Fiche de TP1

Exercice 1

```
> restart;
> f := X^8+X^6-3*X^4-3*X^3+8*X^2+2*X-5;
> g := 3*X^6+5*X^4-4*X^2-9*X+21;
```

$$f := X^8 + X^6 - 3\,X^4 - 3\,X^3 + 8\,X^2 + 2\,X - 5$$

$$g := 3\,X^6 + 5\,X^4 - 4\,X^2 - 9\,X + 21 \quad (1)$$

```
> r:= rem(f,g,X,'q');q;
```

$$r := -\frac{5}{9}\,X^4 + \frac{1}{9}\,X^2 - \frac{1}{3}$$

$$\frac{1}{3}\,X^2 - \frac{2}{9} \quad (2)$$

```
> evalb(f=expand(g*q+r));
```

$$true \quad (3)$$

```
> Delta:=gcdex(f,g,X,'A','B');A;B;
```

$$\Delta := 1$$

$$\frac{13989}{130354}\,X^5 + \frac{9225}{65177}\,X^4 + \frac{20281}{65177}\,X^3 + \frac{67125}{130354}\,X^2 + \frac{5149}{130354}\,X - \frac{1391}{18622}$$

$$-\frac{4663}{130354}\,X^7 - \frac{3075}{65177}\,X^6 - \frac{5206}{65177}\,X^5 - \frac{18275}{130354}\,X^4 + \frac{4944}{65177}\,X^3 + \frac{21579}{130354}\,X^2 + \frac{1910}{65177}\,X + \frac{3889}{130354} \quad (4)$$

```
> evalb(Delta=expand(A*f+B*g));
```

$$true \quad (5)$$

```
> R:=resultant(f,g,X);
```

$$R := 260708 \quad (6)$$

```
> a,b:= R*A,R*B;
```

$$a, b := 27978\,X^5 + 36900\,X^4 + 81124\,X^3 + 134250\,X^2 + 10298\,X - 19474,\ -9326\,X^7 - 12300\,X^6 - 20824\,X^5 - 36550\,X^4 + 19776\,X^3 + 43158\,X^2 + 7640\,X + 7778 \quad (7)$$

```
> evalb(R=expand(a*f+b*g));
```

$$true \quad (8)$$

Exercice 2

```
> restart;
> f:=t*X^2+3*X-1;
```

$$f := X^2\,t + 3\,X - 1 \quad (9)$$

```
> g:=6*X^2+t^2-4;
```

$$g := 6\,X^2 + t^2 - 4 \quad (10)$$

```
> R:=sort(resultant(f,g,X),t);
```

$$R := t^6 - 8\,t^4 + 12\,t^3 + 70\,t^2 - 48\,t - 180 \quad (11)$$

```
> f0:=subs(t=0,f):g0:=subs(t=0,g): #on spécialise en t=0, le degré
  de f chute de p=1
> R0:=resultant(f0,g0,X);subs(t=0,R); # R(0)=lcoeff(g,X)^p*R0
```

$$R0 := -30$$

$$-180 \quad (12)$$

```
> restart;
> f:=t*X^2+3*t*X-1;
```

$$f := X^2 t + 3 X t - 1 \quad (13)$$

```
> g:=6*X^2+t^2-4;
```

$$g := 6 X^2 + t^2 - 4 \quad (14)$$

```
> R:=sort(resultant(f,g,X),t);
```

$$R := t^6 + 46 t^4 + 12 t^3 - 200 t^2 - 48 t + 36 \quad (15)$$

```
> f0:=subs(t=0,f):g0:=subs(t=0,g): #on spécialise en t=0, le degré
  de f chute de p=2
> R0:=resultant(f0,g0,X);subs(t=0,R); # R(0)=lcoeff(g,X)^p*R0
```

$$R0 := 1$$

$$36 \quad (16)$$

```
> restart;
> f:=t*X^2+X-1;
```

$$f := X^2 t + X - 1 \quad (17)$$

```
> g:=t*X^2+X+t^2-4;
```

$$g := X^2 t + t^2 + X - 4 \quad (18)$$

```
> R:=sort(resultant(f,g,X),t);
```

$$R := \left(t^2 - 3\right)^2 t^2 \quad (19)$$

```
> f0:=subs(t=0,f):g0:=subs(t=0,g): #on spécialise en t=0, les
  degrés de f rt de g chutent
> R0:=resultant(f0,g0,X);subs(t=0,R);
```

