TP1

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Exercice 1

```
A = PolynomialRing(QQ,'X')
X = A.gen()
A
```

```
Univariate Polynomial Ring in X over Rational Field
```

```
f = X^8+X^6-3*X^4-3*X^3+8*X^2+2*X-5
g = 3*X^6+5*X^4-4*X^2-9*X+21
f;g
```

```
X^8 + X^6 - 3*X^4 - 3*X^3 + 8*X^2 + 2*X - 5
3*X^6 + 5*X^4 - 4*X^2 - 9*X + 21
```

```
q,r = f.quo_rem(g)
q,r
```

```
(1/3*X^2 - 2/9, -5/9*X^4 + 1/9*X^2 - 1/3)
```

```
f == q*g+ r
```

```
True
```

```
D, A, B = f.xgcd(g)
D;A;B
```

```
1
13989/130354*X^5 + 9225/65177*X^4 + 20281/65177*X^3 + 67125/13035
5149/130354*X - 1391/18622
-4663/130354*X^7 - 3075/65177*X^6 - 5206/65177*X^5 - 18275/130354
4944/65177*X^3 + 21579/130354*X^2 + 1910/65177*X + 3889/130354
```

```
D == A*f+B*g
```

```
True
```

```
R = f.resultant(g)
R
```

```
260708
```

```
a, b = R*A, R*B
a,b
```

```
(27978*X^5 + 36900*X^4 + 81124*X^3 + 134250*X^2 + 10298*X - 19474
-9326*X^7 - 12300*X^6 - 20824*X^5 - 36550*X^4 + 19776*X^3 + 43158
7640*X + 7778)
```

```
R == a*f+ b*g
```

```
True
```

Exercice 2

```
A = PolynomialRing(ZZ,'t')
t = A.gen()
A
```

```
Univariate Polynomial Ring in t over Integer Ring
```

```
R = PolynomialRing(A,'X')
X = R.gen()
R
```

```
Univariate Polynomial Ring in X over Univariate Polynomial Ring i
over Integer Ring
```

1.

```
f=t* X^2+3*X-1
g=6*X^2+t^2-4
f,g
```

```
(t*X^2 + 3*X - 1, 6*X^2 + t^2 - 4)
```

```
f.variables()
```

```
(X,)
```

```
R = f.resultant(g)
R
```

```
t^6 - 8*t^4 + 12*t^3 + 70*t^2 - 48*t - 180
```

```
f(t=0).resultant(g(t=0)),R(t=0)
```

```
(-30, -180)
```

2.

```
f=t*X^2+3*t*X-1
 g=6*X^2+t^2-4
 f,g
```

```
R = f.resultant(g)
R
```

```
t^6 + 46*t^4 + 12*t^3 - 200*t^2 - 48*t + 36
```

```
f(t=0).resultant(g(t=0)),R(t=0)
```

```
(1, 36)
```

3.

```
f=t*X^2+X-1
g=t*X^2+X+t^2-4
f,g
```

```
(t*X^2 + X - 1, t*X^2 + X + t^2 - 4)
```

```
R = f.resultant(g)
R
```

```
t^6 - 6*t^4 + 9*t^2
```

```
f(t=0).resultant(g(t=0)),R(t=0)
```

```
(-3, 0)
```

Exercice 3

```
A = PolynomialRing(QQ,'X,Y')
X,Y = A.gens()
A
```

```
Multivariate Polynomial Ring in X, Y over Rational Field
```

```
f=X^4+Y^4-1
f
```

```
X^4 + Y^4 - 1
```

