Numerical Modeling of Immiscible Compressible Two-Phase Flow in Porous Media by the Concept of Global Pressure

B. Amaziane, **M. Jurak**, **A. Žgaljić-Keko**
IPRA–LMA de Pau CNRS-UMR 5142
brahim.amaziane@univ-pau.fr

Journée MoMaS : Modélisation des Ecoulements Diphasiques Liquide-Gaz en Milieux Poreux : Cas Tests et Résultats

IHP Paris, 23 Septembre 2010

Outline

1. Two-phase immiscible, compressible flow equations
2. Global pressure formulation
3. Compressible flow: Fully equivalent global pressure formulation
4. Treatment of multiple rock types
5. Numerical scheme
6. Results of the BOBG test problem

Flow equations

Mass conservation: for $\alpha \in \{w, g\}$,

$$\Phi \frac{\partial}{\partial t}(\rho_\alpha S_\alpha) + \operatorname{div}(\rho_\alpha \mathbf{q}_\alpha) = \mathscr{F}_\alpha,$$

The Darcy-Muscat law: for $\alpha \in \{w, g\}$ (gravity neglected),

$$\mathbf{q}_\alpha = -\lambda_\alpha(S_\alpha)\mathbb{K}\nabla p_\alpha,$$

Capillary law: $p_c(S_w) = p_g - p_w,$
No void space: $S_w + S_g = 1.$

Water is incompressible $\rho_w = cte$, gas is compressible $\rho_g(p_g) = c_g p_g$.

Global pressure formulation

Goal: Reformulate the flow equations in order to

- Make the coupling between the two differential equations less stronger.
- Give to the system a well defined mathematical structure.

We consider:

- Compressible flow: Fully equivalent formulation
- Introducing the total flow: $\mathbf{Q}_t = \rho_w \mathbf{q}_w + \rho_g \mathbf{q}_g$: leads to conservative form of the equations.

Compressible flow: equations

Rewrite the two-phase flow equations as (keep conservative form):

Total flow: ($\mathbf{Q}_t = \rho_w \mathbf{q}_w + \rho_g \mathbf{q}_g$)

$$\mathbf{Q}_t = -\lambda(S_w, p_g)\mathbb{K}\left(\nabla p_g - f_w(S_w, p_g)\nabla p_c(S_w)\right),$$

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}\left(S_w \rho_w + (1 - S_w)\rho_g(p_g)\right) + \mathrm{div}\left(\mathbf{Q}_t\right) = \mathscr{F}_w + \mathscr{F}_g,$$

Water mass conservation:

$$\Phi \rho_w \frac{\partial S_w}{\partial t} + \mathrm{div}(f_w(S_w, p_g)\mathbf{Q}_t) = \mathrm{div}(\mathbb{K}a(S_w, p_g)\nabla S_w) + \mathscr{F}_w.$$

Compressible case: coefficients

phase mobilities $$\lambda_w(S_w) = \frac{kr_w(S_w)}{\mu_w}, \quad \lambda_g(S_w) = \frac{kr_g(S_w)}{\mu_g},$$

total mobility $$\lambda(S_w, p_g) = \rho_w \lambda_w(S_w) + \rho_g(p_g)\lambda_g(S_w),$$

water fractional flow $$f_w(S_w, p_g) = \frac{\rho_w \lambda_w(S_w)}{\lambda(S_w, p_g)},$$

"diffusivity" coeff. $$a(S_w, p_g) = -\rho_w \rho_g(p_g) \frac{\lambda_w(S_w)\lambda_g(S_w)}{\lambda(S_w, p_g)} p_c'(S_w).$$

Compressible flow: decoupling

In the total flow eliminate the saturation gradient:

$$\mathbf{Q}_t = -\lambda(S_w,p_g)\mathbb{K}\left(\nabla p_g - f_w(S_w,p_g)p_c'(S_w)\nabla S_w\right),$$

- Idea: introduce a new pressure-like variable that will eliminate ∇S_w term (*Chavent (1976)*, *Antontsev-Monakhov (1978)*)
- Introduce a new pressure variable p, called global pressure, such that $p_g = \pi(S_w,p)$. Find functions $\pi(S_w,p)$ and $\omega(S_w,p)$ that satisfy:

$$\nabla p_g - f_w(S_w,\pi(S_w,p))p_c'(S_w)\nabla S_w = \omega(S_w,p)\nabla p \qquad (1)$$

Compressible flow: global pressure

1.

$$\begin{cases} \dfrac{d\pi(S,p)}{dS} = \dfrac{\rho_w \lambda_w(S) p_c'(S)}{\rho_w \lambda_w(S) + \rho_g(\pi(S,p))\lambda_g(S)}, & 0 < S < 1 \\ \pi(1,p) = p. \end{cases}$$

2.

$$\omega(S_w,p) = \exp\left(\int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) \pi(s,p))^2} p_c'(s)\, ds \right),$$

3.

$$p_w \le p \le p_g.$$

Compressible flow: transformed equations

Variables: p (global pressure) and S_w (water saturation).

