, more generally, over partially commutative groups - a prominent class of groups that interpolates between free and free abelian groups.

Kıvanç Ersoy

Splitting automorphisms of finite groups

Abstract. An automorphism α of the group G is called n-splitting if

$$x.x^{\alpha}.x^{\alpha^2}\ldots x^{\alpha^{n-1}} = 1 \text{ for every } x \in G$$

where $|\alpha|$ divides n. A splitting automorphism α is n-splitting with $|\alpha| = n$. Fixed point free automorphisms of finite groups are immediate examples of splitting automorphisms. Kegel generalized Thompson's result of nilpotentcy of a finite group with a fixed point free automorphism of prime order to the finite groups admitting a splitting automorphism of prime order. Moreover, Higman, Kreknin and Kostrikin proved that if a finite group G has a splitting automorphism of prime order p, then the nilpotency class of G is bounded in terms of p.

A finite group having a fixed point free automorphism is solvable. On the contrary, a finite group admitting a splitting automorphism may not be solvable. But Jabara proved that a finite group with a splitting automorphism of order 4 is solvable. Moreover, we proved the following:

Theorem 1 [1] *A finite group with a splitting automorphism of odd order is solvable.*

In this talk, we will present recent results proved in a joint work with Kanta Gupta and Enrico Jabara [2], describing the structure of a finite group with a splitting automorphism of order 2^n.

References

[1] K. Ersoy, "Finite groups with a splitting automorphism of odd order", Arch. Math, doi: 10.1007/s00013-016-0874-6, 2016

[2] K. Ersoy, C.K. Gupta, E. Jabara, "Finite groups with a splitting automorphism of even prime power order", in preparation.

Nadja Hempel

Almost centralizer: a useful tool in Model theory

Abstract. Given a group G, one particular problem we are interested in is to find definable envelopes for arbitrary abelian, nilpotent or solvable subgroups of G which admit the same algebraic properties. Such evelopes exists if G is stable and even if G is merely dependent but sufficiently saturated (with the additional hypothesis of normality in the solvable case). In groups with a simple theory, one obtains definable envelopes up to finite index. We introduce the notion of an almost centralizer and establish some of its basic properties. This enables us to extend the aforementioned results to groups in which any definable section satisfies a chain condition on centralizers up to finite index as well as saturated NTP_2 groups. Moreover we give some other applications to bounded almost nilpotent groups as well as the Fitting subgroup.

Ilya Kazachkov
Elementary theory of well-structured algebras and nilpotent groups

Abstract. One of the fundamental questions of model theory is to classify algebraic structures up to elementary equivalence. The goal of this talk is to describe groups elementary equivalent to a wide class of nilpotent groups. In order to achieve this goal, we will study Lie algebras associated to nilpotent groups, introduce the notion of well-structured algebras and classify rings elementary equivalent to them. We will then apply this classification result to address the classification problem for groups, thereby generalising and giving a common framework for results of Belegradek, Myasnikov and Sohrabi.

Alexandre Lubotzky
Ramanujan complexes and topological expanders

Abstract. Expander graphs in general, and Ramanujan graphs , in particular, have played a major role in computer science in the last 4 decades and more recently also in pure math. In recent years a high dimensional theory of expanders is emerging. A notion of topological expanders was defined by Gromov who proved that the complete d-dimensional simplicial complexes are such. He raised the basic question of existence of such bounded degree complexes of dimension $d > 1$. This question was answered recently (by T. Kaufman, D. Kazdhan and A. Lubotzky for $d = 2$ and by T. Kaufman and S. Evra for general d) by showing that the d-skeleton of $(d+1)$-dimensional Ramanujan complexes provide such topological expanders. We will describe these developments and the general area of high dimensional expanders.

Rizos Sklinos
Definable fields and centralisers in the free group

Abstract. After the work of Sela (and Kharlampovich-Myasnikov) on the first order theory of non abelian free groups, culminating to the positive answer to Tarski's question on whether non abelian free groups share the same common theory, the model theoretic interest for those natural algebraic structures has arisen.

Although on the way of answering Tarski's question a quantifier elimination down to AE formulas had been proven, it is still not easy to determine which subsets of some cartesian power of a non abelian free group are definable.

On the other hand, Sela proved that the common theory of non abelian free groups is stable. Roughly speaking this means that a "well-behaved" independence relation is supported in this first order theory. This independence relation had been proved (by Ould Houcine-Tent and by myself) to be as complicated as possible (technically speaking n-ample for all n), which is usually the case in the presence of a definable field.

