TP6

Fiche de TP 6

exercice 1

```
A = PolynomialRing(RationalField(), 3, 'XYZ', order='lex')
A
```

    Multivariate Polynomial Ring in X, Y, Z over Rational Field

```
show(A)
```

$\mathbf{Q}[X, Y, Z]$

```
(X,Y,Z) = A.gens()
```

```
A.base_ring()
```

    Rational Field

```
f1 = X^2+Y^2+Z^2+1;f1
```

    X^2 + Y^2 + Z^2 + 1

```
parent(f1)
```

    Multivariate Polynomial Ring in X, Y, Z over Rational Field

```
f2 = X^2+2*Y^2-Y*Z-1;f2
```

    X^2 + 2*Y^2 - Y*Z - 1

```
f3 = X+Z^3-1;f3
```

    X + Z^3 - 1

```
I = A.ideal(f1,f2,f3);I
```

    Ideal (X^2 + Y^2 + Z^2 + 1, X^2 + 2*Y^2 - Y*Z - 1, X + Z^3 - 1) of
    Multivariate Polynomial Ring in X, Y, Z over Rational Field

```
B = I.groebner_basis()
B
```

    [X + Z^3 - 1, Y - 1/4*Z^11 + Z^8 - 5/4*Z^7 - 2*Z^5 + 5/2*Z^4 -
    5/4*Z^3 + 2*Z^2 - 5/2*Z, Z^12 - 4*Z^9 + 5*Z^8 + 12*Z^6 - 10*Z^5 +
    5*Z^4 - 16*Z^3 + 18*Z^2 + 16]

```
f4 = X^3+2*X*Y-2;f4
```

    X^3 + 2*X*Y - 2

```
J = A.ideal(f1,f2,f3,f4);J
```

    Ideal (X^2 + Y^2 + Z^2 + 1, X^2 + 2*Y^2 - Y*Z - 1, X + Z^3 - 1, X^
    + 2*X*Y - 2) of Multivariate Polynomial Ring in X, Y, Z over
    Rational Field

```
J.groebner_basis()
```

```
[1]
```

```
1 in J
```

```
True
```

exercice 2

```
A = PolynomialRing(RationalField(), 3, 'XYZ', order='lex')
(X,Y,Z) = A.gens()
```

```
f1 = 3*X^2*Y-Y*Z
f2 = X*Y^2+Z^4
f1;f2
```

```
3*X^2*Y - Y*Z
X*Y^2 + Z^4
```

```
I = ideal(f1,f2);I
```

```
Ideal (3*X^2*Y - Y*Z, X*Y^2 + Z^4) of Multivariate Polynomial Ring
in X, Y, Z over Rational Field
```

```
B = I.groebner_basis();B
```

```
[X^2*Y - 1/3*Y*Z, X*Y^2 + Z^4, X*Z^4 + 1/3*Y^2*Z, Y^4*Z - 3*Z^8]
```

```
f = 3*X^4*Z-2*X^3*Y^4+7*X^2*Y^2*Z^2-8*X*Y^3*Z^2
f;
```

```
3*X^4*Z - 2*X^3*Y^4 + 7*X^2*Y^2*Z^2 - 8*X*Y^3*Z^2
```

```
f in I
```

```
False
```

```
r = f.reduce(B);
r
```

```
3*X^4*Z + 2/3*Y^2*Z^5 + 7/3*Y^2*Z^3 + 8*Y*Z^6
```

```
(f-r) in I
```

```
True
```

```
(f-r).lift(B)
```

```
[0, -2*X^2*Y^2 + 7*X*Z^2 - 8*Y*Z^2, 2*X*Y^2 - 2*Z^4 - 7*Z^2, -2/3*
```

exercice 3.

```
A = PolynomialRing(RationalField(), 5, 'UVXYZ', order='lex')
(U,V,X,Y,Z) = A.gens()
```

```
show(A)
```

$\mathbf{Q}[U, V, X, Y, Z]$

```
I = ideal(X-U*V,Y-U*V^2,Z-U^2)
```

```
show(I)
```

$$\left(-UV + X, -UV^2 + Y, -U^2 + Z\right)\mathbf{Q}[U, V, X, Y, Z]$$

```
GB = I.groebner_basis();GB
```

```
[U^2 - Z, U*V - X, U*X - V*Z, U*Y - X^2, V^2*Z - X^2, V*X - Y, V*Y
- X^3, X^4 - Y^2*Z]
```

```
EL = [g for g in GB if U not in g.variables()  and V not in
g.variables()]
EL
```

```
[X^4 - Y^2*Z]
```

```
H = EL[0]
H
```

```
X^4 - Y^2*Z
```

```
var('x,y,z')
```

