FICHE DE TP 2

Exercice 1

```
> restart;
> F := 3*X^5+7*X+6;
```

$$F := 3X^5 + 7X + 6 \quad (1)$$

```
> Delta := discrim(F,X);
```

$$\Delta := 444219984 \quad (2)$$

```
> ifactor(Delta);
```

$$(2)^4\,(3)^3\,(13)\,(83)\,(953) \quad (3)$$

```
> f := F mod 5;
```

$$f := 3X^5 + 2X + 1 \quad (4)$$

```
> discrim(f,X) mod 5; # on utilise la spécialisation à défaut de
  forme inerte du discriminant
```

$$4 \quad (5)$$

```
> m:=degree(f,X);
```

$$m := 5 \quad (6)$$

```
> df:=diff(f,X) mod 5;
```

$$df := 2 \quad (7)$$

```
> k:=m-1-degree(df,X);
```

$$k := 4 \quad (8)$$

```
> R:=Resultant(f,df,X) mod 5;
```

$$R := 2 \quad (9)$$

```
> c:=lcoeff(f,X);
```

$$c := 3 \quad (10)$$

```
> (-1)^(m*(m-1)/2)*c^(k-1)*R mod 5;
```

$$4 \quad (11)$$

Exercice 2

```
> restart;
> f := X^4 -4*X^3-X^2+ 16*X -12;
```

$$f := X^4 - 4X^3 - X^2 + 16X - 12 \quad (12)$$

```
> g := X^3 +2*X^2-X-2;
```

$$g := X^3 + 2X^2 - X - 2 \quad (13)$$

```
> resultant(f,g,X);
```

$$0 \quad (14)$$

```
> gcdex(f,g,X);
```

$$X^2 + X - 2 \quad (15)$$

```
> with(LinearAlgebra):
> S_:=SylvesterMatrix(f,g,X):
> S:=Transpose(S_);
```

$$S := \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & -4 & 1 & -1 & 2 & 1 & 0 \\ 16 & -1 & -4 & -2 & -1 & 2 & 1 \\ -12 & 16 & -1 & 0 & -2 & -1 & 2 \\ 0 & -12 & 16 & 0 & 0 & -2 & -1 \\ 0 & 0 & -12 & 0 & 0 & 0 & -2 \end{bmatrix} \quad (16)$$

```
> H:=Transpose(ReducedRowEchelonForm(S_));
```

$$H := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 21 & -11 & 5 & -3 & 1 & 0 & 0 \\ -22 & 10 & -6 & 2 & -2 & 0 & 0 \end{bmatrix} \quad (17)$$

```
> n,n:=Dimension(H);
```

$$n, n := 7, 7 \quad (18)$$

```
> r:=Rank(H);
```

$$r := 5 \quad (19)$$

```
> [seq(add(Column(H,j)[i]*X^(n-i),i=1..n),j=1..r)];
```

$$[X^6 + 21X - 22, X^5 - 11X + 10, X^4 + 5X - 6, X^3 - 3X + 2, X^2 + X - 2] \quad (20)$$

Exercice 3

```
> restart;
> f := 3*X^5+7*X+6;
```

$$f := 3X^5 + 7X + 6 \quad (21)$$

```
> g := 2*X^5 -3*X^2+X-1;
```

$$g := 2X^5 - 3X^2 + X - 1 \quad (22)$$

```
> resultant(f,g,X);
```

$$-165871 \quad (23)$$

```
> P:=expand(quo(f*subs(X=Y,g)-subs(X=Y,f)*g,X-Y,X));
```

$$P := -9X^4Y^2 - 11X^4Y - 15X^4 - 9X^3Y^3 - 11X^3Y^2 - 15X^3Y - 9X^2Y^4 - 11X^2Y^3 - 15X^2Y^2 + 21XY + 18X - 11XY^4 - 15XY^3 - 13 + 18Y - 15Y^4 \quad (24)$$

```
> with(LinearAlgebra):
> n:=degree(f,X);
```

$$n := 5 \quad (25)$$

```
> B:=Matrix(n,n,(i,j)->coeff(coeff(P,X,n-i),Y,n-j));
```

$$B := \begin{bmatrix} 0 & 0 & -9 & -11 & -15 \\ 0 & -9 & -11 & -15 & 0 \\ -9 & -11 & -15 & 0 & 0 \\ -11 & -15 & 0 & 21 & 18 \\ -15 & 0 & 0 & 18 & -13 \end{bmatrix} \quad (26)$$

```
> Determinant(B);
```

$$-165871 \quad (27)$$

```
> H:=Transpose(ReducedRowEchelonForm(B));
```

$$H := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

```
> BezoutMatrix(f,g,X,method=symmetric);
```

$$\begin{bmatrix} 0 & 0 & 9 & 11 & 15 \\ 0 & 9 & 11 & 15 & 0 \\ 9 & 11 & 15 & 0 & 0 \\ 11 & 15 & 0 & -21 & -18 \\ 15 & 0 & 0 & -18 & 13 \end{bmatrix} \quad (29)$$