Total flow:

$$\mathbf{Q}_t = -\lambda^n(S_w,p)\omega(S_w,p)\mathbb{K}\nabla p.$$

Total mass conservation:

$$\Phi\frac{\partial}{\partial t}(S_w\rho_w + c_g(1-S_w)\pi(S_w,p)) + \mathrm{div}\mathbf{Q}_t = \mathscr{F}_w + \mathscr{F}_g.$$

Water mass conservation:

$$\Phi\rho_w\frac{\partial S_w}{\partial t} + \mathrm{div}(f_w^n(S_w,p)\mathbf{Q}_t) = \mathrm{div}(\mathbb{K}a^n(S_w,p)\nabla S_w) + \mathscr{F}_w.$$

These equations are fully equivalent to original equations.

Compressible flow: Coefficients

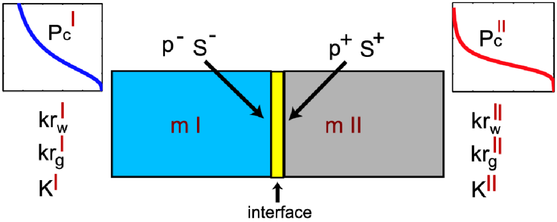
New coefficients are obtained from the old ones by replacing gas pressure p_g by $\pi(S_w, p)$:

total mobility	$\lambda^n(S_w, p) = \lambda(S_w, \pi(S_w, p)),$
water fractional flow	$f_w^n(S_w, p) = f_w(S_w, \pi(S_w, p)),$
"diffusivity" coeff.	$a^n(S_w, p) = a(S_w, \pi(S_w, p)).$

Multiple rock types

For simplicity, we are assuming situations when $p_c(1) = 0$.

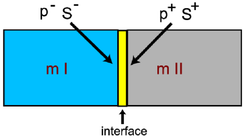
- Different porosity, permeability, and two-phase flow function parameters at each rock type.
- Capillary and phase pressures are continuous at the interface point.
- The consequence is: Saturation and global pressure are discontinuous at the interface.



Interface conditions

- The limit values of the saturation are connected through the equation

$$p_c^{I}(S^-) = p_c^{II}(S^+)$$
$$\Downarrow$$
$$S^+ = (p_c^{II})^{-1}(p_c^{I}(S^-))$$



- The limit values of the global pressure are connected through the equation

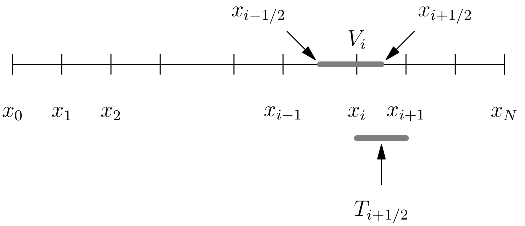
$$P_g^- = P_g^{I}(S^-, p^-) = P_g^{II}(S^+, p^+)$$

After S^+ is found the p^+ is solution of the following nonlinear equation

$$P_g^{II}(S^+, p^+) - P_g^- = 0$$

Vertex centered finite volume method

- Time interval $J = [0, T]$
  - $J = \cup_k J_k$, $J_k := [t_k, t_{k+1}]$, $k = 0, 1, \ldots, N_T$.
- Spatial domain $I = [x_0, x_N]$



$V_i = (x_{i-1/2}, x_{i+1/2})$

$T_{i+1/2} = (x_i, x_{i+1})$

  - $\triangle x_i := x_{i+1} - x_i$
  - control volumes $V_i = \left[x_{i-1/2}, x_{i+1/2}\right]$, $i = 0, 1, \ldots, N$.
  - $x_{-1/2} := x_0$, $x_{N_x+1/2} := x_N$,
- p_i^k (resp. S_i^k) is approximation of $p(x_i, t_k)$, (resp. $S(x_i, t_k)$).

