

**Exercise sheet # 7**  
Quantum non-locality

**Exercise 7.1.**

Show directly from the definition that the set  $QC_{m,n}$  of quantum correlations is convex.

**Exercise 7.2.**

Give an explicit family of observables on  $\mathbb{C}^2$ , as well as a state on  $\mathbb{C}^2 \otimes \mathbb{C}^2$ , which show that the matrix  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$  belongs to  $QC_{2,2}$ .

**Exercise 7.3. Optimality of the CHSH violation**

1. Show that if  $X_1, X_2, Y_1, Y_2$  are self-adjoint operators satisfying  $X_1^2 = X_2^2 = \text{Id}_{\mathcal{H}_A}$  and  $Y_1^2 = Y_2^2 = \text{Id}_{\mathcal{H}_B}$  then the operator  $B = X_1 \otimes Y_1 + X_1 \otimes Y_2 + X_2 \otimes Y_1 - X_2 \otimes Y_2$  satisfies  $\|B\| \leq 2\sqrt{2}$ . (Hint : compute  $B^2 + [X_1, X_2] \otimes [Y_1, Y_2]$ ).
2. Deduce that any matrix  $(a_{ij})$  in  $LC_{2,2}$  satisfies the inequality  $a_{11} + a_{12} + a_{21} - a_{22} \leq 2\sqrt{2}$ .