TP8

TP 8

Exercice 1

```
restart
```

```
pretty_print_default()
```

```
A = PolynomialRing(RationalField(), 2, 'XY', order='lex')
A
```

$\mathbf{Q}[X, Y]$

```
X,Y = A.gens()
```

```
F = X^2*Y^5-X^5*Y^2-2*X*Y^5+X^5+Y^5-Y*X^3-X^2*Y^2-Y^3*X
F
```

$-X^5Y^2 + X^5 - X^3Y + X^2Y^5 - X^2Y^2 - 2XY^5 - XY^3 + Y^5$

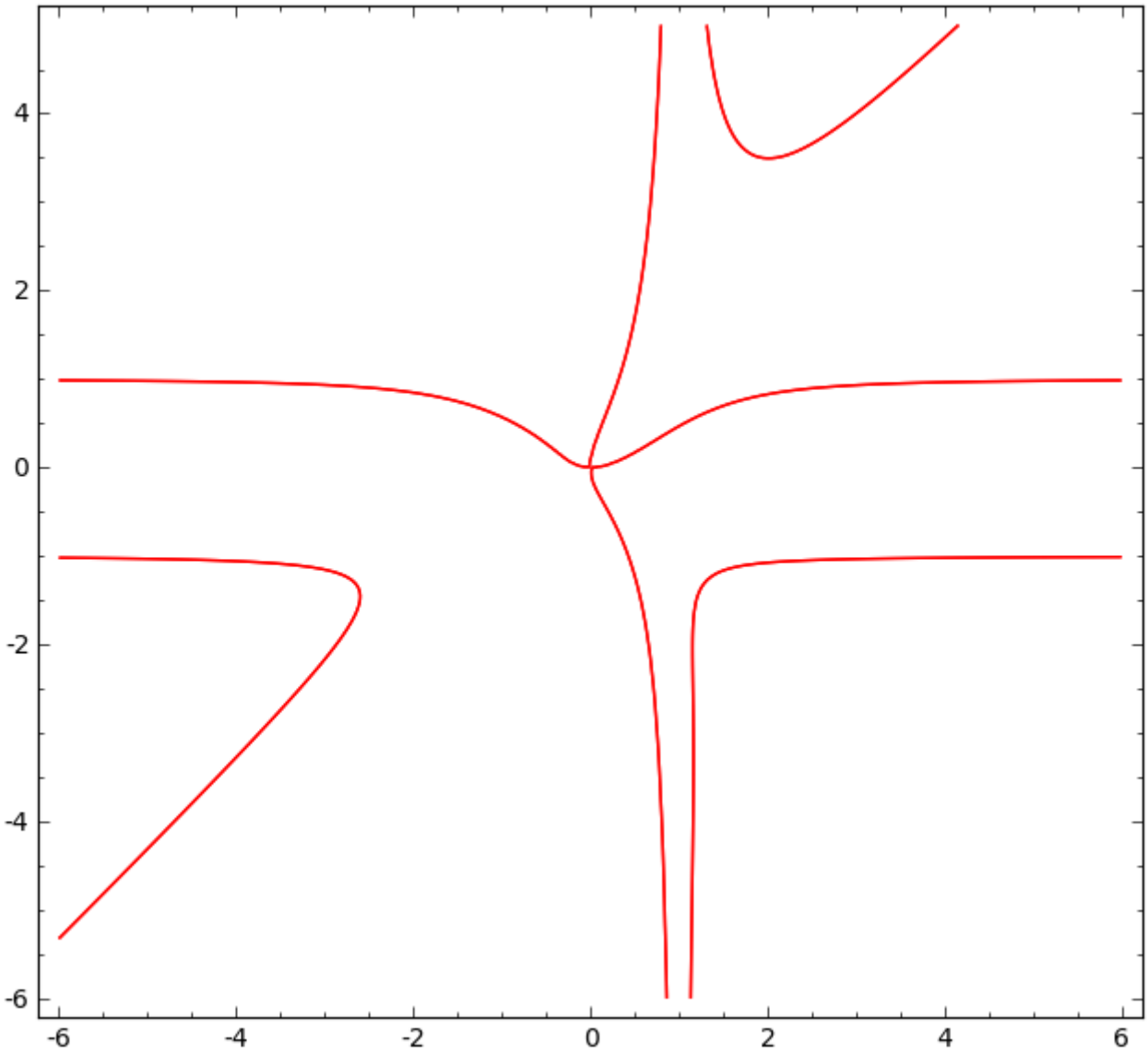
```
Fbar = F.change_ring(QQbar)
```

```
Fbar.factor()
```

```
Traceback (click to the left of this block for traceback)
...
NotImplementedError: proof = True factorization not implemented.
Call factor with proof=False.
```

```
des = implicit_plot(F, (X, -6, 6), (Y, -6, 5), plot_points = 250,
color = 'red')
```

```
show(des)
```




```
I = Ideal(F,F.derivative(X),F.derivative(Y))
I
```