Discretization

Integrating the sytem over the set $V_i \times J_k$ we obtain the following implicit numerical scheme, m is material number (I or II).

$$|\Phi V_i| \frac{(M^m)_i^{k+1} - (M^m)_i^k}{\Delta t_k} = R_{p,i}^{k+1} \tag{2}$$

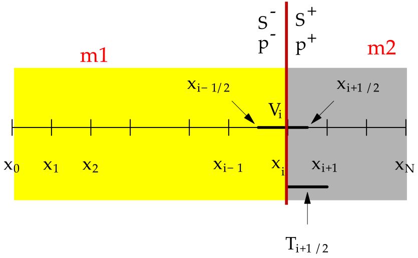
$$|\Phi V_i| \frac{(N^m)_i^{k+1} - (N^m)_i^k}{\Delta t_k} = R_{S,i}^{k+1} \tag{3}$$

$M := M(S,p) = \rho_w(p_w)S + \rho_g(p_g)(1-S)$ and $N := N(S,p) := \rho_w S$.
and for $i = 1, \ldots, N-1$

$$R_{p,i}^{k+1} = (\lambda^m \omega^m)_{i+\frac{1}{2}}^{k+1} T_{i+\frac{1}{2}} (p_{i+1}^{k+1} - p_i^{k+1}) - (\lambda^m \omega^m)_{i-\frac{1}{2}}^{k+1} T_{i-\frac{1}{2}} (p_i^{k+1} - p_{i-1}^{k+1})$$

$$R_{S,i}^{k+1} = (\alpha^m)_{i+\frac{1}{2}}^{k+1} T_{i+\frac{1}{2}}^m (p_c^m(S_{i+1}^{k+1}) - p_c^m(S_i^{k+1})) - (\alpha^m)_{i-\frac{1}{2}}^{k+1} T_{i-\frac{1}{2}}^m (p_c^m(S_i^{k+1}) - p_c^m(S_{i-1}^{k+1}))$$
$$- q_{i+\frac{1}{2}}^{k+1} (f_w^m)_{i+\frac{1}{2}}^{up,k+1} + q_{i-\frac{1}{2}}^{k+1} (f_w^m)_{i-\frac{1}{2}}^{up,k+1}$$

Interface



- Node x_i of the spatial mesh is situated at the interface point.
- We choose one of the limit saturation and global pressure (for example $S_i^- = S_i, p_i^- = p_i$) to be global variable.
- on element $T_{i-\frac{1}{2}}$ we calculate $S_i^+ = S^+(S_i), p_i^+ = p^+(S_i, p_i)$
- During the Newton procedure we need to calculate derivatives over S_i, p_i.

BOBG test problem - function parameters

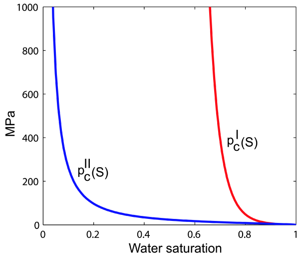
Domain $=[-0.5, 0.5]$, Interface $x = 0.0$

	Pr MPa	m -	A -	B -	C -	D -	ϕ -	k m^2
Ω_1	1.5	0.06	0.25	16.67	1.88	0.5	0.3	10^{-20}
Ω_2	10	0.412	1.0	2.429	1.176	1.0	0.05	10^{-20}

$$S^{\Omega_i}(P_c) = \left(1 + \left(\frac{P_C}{Pr}\right)^{\frac{1}{1-m}}\right)^{-m}$$