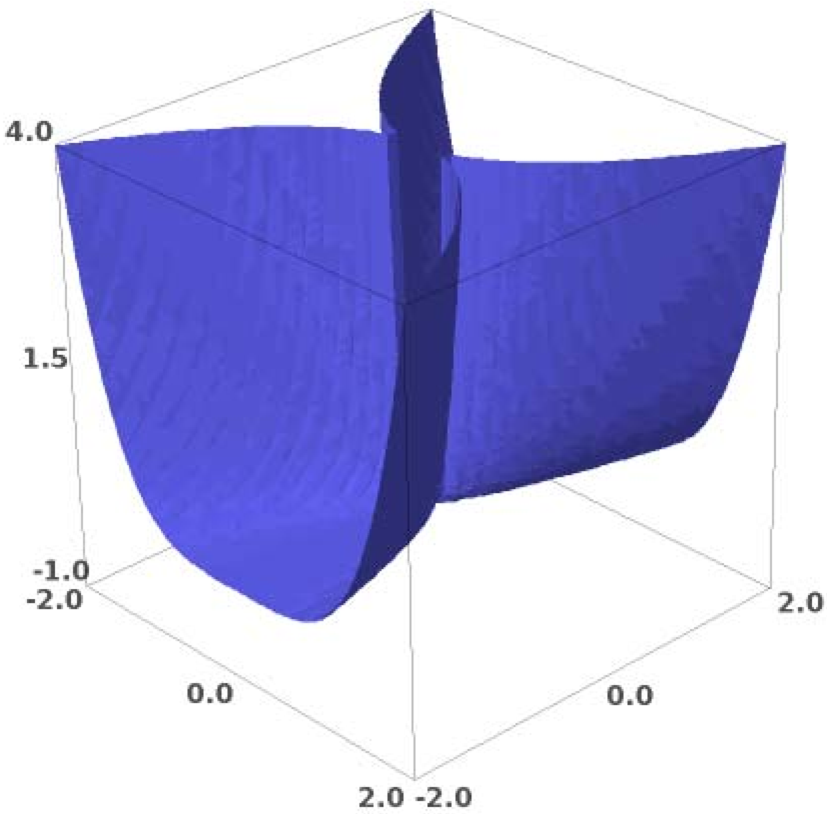
```
(x, y, z)
```

```
h = lambda x,y,z:H(X=x,Y=y,Z=z)
```

```
d = implicit_plot3d(h,(x,-2,2),(y,-2,2),(z,-1,4))
```

```
show(d)
```

Sleeping... Make Interactive



Exercice 4

```
A = PolynomialRing(RationalField(), 3, 'XYZ', order='degrevlex')
(X,Y,Z) = A.gens()
```

```
def S(R,f,g):
   mu = R.monomial_lcm(f.lm(),g.lm())
   return R.monomial_quotient(mu,f.lm())/f.lc()*f-
R.monomial_quotient(mu,g.lm())/g.lc()*g
```

```
G = [Y^2-Z*X,Y*X-Z,X^2-Y]
G
```

```
[Y^2 - X*Z, X*Y - Z, X^2 - Y]
```

```
S01 = S(A,G[0],G[1])
S01
```

```
-X^2*Z + Y*Z
```

on peut aussi procéder ainsi:

```
from sage.rings.polynomial.toy_buchberger import spol
spol(G[0],G[1])
```

```
-X^2*Z + Y*Z
```

```
S01.reduce(G)
```

```
0
```

```
Q = S01.lift(G)
Q
```

```
[0, 0, -Z]
```

```
QG = map(mul,zip(Q,G))
QG
```

```
[0, 0, -X^2*Z + Y*Z]
```

```
S01 == sum(QG)
```

```
True
```

```
map(lambda g: g.lm()<=S01.lm(), QG)
```

```
[True, True, True]
```

```
S02 = S(A,G[0],G[2])
S02
```

```
-X^3*Z + Y^3
```

```
S02.reduce(G)
```

```
0
```

```
Q = S02.lift(G)
Q
```

```
[Y, 0, -X*Z]
```

```
QG = map(mul,zip(Q,G))
QG
```

```
[Y^3 - X*Y*Z, 0, -X^3*Z + X*Y*Z]
```

```
S02 == sum(QG)
```

```
True
```

```
map(lambda g: g.lm()<=S02.lm(), QG)
```

```
[True, True, True]
```

```
S12 = S(A,G[1],G[2])
S12
```

```
Y^2 - X*Z
```

```
S12.reduce(G)
```

```
0
```

```
Q = S12.lift(G)
Q
```

```
[1, 0, 0]
```

```
QG = map(mul,zip(Q,G))
QG
```

```
[Y^2 - X*Z, 0, 0]
```

```
S12 == sum(QG)
```

```
True
```

```
map(lambda g: g.lm()<=S12.lm(), QG)
```

```
[True, True, True]
```

le critère de Buchberger est vérifié; $\mathbf{G}$ est une base de Gröbner de l'idéal $\langle G \rangle$

```
B = PolynomialRing(RationalField(), 3, 'XYZ', order='lex')
(X,Y,Z) = B.gens()
```

```
F = [X*Y^2-Z*X+Y,Y*X-Z^2,X-Z^4*Y]
F
```

```
[X*Y^2 - X*Z + Y, X*Y - Z^2, X - Y*Z^4]
```

```
S01 = S(B,F[0],F[1])
S01
```

```
-X*Z + Y*Z^2 + Y
```

```
S01.reduce(F)
```

```
-Y*Z^5 + Y*Z^2 + Y
```

l'un des S-polynômes ne se réduit pas 0, F n'est pas une base de Gröbner