TP2

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Exercice 1

```
R = PolynomialRing(ZZ,'X')
X = R.gen()
R
```

```
Univariate Polynomial Ring in X over Integer Ring
```

```
F = 3*X^5+7*X+6
F
```

```
3*X^5 + 7*X + 6
```

```
D = F.discriminant()
D
```

```
444219984
```

```
D.factor()
```

```
2^4 * 3^3 * 13 * 83 * 953
```

```
f = F.change_ring(GF(5))
f
```

```
3*X^5 + 2*X + 1
```

```
f.discriminant()
```

```
4
```

```
D.mod(5)
```

```
4
```

```
m = f.degree()
m
```

```
5
```

```
df = f.derivative()
df
```

```
2
```

```
k = m-1-df.degree()
k
```

```
4
```

```
R = f.resultant(f.derivative())
R
```

```
2
```

```
c = f.leading_coefficient()
c
```

```
3
```

```
(-1)^(m*(m-1)/2)*c^(k-1)*R
```

```
4
```

```
```

Exercice 2

```
A = PolynomialRing(QQ,'X')
X = A.gen()
A
```

```
Univariate Polynomial Ring in X over Rational Field
```

```
f = X^4 -4*X^3-X^2+ 16*X -12
f
```

```
X^4 - 4*X^3 - X^2 + 16*X - 12
```

```
g = X^3 +2*X^2-X-2
g
```

```
X^3 + 2*X^2 - X - 2
```

```
f.resultant(g)
```

```
0
```

```
d, u,v = f.xgcd(g)
d
```

```
X^2 + X - 2
```

```
d == u*f+v*g
```

```
True
```

```
S_ = f.sylvester_matrix(g)
S = S_.transpose()
show(S)
```

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -4 & 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & -4 & 1 & -1 & 2 & 1 & 0 \\ 16 & -1 & -4 & -2 & -1 & 2 & 1 \\ -12 & 16 & -1 & 0 & -2 & -1 & 2 \\ 0 & -12 & 16 & 0 & 0 & -2 & -1 \\ 0 & 0 & -12 & 0 & 0 & 0 & -2 \end{pmatrix}$$

```
H = S_.echelon_form('classical').transpose()
show(H)
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 21 & -11 & 5 & -3 & 1 & 0 & 0 \\ -22 & 10 & -6 & 2 & -2 & 0 & 0 \end{pmatrix}$$

```
col = H.columns()
col
```

```
[(1, 0, 0, 0, 0, 21, -22), (0, 1, 0, 0, 0, -11, 10), (0, 0, 1, 0,
```

```
5, -6), (0, 0, 0, 1, 0, -3, 2), (0, 0, 0, 0, 1, 1, -2), (0, 0, 0,
0, 0, 0), (0, 0, 0, 0, 0, 0, 0)]
```

```
n,n = H.dimensions()
n
```

```
7
```

```
r = H.rank()
r
```

```
5
```

```
[add(col[j][i]*X^(n-1-i) for i in range(0,n))  for j in range(0,r)]
```

```
[X^6 + 21*X - 22, X^5 - 11*X + 10, X^4 + 5*X - 6, X^3 - 3*X + 2, X
+ X - 2]
```

Exercice 3

```
A = PolynomialRing(QQ,'X,Y')
X,Y = A.gens()
A
```

```
Multivariate Polynomial Ring in X, Y over Rational Field
```

```
f = 3*X^5+7*X+6
f
```

```
3*X^5 + 7*X + 6
```

```
g = 2*X^5 -3*X^2+X-1
g
```

```
2*X^5 - 3*X^2 + X - 1
```

```
n = f.degree()
n
```

```
5
```

```
parent(f)
```

```
Multivariate Polynomial Ring in X, Y over Rational Field
```

```
f.polynomial(X).resultant(g.polynomial(X))
```

```
-165871
```

```
P,R = (f*g(X=Y)-f(X=Y)*g).quo_rem(X-Y)
P
```

```
-9*X^4*Y^2 - 9*X^3*Y^3 - 9*X^2*Y^4 - 11*X^4*Y - 11*X^3*Y^2 -
11*X^2*Y^3 - 11*X*Y^4 - 15*X^4 - 15*X^3*Y - 15*X^2*Y^2 - 15*X*Y^3
15*Y^4 + 21*X*Y + 18*X + 18*Y - 13
```

```
parent(P)
```

```
Multivariate Polynomial Ring in X, Y over Rational Field
```

```
P.coefficient({X:n-2,Y:n-4})
```

```
-15
```

```
MS =MatrixSpace(QQ,n,n)
MS
```

```
Full MatrixSpace of 5 by 5 dense matrices over Rational Field
```

```
B = matrix(n,n)
```

```
for i in range(0,n):
    for  j in range(0,n):
        B[i,j] = P.coefficient({X:n-i-1,Y:n-j-1})
show(B)
```

$$\begin{pmatrix} 0 & 0 & -9 & -11 & -15 \\ 0 & -9 & -11 & -15 & 0 \\ -9 & -11 & -15 & 0 & 0 \\ -11 & -15 & 0 & 21 & 18 \\ -15 & 0 & 0 & 18 & -13 \end{pmatrix}$$

```
B.determinant()
```

```
-165871
```

```
H = B.echelon_form().transpose()
show(H)
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 155310 & 102316 & 119292 & 83349 & 165871 \end{pmatrix}$$