In a joint work with A. Byron we prove that no infinite field is definable in a non abelian free group. Part of the proof consists of showing that the independence relation restricted to centralisers of non trivial elements is as tame as possible (i.e. it is not 1-ample). I will present this part of the proof.

Tutorials

Adrien Deloro

Groups and representations of finite Morley rank

Abstract. Groups of finite Morley rank are a natural way to model-theoretically describe linear algebraic groups over algebraically closed fields. The tutorial will be an invitation to study their representations, with a focus on so-called definable representations of algebraic groups. The basic tools and questions will be discussed, with tentative program as follows.

- The first lecture will mostly describe the setting: definable sets and rank functions (very basic concepts from model theory, which loosely mimick behaviour of constructible sets and the Zariski dimension in algebraic geometry), and a rough overview of the "atomic" bricks appearing in groups of finite Morley rank.
- The second lecture should focus on modules, i.e. group representations of finite Morley rank. We shall present linearity results explaining why fields and vector spaces can often be retrieved. As an application, we should classify modules for algebraic groups in characteristic 0.
- The third lecture might — time permitting — focus on positive characteristic where things are more subtle. Here there are mostly questions, and we hope to explain why the topic is both non-trivial and promising.

It should be added that the question of representations is different from the more classical topic of identification of abstract groups of finite Morley rank, both in spirit and methods. The talks will therefore be self-contained (and not too heavy on group-theoretic aspects).

Ergün Yalçın

Homology decompositions for classifying spaces of finite groups

Abstract. In this series of two talks, I will explain the theory of mod-p homology decompositions for finite groups using an approach due to Dwyer, and give some (more recent) applications of this theory to constructions of finite group actions on homotopy spheres.

For a discrete group G, the n-th cohomology group of G is defined algebraically as an ext-group over the group ring $\mathbb{Z}G$, or topologically as the n-th cohomology group of the classifying space BG. The cohomology of a finite group G with integer coefficients, at positive degrees, is a direct sum of its cohomology in p-local coefficients over all primes p dividing the order of G. This allows us to study the classifying space of G, and hence the group cohomology, a prime at a time.

For a fixed prime p, dividing the order of G, a mod-p homology decomposition of BG is a mod-p homology isomorphism between BG and a homotopy colimit hocolim F of a functor F from a category D to the category of spaces, such that for each object d in D, $F(d)$ is homotopy equivalent to BH_d for some subgroup H_d of G. There are mainly three types of homology decompositions: subgroup decomposition, centraliser decomposition, and normaliser decomposition.

In all these decompositions, the category D is described in terms of an ample collection C of subgroups. A collection C is ample if mod-p equivariant cohomology of the topological realisation of the poset of subgroups in C, is isomorphic to the mod-p cohomology of the group G. The collection of all nontrivial p-subgroups, the collection of all nontrivial elementary abelian p-subgroups, and the collection of all p-centric and p-radical subgroups are ample collections. The homology decompositions that we obtain for these collections can be considered as generalisations of homology decompositions for finite groups of Lie type coming from the Tits building.

In the first talk, I will start with basic definitions related to group cohomology, classifying spaces, (saturated) fusion systems, and state the Cartan-Eilenberg theorem for cohomology of groups which is an important theorem for understanding group cohomology with p-local coefficients. Then I will introduce basic definitions for realization of categories and homotopy colimits.

In the second talk, I will state the theorems for the three types of homology decompositions and discuss sharpness of these decompositions for various collections of subgroups. These results are due to Webb, Jackowski-McClure, and Dwyer. Finally in the last part to the talk I will discuss some more recent applications of homology decompositions to constructing group actions on homotopy spheres. The results that I will mention are due to Adem-Smith, Jackson, and Hambleton-Yalçın.

Contributed talks

Benjamin Brück
Weight in non-standard models of the theory of free groups

Abstract. As the common first order theory T_{fg} of non-abelian free groups is stable, we can use the notion of forking independence in order to ask whether a set of elements in models of T_{fg} is independent or not. In this theory, there is a unique generic type p_0 over the empty set. The interest in this type comes from the fact that its realisations in free groups are exactly the primitive elements and in a free group of finite rank, a set is a maximal independent set of realisations of p_0 if and only if it forms a basis. In particular, all those sets have the same cardinality. The aim of this talk is to look at the analogues of this in groups that share the same theory as free groups but are not free themselves. Using hyperbolic towers, I will firstly present a criterion for the maximality of independent sets of realisations of p_0 in those non-standard models and afterwards give a construction of models of T_{fg} that contain such maximal independent sets with arbitrarily large differences in their sizes. The existence of such sets of different cardinalities can be expressed by the fact that the type p_0 has infinite weight.