Corrections to

Functional Analysis, Calculus of Variations and Optimal Control

by Francis Clarke

(version of February 2017)

Regarding Exer. 1.38. On page 21, the statement of part (c) should have just inclusions; the result should read

$$T_{A\times E}(x,y) \subset T_A(x) \times T_E(y), \quad N_{A\times E}(x,y) \supset N_A(x) \times N_E(y).$$

Accordingly, the parenthetical remark at the end of Exer. 10.33 (p. 212) should now read as follows:

"(When the sets in question are *regular* (see p. 215), for example when they are convex or consist of smooth manifolds with or without boundary, it follows that equality holds in the inclusions noted in Exer. 1.38(c).)"

Page 22. Replace line -9 by: "When $Z = \mathbb{R}$, one may use the adjoint to write instead"

Page 37. Line 9: replace $x_i(a)$ and $x(a)$ by $x_i(0)$ and $x(0)$.

Page 80. In Exer. 5.13, replace "subsequence" by "sequence".

Page 81. In the proof of Prop. 5.15, in the first displayed formula, delete the term $\|x\|$ on the right.

Page 84. Line -10: replace $f(v)$ by $f(z)$.

Page 107. In line 6, on the extreme right, replace a and b by their absolute values.

Page 118. Line -2, replace $+i^{-1}$ by $-i^{-1}$.

Page 123. In line 5, W is a subset of $\mathbb{R}$, not $\mathbb{R}^n$.

Page 124. In line 8, say that Ω is a *bounded* open subset of $\mathbb{R}^m$. This is needed later (on p. 127) when invoking the weak convergence of the sequence z_i, in order to have $L^\infty(\Omega,\mathbb{R}^n) \subset L^{r_*}(\Omega,\mathbb{R}^n)$.

Page 125. Line 16, replace $z \in \mathbb{R}^n$ by $p \in \mathbb{R}^n$; in line -10, replace $\mathbb{R}^m$ by $\mathbb{R}^n$.

Page 127. In lines 8, -2, and -1, replace δ by δ/r. Oh, and in line -8, there's a needless space before the apostrophe in Fatou's.

Page 134. To the statement of Theorem 7.2, for the sake of clarity, let us add the sentence: "Conversely, every $u \in X$ generates an element ζ of X^* in this way."

Page 137. In line -14, replace $\langle v, u \rangle$ by $\langle v, u_1 \rangle$

Page 139. Line -11: refer to Prop. 7.16, not Exer. 7.16.

Page 148. Line 6, replace $y, z \in B(x, \delta)$ by $u \in B(x, \delta)$.

Page 168, Exer. 8.56. In the statement of Lemma 1, add the words "in P" after the phrase "extreme sets".

Page 213. Line -7, replace $\partial_C f(x)$ by $\partial d_S(x)$.

Page 233. Line -15, replace x^2 by y^2; in line -8, add the closing parenthesis to $\partial_P f(0,0)$.

Page 241. Line -10, replace $t(y-x)$ by $t(z-x)$.

Page 248. Line 6: replace the second $\langle \zeta_i, \alpha \rangle$ by $\langle \zeta_i, \alpha_i \rangle$; replace line 13 by

$$V(\alpha_i) = f(x_i) + |x_i - x_*|^2.$$

Page 250. The reference to Exer. 1.38(c) (near the middle) should be to Exer. 1.41 instead.

Page 262. In line 2, replace $\forall t \geqslant 0$ by $\forall t \in [0, T]$.

Page 269. Replace the text between "We define" on line 15 and "invariant." on line 22 by the following:

We define a multifunction $\widetilde{F}(u)$ on $B(x, \rho)$ that equals $\{f(u)\}$ when u lies in $B^\circ(x, \rho)$, and otherwise equals $\operatorname{co}\{0, f(u)\}$. We also define the closed set $\widetilde{S} = S \cap B(x, \rho)$. It is routine to check that $\widetilde{F}$ satisfies Hypothesis 12.1 on $\widetilde{S}$. The strong invariance of (S, F) can be used to show that the system $(\widetilde{S}, \widetilde{F})$ is weakly invariant. The argument, whose details are omitted, calls upon Cor. 12.4 for initial points α in S that lie in $B^\circ(x, \rho)$, and for initial points α on the boundary of $B^\circ(x, \rho)$, the observation that $0 \in \widetilde{F}(\alpha)$.

Page 296. Line -5: at the beginning, replace $-2[$ by $-2K[$.

Page 326. In line 7, the reference should be to "Exer. 14.24" rather than "the second problem of Exer. 14.23".

Page 329. Let us add to the remark at the bottom of the page the following sentence: "In fact, this can be further improved upon; see the remark following Theorem 18.13 (page 364)."

Page 330. Line -6: put x_* for x.

Page 331. Line -6: we invoke Theorem 6.32, not 6.31.

Page 334. Exer. 16.22 : in the integral to be minimized, the term $x'(t)^2$ should be multiplied by $\frac{1}{2}$.

Page 340. Line -10: replace x'' by x'.

Page 341. Line 7: replace $\psi(x)$ by $\psi(u)$.

