

C¹ Fractals

Pictures !

Flat 2-Tori in \mathbb{E}^3

Vincent Borrelli

Université Lyon 1





Implementing the Convex Integration Process

C¹ Fractals

Pictures !





The Hevea Project



Francis Lazarus Gipsa-Lab, Grenoble Boris Thibert LJK, Grenoble Saïd Jabrane ICJ, Lyon

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The initial map



Step 1 : decomposition of Δ



Integration Process C¹ Fractals

the Convex

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$$\ell_1(.) = \langle \boldsymbol{e}_1, . \rangle_{\mathbb{E}^2}$$

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$$\ell_2(.) = \langle \frac{1}{5}(e_1 + 2e_2), . \rangle_{\mathbb{E}^2}$$

 $\underset{\mathbb{E}^{3}}{\text{Flat 2-Tori in}}$

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$$\ell_3(.) = \langle \frac{1}{5}(\textbf{e}_1 - 2\textbf{e}_2), . \rangle_{\mathbb{E}^2}$$

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Step 2 : Three convex integrations

• From f_0 and by applying three convex integrations along the directions given by ℓ_1 , ℓ_2 and ℓ_2 , we build iteratively three maps (a *salvo*) :

$$f_{1,1}, f_{1,2}, f_{1,3}$$

such that

- $$\begin{split} g &- f_{1,1}^* \langle ., . \rangle_{\mathbb{E}^q} = \rho_2 \ell_2^2 + \rho_3 \ell_3^2 &+ O(\frac{1}{N_{1,1}}) \\ g &- f_{1,2}^* \langle ., . \rangle_{\mathbb{E}^q} = \rho_3 \ell_3^2 &+ O(\frac{1}{N_{1,1}}) + O(\frac{1}{N_{1,2}}) \\ g &- f_{1,3}^* \langle ., . \rangle_{\mathbb{E}^q} = 0 &+ O(\frac{1}{N_{1,1}}) + O(\frac{1}{N_{1,2}}) + O(\frac{1}{N_{1,3}}) \end{split}$$
- We choose $N_{1,1}$, $N_{1,2}$ and $N_{1,3}$ such that :

$$\|\langle .,.\rangle_{\mathbb{E}^2} - f_{1,3}^* \langle .,.\rangle_{\mathbb{E}^3} \|_{C^0} \leq \frac{1}{2} \|\langle .,.\rangle_{\mathbb{E}^2} - f_0^* \langle .,.\rangle_{\mathbb{E}^3} \|_{C^0}.$$



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Convex Integration in the First Direction





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Convex Integration in the Second Direction





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Convex Integration in the Third Direction





Three directions

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The flow in direction 1

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Three directions



The flow in direction 2



The flow in direction 3

A zoom

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The vertical scale is exaggerated for emphasis.

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Step 3 : Iterating the process

• We continue the process to create infinite sequence of maps

 $f_0, f_{1,1}, f_{1,2}, f_{1,3}, \dots f_{k,1}, f_{k,2}, f_{k,3}, \dots$

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Step 3 : Iterating the process

• We continue the process to create infinite sequence of maps

$$f_0, f_{1,1}, f_{1,2}, f_{1,3}, \dots f_{k,1}, f_{k,2}, f_{k,3}, \dots$$

• After the salvo n° k, the isometric default is divided by 2^k

$$\|\langle .,.\rangle_{\mathbb{E}^2}-f_{k,3}^*\langle .,.\rangle_{\mathbb{E}^3}\|_{C^0}\leq \frac{1}{2^k}\|\langle .,.\rangle_{\mathbb{E}^2}-f_0^*\langle .,.\rangle_{\mathbb{E}^3}\|_{C^0}.$$

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Step 3 : Iterating the process

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$$\|\langle .,.\rangle_{\mathbb{E}^2}-f_{k,3}^*\langle .,.\rangle_{\mathbb{E}^3}\|_{C^0}\leq \frac{1}{2^k}\|\langle .,.\rangle_{\mathbb{E}^2}-f_0^*\langle .,.\rangle_{\mathbb{E}^3}\|_{C^0}.$$

• The limit

$$f_{\infty} := \lim_{k \longrightarrow +\infty} f_{k,3}$$

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is a C^1 isometric immersion of the flat torus.



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Another CI in the first direction





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And so on... up to the Flat Torus



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Nash-Kuiper in a 1D setting !



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Nash-Kuiper in a 1D setting !



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Nash-Kuiper in a 1D setting !



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Kuiper in a 1D setting !



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Moscow University





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Riesz Products

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Frigyes Riesz

• These spectra arise from the study of *Riesz products* :

$$u \mapsto \prod_{j=0}^{\infty} (1 + \alpha_j \cos(2\pi b^j u)).$$

In that product, $b \ge 3$ is an integer and $(\alpha_j)_{j \in \mathbb{N}}$ is a sequence of real numbers such that, for every $j \in \mathbb{N}$, $|\alpha_j| \le 1$.



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Riesz Products

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In that product, $b \ge 3$ is an integer and $(\alpha_j)_{j \in \mathbb{N}}$ is a sequence of real numbers such that, for every $j \in \mathbb{N}$, $|\alpha_j| \le 1$.