$$kr_g(S) = (1-S)^2(1-S^{\frac{5}{3}})$$

$$kr_w(S) = (1 + A(S^{-B} - 1)^C)^{-D}$$

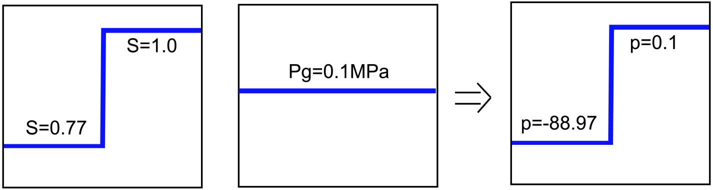


BOBG test problem - boundary and initial conditions

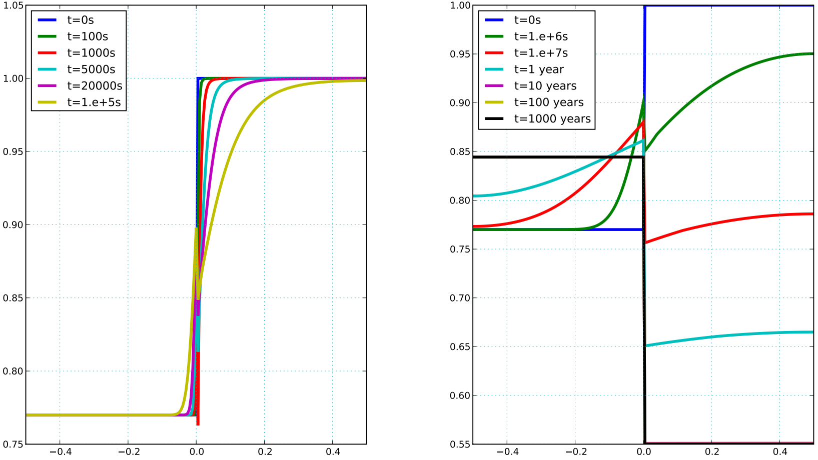
- Fluid properties

ρ_w	c_g	μ_w	μ_n
1000	0.808	1.0	0.018

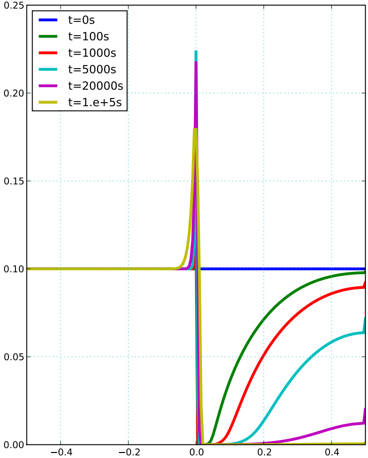
- No flow at the boundaries $q_w = q_g = 0.$
- Inital conditions

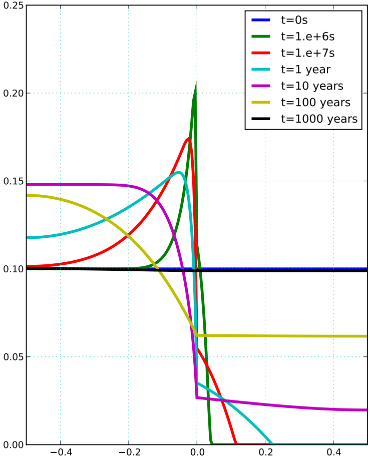


BOBG test problem – Water saturation at different times

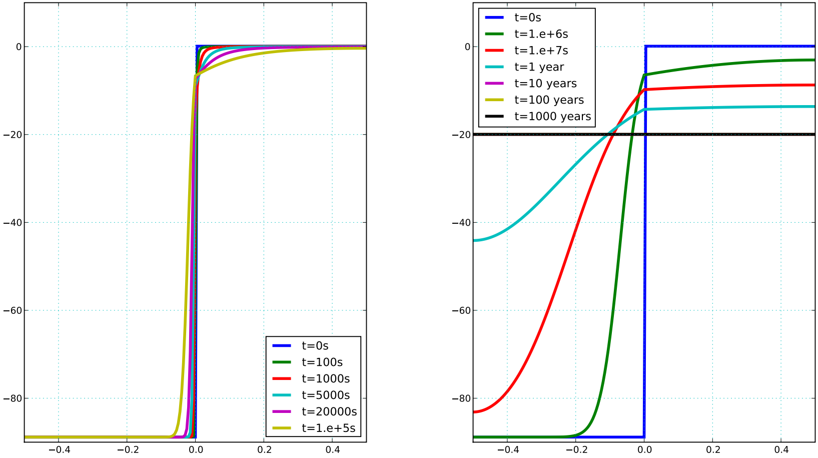


BOBG test problem – Gas pressure at different times

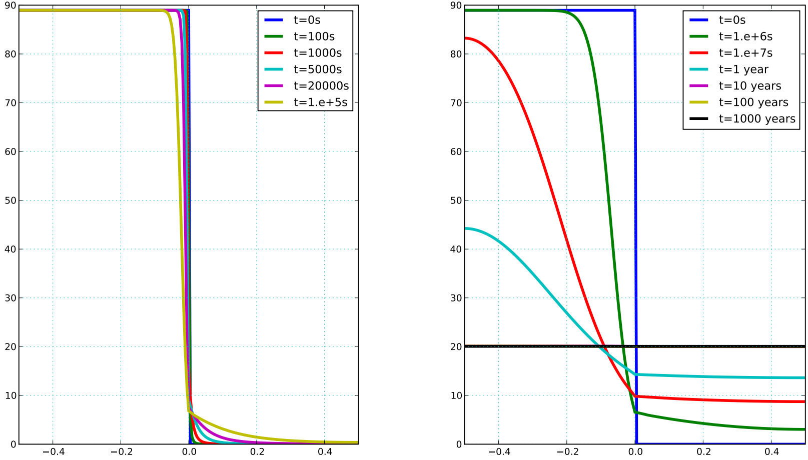




BOBG test problem – Global pressure at different times



Simulation 1 -Capillary pressure at different times



Work in progress & References

- An extension to multiphase, multicomponent models.
- Implementation of FV method in two and three dimensions.
- Treatment of multiple rock types in higher dimensions.

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