Page 342. Line 6, replace $n-k$ by k; in lines 10 and -14, replace ϕ by φ.

Page 352. In lines 9, 10, and 16, the range of m should be given by $m = 0, 1, \ldots n$.

Page 353. Line -1: p_i should be p_i'.

Page 376. In the first line, $v + 2\sqrt{x}$ should be $v - 2\sqrt{x}$.

Page 382. For clarity's sake, Exer. 19.8 should have an initial phrase recalling that the continuity of H is being assumed at all times (see p.380). Then the reference in (b) to that continuity becomes superfluous.

Page 388. Lemma 1: the inequality stated in the conclusion is incorrect, and not required; just $|x(t)| \leqslant L$ will do for later purposes. One must redefine Δ and Δ^+ accordingly on page 389.

Page 390. The displayed formula of lines 9 to 10 should read as follows:

$$\begin{aligned} u\big(\tau - t, \tilde{x}(t)\big) + \int_0^t \Lambda\big(\tilde{x}(s), -\tilde{x}'(s)\big)\, ds &\leqslant \varphi\big(\tau - t, \tilde{x}(t), y(t)\big) \\ &\leqslant \varphi(\tau, \beta, 0) = u(\tau, \beta). \end{aligned}$$

In the next line, replace $t = t - \tau$ by $t = \tau - t$.

Page 394. Line -3, replace "in t" by "in x".

Page 399. Line -1: replace $u_1(z)$ by $u_1(x)$.

Page 403. In line 9, the reference should be to Prop. 20.18.

Page 415, Exer. 21.2 Replace $\Lambda(x,v)$ by $\Lambda(v)$.

Page 422, Exer. 21.24 Replace $C^1[a,b]$ by $C^1[0,1]$.

Page 449. Line 6: replace $[\tau_1,\tau_2]$ by $[\tau_2,T]$.

Page 470. In the first line, replace α by r.

Page 492. Line 15: replace $M(t,x(t),u(t))$ by $M(t,x(t),p(t))$.

Page 493. Corollary 24.2 is quite incorrect as stated. (Apologies for the blunder, most likely due to author fatigue.)
It is the concavity of the function $y \mapsto M\big(t, x_*(t)+y,\, p(t)\big)$ that is needed. Thus, the corollary should be replaced by:

"**24.2 Corollary.** Suppose that for almost every t, the function

$$x \mapsto M\big(t, x, p(t)\big)$$

is concave on $B(x_*(t),\delta)$. Then the conclusion of Theorem 24.1 holds if the hypothesis (A*) is replaced by the adjoint inclusion (A) of Theorem 22.26.

Proof. It suffices to verify that condition (A*) holds. When M has the stated concavity property, the function

$$\varphi(x) = (-M)\big(t, x, p(t), u_*(t)\big) + I_{B(x_*(t),\delta)}(x)$$

(for fixed t) is convex. In this case we may write (for almost every t), with the help of Theorem 10.8 and Prop. 10.11:

$$\partial\varphi\big(x_*(t)\big) = \partial_C\,\varphi\big(x_*(t)\big) = -\partial_C(-\varphi)\big(x_*(t)\big) \ni p'(t),$$

by (A). It now follows that (A*) holds: it is simply the subgradient inequality at $x_*(t)$ corresponding to $p'(t)$ and the convex function φ ." $\square$

And the remark that follows the corollary should be replaced by :

"Remark. Note that no convexity of Λ is postulated in Theorem 24.1. It is clear, however, from the definition of H, that the concavity property cited in the corollary will hold if the dynamics of the problem are affine in x and u, Λ is convex in (x,u), and M is finite-valued (see Exer. 8.10). In such cases, then, the maximum principle is rather close to being a sufficient, as well as a necessary, condition. "

The incorrect result is used only in Example 24.3. To fix this, it suffices to replace the last complete sentence on p. 493 by :

"We have $H = pu - |x| - g(|u|)$, from which if follows that M is concave with respect to x."

Page 510. Line -8: replace $x_i'(t)$ by v.

Page 519. Line -7, replace $[c,a]$ by $[c,b]$.

Page 527. In lines -12 and -14, replace $\|p\|$ by $\|p_i\|$.

Page 530. Line -4: replace " $\leqslant r+(1-r)^2 =$" by " $\leqslant 2r+(1-r)^2 =$".

Page 537, Lemma 1 At the end of the statement, in the final inequality, replace K by M.

Page 565. The hint for Exer. 1.42(b) could usefully refer to the new parenthetical remark at the end of Exer. 10.33 (p. 212), as given in the first correction above "Regarding Exer. 1.38."

Page 566, solution to Exer. 4.9. There's a missing factor of meas Ω; the correct inequality is:

$$\varphi\left(\frac{1}{\operatorname{meas}\Omega}\int_\Omega g(x)\,dx\right) \leqslant \frac{1}{\operatorname{meas}\Omega}\int_\Omega \varphi\big(g(x)\big)\,dx \quad \forall g \in L^1(\Omega)\,.$$

In the index, add: "function / convex / strictly, 66" (p. 585), and "strictly convex function, 66" (p. 588).

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