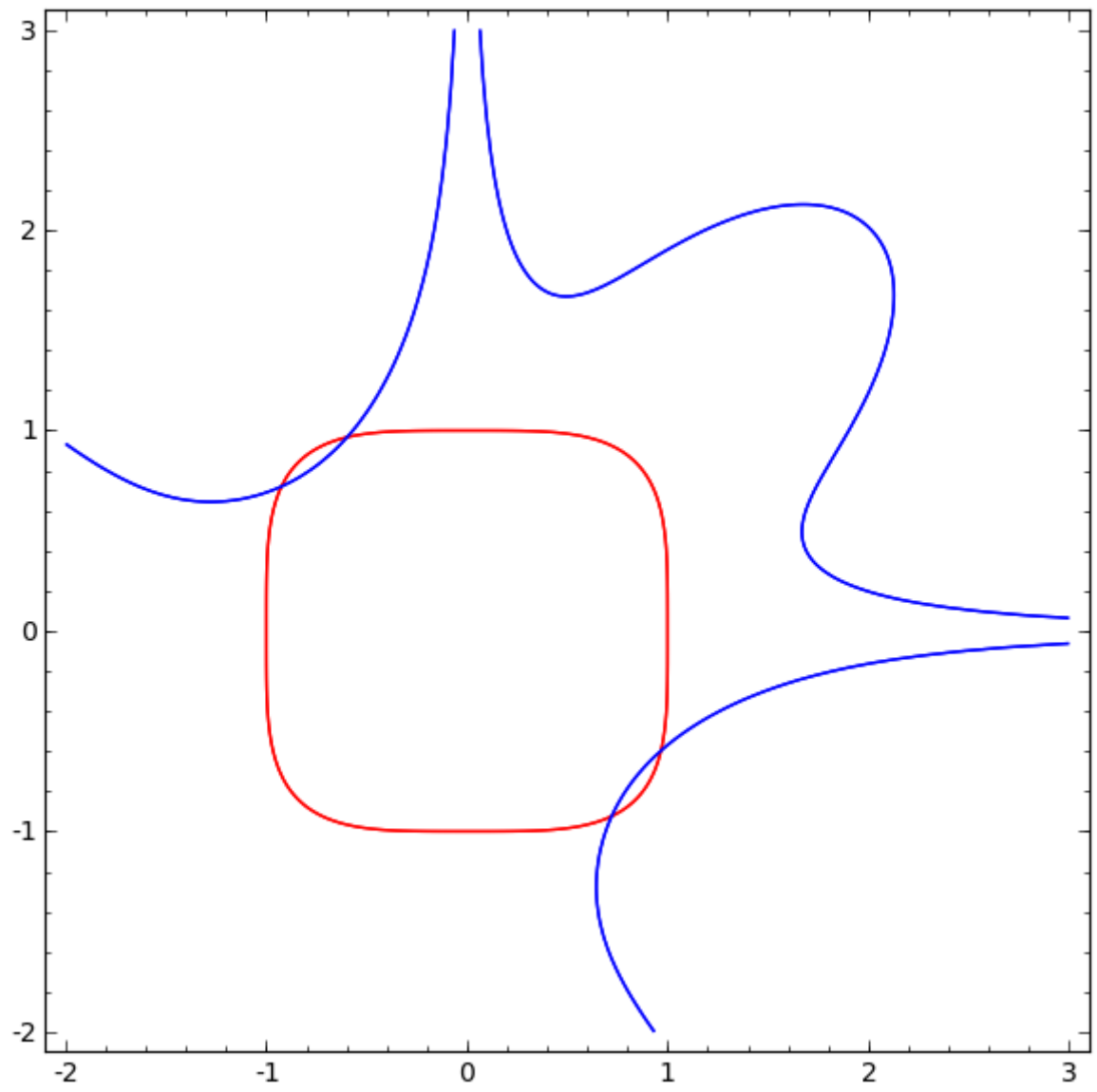
```
g=X^5*Y^2-4*X^3*Y^3+X^2*Y^5-1
g
```

```
X^5*Y^2 + X^2*Y^5 - 4*X^3*Y^3 - 1
```

```
des1 = implicit_plot(f, (X, -2, 3), (Y, -2, 3), plot_points = 250,
color = 'red')
```

```
des2 = implicit_plot(g, (X, -2, 3), (Y, -2, 3), plot_points = 250,
color = 'blue')
```

```
show(des1+des2)
```




```
RX =f.polynomial(Y).resultant(g.polynomial(Y))
RX
```

```
2*X^28 - 16*X^27 + 32*X^26 + 249*X^24 + 48*X^23 - 128*X^22 + 4*X^
757*X^20 - 112*X^19 + 192*X^18 - 12*X^17 + 758*X^16 + 144*X^15 -
126*X^14 + 28*X^13 - 251*X^12 - 64*X^11 + 30*X^10 - 36*X^9 - X^8
16*X^5 + 1
```

```
RX.is_irreducible()
```

```
True
```

```
rac = RX.real_roots()
rac
```

```
[-0.924209668349044, -0.597428986963397, 0.721113386166218,
0.966506296874216]
```

```
P = CartesianProduct(rac,rac)
P
```

```
Cartesian product of [-0.924209668349044, -0.597428986963397,
0.721113386166218, 0.966506296874216], [-0.924209668349044,
-0.597428986963397, 0.721113386166218, 0.966506296874216]
```

```
inter = []
for (a,b) in P:
    if abs(f(X=a,Y=b))<e^(-10) and abs(g(X=a,Y=b))<e^(-10):
        inter.append([a,b])
inter
```

```
[[-0.924209668349044, 0.721113386166218], [-0.597428986963397,
0.966506296874216], [0.721113386166218, -0.924209668349044],
[0.966506296874216, -0.597428986963397]]
```

```
len(inter)
```

```
4
```