TD n°04

1 mars 2018

Exercice 1 - 100 lancers

We throw a balanced die 100 times. Let X be the sum of numbers that appear during all 100 throws. Upper bound $\mathbf{P}\{|X - 350| \geq 50\}$.

Exercice 2 - Tester La Pièce

Consider a coin that turns up head with probability p. How many times does one need to toss this coin to approximate $p \pm 0.1$ with probability 0.9 ?

Exercice 3 - Comparons !

We throw a fair die n times. Let X be the number of times the die turned up 6. Let q be the probability of the event $X \geq n/4$. Compare the upper bound for q when you use Markov's inequality, Chebyshev's inequality, and Chernoff bound.

Exercice 4 - Pour les plus rapides

Let Y be a random variable that takes either a positive integer value or 0. Its expected value is strictly positive. The aim of this exercise is to show that

$$\frac{\mathbf{E}\{Y\}^2}{\mathbf{E}\{Y^2\}} \leq \mathbf{P}\{Y \neq 0\} \leq \mathbf{E}\{Y\} \ .$$

4.1 We would like to define a random variable that informally corresponds to $(Y|Y \neq 0)$. How would you properly define such a random variable (one might need to change the probability space) ?

4.2 Compare $\mathbf{E}\{X\}^2$ with $\mathbf{E}\left\{X^2\right\}$.

4.3 Conclude on the inequalities above.

Exercice 5 - Formule de Jensen

Let f be a convex function and X be a random variable that takes real values. Jensen's inequality states that

$$\mathbf{E}\{f(X)\} \geq f(\mathbf{E}\{X\}).$$

Suppose $f \in \mathcal{C}^2$ (i.e., its first and second derivatives both exist and continuous.) Show that Jensen's inequality holds.