$$R0 := -3$$

$$0 \quad (20)$$

Exercice 3

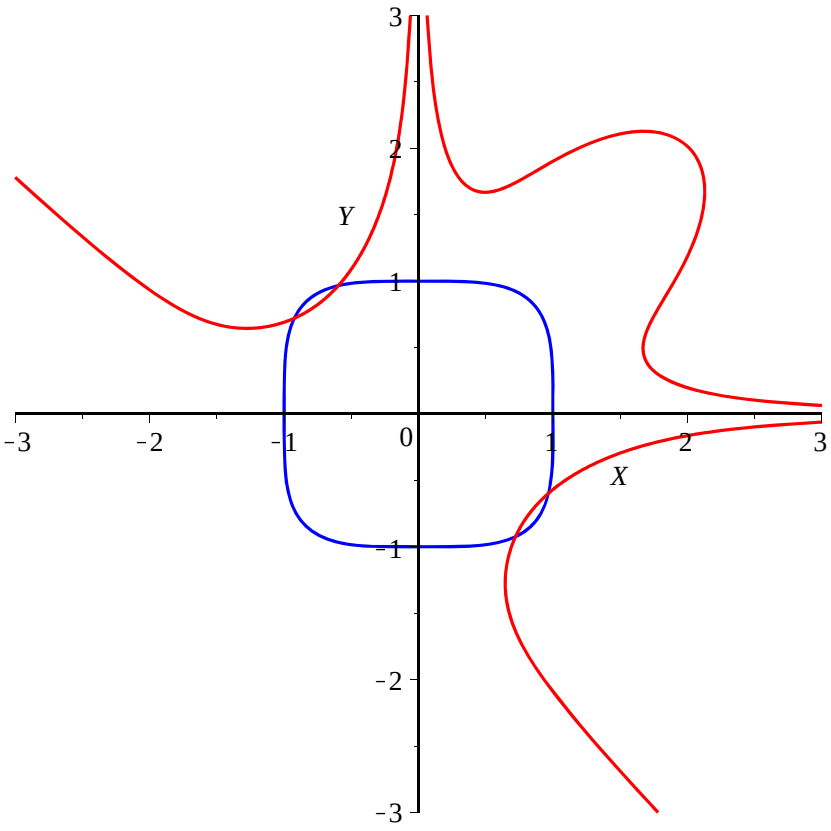
```
> restart;
> with(plots):
> f:=X^4+Y^4-1;
```

$$f := X^4 + Y^4 - 1 \quad (21)$$

```
> g:=X^5*Y^2-4*X^3*Y^3+X^2*Y^5-1;
```

$$g := X^5 Y^2 + X^2 Y^5 - 4 X^3 Y^3 - 1 \quad (22)$$

```
> D1:=implicitplot(f,X=-3..3,Y=-3..3,grid=[200,200],color=blue):
> D2:=implicitplot(g,X=-3..3,Y=-3..3,grid=[200,200],color=red):
> display({D1,D2});
```




```
> R:=resultant(f,g,Y); # le degré de f en Y ne chute pas: équation
  de la projection
     # de l'intersection des courbes sur l'axe des X
```

$$R := 2X^{28} - 16X^{27} + 32X^{26} + 249X^{24} + 48X^{23} - 128X^{22} + 4X^{21} - 757X^{20} - 112X^{19} + 192X^{18} - 12X^{17} + 758X^{16} + 144X^{15} - 126X^{14} + 28X^{13} - 251X^{12} - 64X^{11} + 30X^{10} - 36X^{9} - X^{8} + 16X^{5} + 1 \quad (23)$$

```
> x:=fsolve(R,X); # les ordonnées approchées des points
  d'intersection
```

$$x := -0.9242096683, -0.5974289870, 0.7211133862, 0.9665062969 \quad (24)$$

```
> # il y a symétrie de l'intersection par rapport à la 1ere
  bissectrice puisque
  # les polynômes f et g sont symétriques.
> with(combinat, cartprod):
> pt:=[]:T:=cartprod([[x], [x]]):
> while not T[finished] do
>     a,b:=op(T[nextvalue]());
>     if abs(subs({X=a,Y=b},f))<10^(-7) and abs(subs({X=a,Y=b},g))
  <10^(-8) then
>                pt:=[op(pt),[a,b]] end if;
>   end do:
```

```
> pt; # les coordonnées approchées des points d'intersection
[[-0.9242096683, 0.7211133862], [-0.5974289870, 0.9665062969], [0.7211133862,
    -0.9242096683], [0.9665062969, -0.5974289870]]
```

(25)

```
>
>
>
>
```