• Here, products of *corrugation matrices* take the place of Riesz products.

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Corrugation matrices

• Let $C_k : \mathbb{S}^1 \longrightarrow O(2)$ be the matrix valued map such that

$$\forall u \in \mathbb{S}^1, \quad \left(\begin{array}{c} t_k(u) \\ n_k(u) \end{array}\right) = \mathcal{C}_k(u) \cdot \left(\begin{array}{c} t_{k-1}(u) \\ n_{k-1}(u) \end{array}\right)$$



We call C_k a corrugation matrix.

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Structure of the Gauss Map

• Corrugation matrices encode the data of the successive convex integrations :

$$\mathcal{C}_k(u) := \begin{pmatrix} \cos \theta_k(u) & \sin \theta_k(u) \\ -\sin \theta_k(u) & \cos \theta_k(u) \end{pmatrix}$$

with

$$\theta_k(u) = \arg h_u(\{N_k u\}) = \alpha_k \cos(2\pi N_k u).$$

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Structure of the Gauss Map

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ight)$$

with

$$\theta_k(u) = \arg h_u(\{N_k u\}) = \alpha_k \cos(2\pi N_k u).$$

 \bullet The Gauss map n_∞ of the limit embedding is given by a Riesz-like product :

$$\left(\begin{array}{c}t_{\infty}\\n_{\infty}\end{array}\right) = \left(\prod_{k=1}^{\infty} \mathcal{C}_{k}\right) \cdot \left(\begin{array}{c}t_{0}\\n_{0}\end{array}\right)$$



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From the circle to the torus



We denote by $C_{k,j}$ the SO(3) matrix such that :

$$\left(\begin{array}{c} \mathbf{v}_{k,j}^{\perp} \\ \mathbf{v}_{k,j} \\ \mathbf{n}_{k,j} \end{array}\right) = \mathcal{C}_{k,j} \cdot \left(\begin{array}{c} \mathbf{v}_{k,j-1}^{\perp} \\ \mathbf{v}_{k,j-1} \\ \mathbf{n}_{k,j-1} \end{array}\right)$$

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From the circle to the torus

Let $f_{\infty}: \mathbb{T}^2 \longrightarrow \mathbb{E}^3$ be the limit of the maps :

$$f_0, \quad f_{1,1}, f_{1,2}, f_{1,3}, \quad f_{2,1}, f_{2,2}, f_{2,3}, \quad \dots$$

We have

$$\left(\begin{array}{c} v_{\infty}^{\perp} \\ v_{\infty} \\ n_{\infty} \end{array}\right) = \prod_{k=1}^{\infty} \left(\prod_{j=1}^{3} \mathcal{C}_{k,j}\right) \cdot \left(\begin{array}{c} v_{0}^{\perp} \\ v_{0} \\ n_{0} \end{array}\right)$$

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From the circle to the torus

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We have

$$\begin{pmatrix} \mathbf{v}_{\infty}^{\perp} \\ \mathbf{v}_{\infty} \\ \mathbf{n}_{\infty} \end{pmatrix} = \prod_{k=1}^{\infty} \begin{pmatrix} \mathbf{3} \\ \prod_{j=1}^{3} \mathcal{C}_{k,j} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v}_{0}^{\perp} \\ \mathbf{v}_{0} \\ \mathbf{n}_{0} \end{pmatrix}$$

Beware ! – Unlike the 1-dimensional case, the analytic expressions of the $C_{k,j}$'s are simply ugly...



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From the circle to the torus

Let $f_{\infty} : \mathbb{T}^2 \longrightarrow \mathbb{E}^3$ be the limit of the maps :

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We have

$$\left(\begin{array}{c} \mathbf{v}_{\infty}^{\perp} \\ \mathbf{v}_{\infty} \\ \mathbf{n}_{\infty} \end{array}\right) = \prod_{k=1}^{\infty} \left(\prod_{j=1}^{3} \mathcal{C}_{k,j}\right) \cdot \left(\begin{array}{c} \mathbf{v}_{0}^{\perp} \\ \mathbf{v}_{0} \\ \mathbf{n}_{0} \end{array}\right)$$

Beware ! – Unlike the 1-dimensional case, the analytic expressions of the $C_{k,j}$'s are simply ugly... but fortunately, their asymptotic expressions are nice.


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Loss of derivatives

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• In the one dimensional setting we have :

$$f(t) := f_0(0) + \int_0^t r(u) \mathbf{e}^{i\alpha(u)\cos 2\pi N u} \, \mathrm{d}u.$$

where
$$\mathbf{e}^{i\theta} := \cos \theta \mathbf{t} + \sin \theta \mathbf{n}$$
 and $\mathbf{t} := \frac{f'_0}{\|f'_0\|}$.



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Loss of derivatives

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• In the one dimensional setting we have :

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where
$$\mathbf{e}^{i\theta} := \cos \theta \mathbf{t} + \sin \theta \mathbf{n}$$
 and $\mathbf{t} := \frac{f'_0}{\|f'_0\|}$.

