$\boxed{\text{Ex 1}}$

1) $f_n(x) \to \arctan(x)$, $g_n(x) \to \begin{cases} 1 & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

2) $|f_n(x) - \arctan x| = \dfrac{x}{x+n} \leq \dfrac{1}{n} \Rightarrow$ cv unif sur $[0,1]$, $]0,1]$ et $[a,1]$.

$\left| g_n(x) - \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases} \right| = \dfrac{1}{1+nx} \quad (x \neq 0)$, $\sup\limits_{x \in [0,1]} = 1$ pas de cv unif (la lim n'est pas cont. donc aucune chance)

$\sup\limits_{]0,1]} = 1$ idem. pas cv unif.

$\sup\limits_{[a,1]} = \dfrac{1}{1+na} \xrightarrow[n]{} 0 \Rightarrow$ cv unif.

3) $\dfrac{x}{x+n} \to 0$, $\sup\limits_{x \geq 1} \dfrac{x}{x+n} = 1$ pas de cv unif.

$\dfrac{x+n-x}{(x+n)^2} = \dfrac{n}{(x+n)^2} \Rightarrow \nearrow$

$\sup\limits_{x \geq 1} \dfrac{1}{1+nx} = \dfrac{1}{1+n} \Rightarrow$ cv. unif.

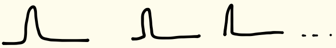
$\boxed{\text{Ex 2}}$

1) $f_n(x) \to 0 \quad \forall x$.

2) $\sup\limits_{[0,1]} |\sin nx^2| e^{-nx^2} =$ $\quad 2nx \cos nx^2 \, e^{-nx^2} + \sin nx^2 \, (-2nx) e^{-nx^2}$

$$2nx(\cos nx^2 - \sin nx^2) e^{-nx^2} \qquad nx^2 = \pi/4 \qquad \begin{matrix} + & - \\ \nearrow & \searrow \end{matrix}$$

$\sin \frac{\pi}{4} e^{-\pi/4}$ en $x = \sqrt{\frac{\pi}{4n}}$



3) $\sup\limits_{[a,1]} |\sin nx^2| e^{-nx^2}$ $\quad \sqrt{\frac{\pi}{4n}} > a \quad \frac{\pi}{4n} > a^2 \quad n \le \frac{\pi}{4a^2}$

Dès que $n > \frac{\pi}{4a^2}$ $\searrow f \Rightarrow \sup = |\sin na^2| e^{-na^2} \Rightarrow$ cv. unif.

$\boxed{Ex\,3}$ $x \neq 1 \to 0$, $x = 1 \to \frac{e^{-1}}{2}$

Sur $[0,1]$ non car lim pas continue

Autre façon: $|f(x)| = \frac{x^n e^{-x}}{1+x^n}$, $\sup\limits_{[0,1[} = \frac{e^{-1}}{2}$.

$$\frac{e^{-x}}{1+1/x^n}$$

Idem $[0,1[$.

$[0,a]$: $\sup \frac{e^{-a}}{1+\frac{1}{a^n}} \to 0$ cv unif.

$\boxed{\text{Ex } 4}$

1) $\dfrac{x}{nx^2 + 1/2^n} \to 0$.

2) $\displaystyle\int_0^1 \frac{2^n x}{1+2^n n x^2}\,dx = \frac{1}{2n} \int_0^1 \frac{2n 2^n x}{1+2^n n x^2}\,dx = \frac{1}{2n}\left[\ln(1+2^n n x^2)\right]_0^1 =$

$\displaystyle = \frac{1}{2n} \ln(1+2^n n) = \frac{\ln(2^n) + \ln\left(\frac{1}{2^n}+n\right)}{2n} = \frac{n\ln 2 + \ln\left(\frac{1}{2^n}+n\right)}{2n}$

$\to \dfrac{\ln 2}{2}$. Pas conv unif.

3) $\displaystyle\sup_{[0,1]} \frac{2^n x}{1+2^n n x^2}$

Dérivée : $\dfrac{2^n(1+2^n n x^2) - 2^n x \, 2n 2^n x}{(\ \cdots\ )^2} = \dfrac{2^n + 2^{2n} n x^2 - 2n 2^{2n} x^2}{(\quad)^2} = \dfrac{2^n - n 2^{2n} x^2}{(\quad)^2}$

$2^n - n 2^{2n} x^2 \geq 0 \iff 2^n \geq n 2^{2n} x^2 \iff \dfrac{1}{n 2^n} \geq x^2 \iff x \leq \dfrac{1}{\sqrt{n 2^n}}$

$+ \nearrow \ - \searrow \quad x = \dfrac{1}{\sqrt{n 2^n}} \to \dfrac{2^n/\sqrt{n2^n}}{1+2^n n \frac{1}{n 2^n}} = \dfrac{1}{2}\sqrt{\dfrac{2^n}{n}} \not\to 0$.

$\boxed{\text{Ex 5}}$

$[-1,1] \quad f_n(x) = \dfrac{x}{1+n^2x^2}$

1) $\to 0$ ✓ $\quad \dfrac{1+n^2x^2 - x\,2n^2x}{(1+n^2x^2)^2} = \dfrac{1-n^2x^2}{(\quad)^2} : x = \pm\dfrac{1}{\sqrt{n}}$

	$-\frac{1}{\sqrt{n}}$		$\frac{1}{\sqrt{n}}$	
$-$		$+$		$-$
$\searrow$		$\nearrow$		$\searrow$

$f_n(-1) = \dfrac{-1}{1+n^2}$, $f_n\left(\dfrac{1}{\sqrt{n}}\right) = \dfrac{1/\sqrt{n}}{2} = \dfrac{1}{2\sqrt{n}}$: $\sup\limits_{[-1,1]} |f_n| = \dfrac{1}{2\sqrt{n}} \Rightarrow$ cv. unif.

2) $f_n'(x) = \dfrac{1-n^2x^2}{(1+n^2x^2)^2} \xrightarrow[n]{} 0$ sauf $x=0 \to 1$

$\sup\limits_x |f_n' - \lim| = 1$ pas de cv. unif.

car $\lim\limits_{x\to 0^+} f_n(x) = 1$

3) $g_n = \int f_n$. / conv simple : $\to 0$, (dérivée $= f_n$ $\overset{-\;+}{\vee}$) $\Rightarrow |g_n| \leq \dfrac{\ln(1+n^2)}{2n^2}$

cv unif.

$\boxed{\text{Ex 6}}$

$f_n = \frac{1}{n}\arctan\left(\frac{x}{n}\right)$, $f_n' = \frac{1}{n}\,\frac{1}{n}\,\frac{1}{1+\frac{x^2}{n^2}} = \frac{1}{n^2+x^2}$ cv unif vers 0 sur $\mathbb{R}$.

$f_n(x) = \int_0^x f_n'(u)\,du$, $|f_n(x)| \leq \int_0^x |f_n'(u)|\,du \leq \frac{1}{n^2}x$. cv unif sur $[-a,a]$.

$\boxed{\text{Ex 7}}$

$$\frac{ne^{-x}+x^2}{n+x^2}$$

1) $\to e^{-x}$ 0,5

2) $\frac{ne^{-x}+x^2}{n+x^2} - e^{-x} = \frac{x^2(1-e^{-x})}{n+x^2} \leq \frac{c}{n}$ si $x \in [-a,a]$. 1

3) $\sup_{[a,+\infty[} \frac{x^2(1-e^{-x})}{n+x^2}$? $\lim_{x\to+\infty}$ est 1 donc sup $=1$ pas de conv unif. 1,5

4) $\int_0^1 f_n$: Conv. dominée $\Rightarrow \int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1 = 1-e^{-1}$

1

$\boxed{\text{Ex 8}}$

$$f_n = \begin{cases} -n^3x^2 + 2n^2x &, \ x \in [0, 2/n] \\ 0 &, \ x \in [\frac{2}{n}, 1] \end{cases}$$

1) $f_n(x) \to 0 \ \forall x$ car forcément $\frac{2}{n} < x$ pour n assez grand.

2) $\int_0^1 f_n = \int_0^{2/n} (-n^3x^2 + 2n^2x) = \left[-\frac{n^3}{3}x^3 + n^2x^2\right]_0^{2/n} = \frac{-n^3}{3}\frac{8}{n^3} + n^2\frac{4}{n^2} = -8/3 + 4 = \frac{4}{3}$

3) $\lim \int f_n \neq \int \lim f_n$.