• In particular

$$\frac{\partial f}{\partial t}(t) = r(t) \mathbf{e}^{i\alpha(t)\cos 2\pi Nt},$$

therefore, if f_0 is C^k then f is C^k also.



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Loss of derivatives

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• In the two dimensional setting we have :

$$f(t,s) := f_0(0,s) + \int_0^t r(u,s) \mathbf{e}^{i lpha(u,s) \cos 2\pi N u} \, \mathrm{d}u + \mathrm{gluing \ term}$$

where $\mathbf{e}^{i\theta} := \cos\theta \mathbf{t} + \sin\theta \mathbf{n}$ with

$$\mathbf{t} := \frac{\partial_t f_0}{\|\partial_t f_0\|} \quad \text{and} \quad \mathbf{n} := \frac{\partial_t f_0 \wedge \partial_s f_0}{\|\partial_t f_0 \wedge \partial_s f_0\|}$$



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Loss of derivatives

• In the two dimensional setting we have :

$$f(t,s) := f_0(0,s) + \int_0^t r(u,s) \mathbf{e}^{i lpha(u,s) \cos 2\pi N u} \, \mathrm{d}u + \mathrm{gluing \ term}$$

where $\mathbf{e}^{i\theta} := \cos\theta \mathbf{t} + \sin\theta \mathbf{n}$ with

$$\mathbf{t} := \frac{\partial_t f_0}{\|\partial_t f_0\|} \quad \text{and} \quad \mathbf{n} := \frac{\partial_t f_0 \wedge \partial_s f_0}{\|\partial_t f_0 \wedge \partial_s f_0\|}$$

• The integral over the variable *t* can not recover the loss of derivative due to the presence of the partial derivative $\partial_s f$ in the definition of **n**. Therefore if f_0 is C^k then, generically, *f* is C^{k-1} only.



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Corrugation Theorem

Corrugation Theorem (~, Jabrane, Lazarus, Thibert).– For every $p \in \mathbb{T}^2$, we have

$$\mathcal{C}_{k,j+1}(p) = \mathcal{L}_{k,j+1}(p) \cdot \mathcal{R}_{k,j}(p)$$

where

$$\mathcal{L}_{k,j+1} := \begin{pmatrix} \cos \theta_{k,j+1} & 0 & \sin \theta_{k,j+1} \\ 0 & 1 & 0 \\ -\sin \theta_{k,j+1} & 0 & \cos \theta_{k,j+1} \end{pmatrix} + O\left(\frac{1}{N_{k,j+1}}\right)$$

with $\theta_{k,j+1}(p) := \alpha_k(p) \cos(2\pi N_k X_{j+1})$ and

$$\mathcal{R}_{k,j} := egin{pmatrix} -\sineta_j & -\coseta_j & 0\ \coseta_j & -\sineta_j & 0\ 0 & 0 & 1 \end{pmatrix} + O(\Omega_{k,j})$$

where $\Omega_{k,j} = \|\langle ., . \rangle_{\mathbb{E}^2} - f_{k,j}^* \langle ., . \rangle_{\mathbb{E}^3} \|$ is the isometric default.

It is time for pictures !



The new CSCS (Swiss National Supercomputing Centre) building in Lugano-Cornaredo.

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Pictures !

A wide-angle view



As an arch



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"Cinq colonnes à la une"



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In the belly of the beast



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Flexible electrical conduits



The truck suspension

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Velodrome

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The landing



Process

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Pictures !

The rope

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YouTube : Flat Torus



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A part of the initial torus of revolution

Zoom!



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First integration : 8 oscillations



Zoom!



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Zoom!



Second integration : 64 oscillations



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Zoom in on the second integration

Zoom!

Zoom!

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More closely



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Third integration : 4096 oscillations



Third integration : 4096 oscillations

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Zoom!

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Zoom in on the third integration



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More closely



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The fourth integration : 524 288 oscillations



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The fourth integration : 524 288 oscillations



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The fourth integration : 524 288 oscillations



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The fifth integration : 2 097 152 oscillations



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The fifth integration : 2 097 152 oscillations


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The fifth integration : 2 097 152 oscillations



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The sixth integration : 16 777 216 oscillations

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Process

Pictures !

Zoom!

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Zoom in on the sixth integration



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Pictures !



Zoom in on the sixth integration

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Zoom!



The seventh integration : 536 870 912 oscillations







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"Dites 33 !"

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"Dites 33 !"

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Artefacts



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Artefacts



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Corrugations in real life?



An ammonite : Douvilleiceras Mammillatum

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Corrugations in real life?



A whelk

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Implementing the Convex Integration Process

Flat 2-Tori in \mathbb{R}^3

V.Borrelli

C¹ Fractals



Implementing the Convex Integration Process

C¹ Fractals

Pictures !

Corrugations in real life?



Close up picture of a whelk



Implementin the Convex Integration Process

C¹ Fractals

Pictures !

Corrugations in real life?

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dogmaticequation 26 Apr 2012 12:09 AM 5

Leave it to the French to try and pass off baked goods as an advancement in science. promoted by FrankN.Stein





Implementin the Convex Integration Process

C¹ Fractals

Pictures !

The Hevea Team