$$\left(-X^5Y^2 + X^5 - X^3Y + X^2Y^5 - X^2Y^2 - 2XY^5 - XY^3 + Y^5, -5X^4Y^2 + 5X^4 - 3X^2Y + 2\right.$$

```
I.variety()
```

$$[\{Y : 0, X : 0\}]$$

```
I.variety(QQbar)
```

$$[\{Y : 0, X : 0\}]$$

```
ct = A(taylor(F,(X,0),(Y,0),4))
ct
```

$$-X^3Y - X^2Y^2 - XY^3$$

```
ctf = ct.factor()
ctf
```

$$(-1) \cdot Y \cdot X \cdot (X^2 + XY + Y^2)$$

```
ctf1 = list(ctf)
ctf1
```

$$\left[(Y, 1), (X, 1), \left(X^2 + XY + Y^2, 1\right)\right]$$

```
f = ctf1[2][0]
f
```

$X^2 + XY + Y^2$

```
p = f(Y=1)
p
```

$X^2 + X + 1$

```
t = var('t')
p(X=t).solve(t)
```

$\left[t = -\frac{1}{2}\,i\sqrt{3} - \frac{1}{2}, t = \frac{1}{2}\,i\sqrt{3} - \frac{1}{2}\right]$

```

```

```
K.<alpha> = QQ.extension(p.univariate_polynomial())
K
```

$\mathbf{Q}[\alpha]/(\alpha^2 + \alpha + 1)$

```
ctK = ct.change_ring(K)
ctK
```

$-X^3Y - X^2Y^2 - XY^3$

```
ctK.factor()
```

$(-1) \cdot Y \cdot X \cdot (X - \alpha Y) \cdot (X + (\alpha + 1)Y)$

```

```

Exercice 2

```
restart
```

```
pretty_print_default()
```

```
m = 3
n = 4
MS = MatrixSpace(ZZ,3,4)
MS
```

$\mathrm{Mat}_{3\times 4}(\mathbf{Z})$

```
M = matrix([[6,8,4,20],[12,12,18,30],[18,4,4,10]])
M
```

$$\begin{pmatrix} 6 & 8 & 4 & 20 \\ 12 & 12 & 18 & 30 \\ 18 & 4 & 4 & 10 \end{pmatrix}$$

```
S,U,V = M.smith_form()
S
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 174 & 0 \end{pmatrix}$$

```
U*M*V == S
```

True

```
d = [gcd(M.minors(k)) for k in range(1,min(m,n)+1)]
d
```

$[2, 4, 696]$

```
d[0],d[1]/d[0],d[2]/d[1]
```

$(2, 2, 174)$

```
Uinv = U^(-1)
Uinv
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 30 & -1 \\ 22 & 121 & -4 \end{pmatrix}$$

```
Uinv.columns()
```

$[(1, 0, 22), (0, 30, 121), (0, -1, -4)]$

```

```

Exercice 3

```
restart
```

```
pretty_print_default()
```

```
n = 6
MS = MatrixSpace(QQ,n,n)
MS
```

$\mathrm{Mat}_{6\times 6}(\mathbf{Q})$

```
M = matrix([[1,-2,4,-2,5,-4],[0,1,5/2,-7/2,2,-5/2],[1,-5/2,2,-1
/2,5/2,-3],[0,-1,9/2,-7/2,3,-7/2],[0,0,2,-2,3,-1],
[1,-3/2,-1/2,1,3/2,1/2]])
M
```

$$\begin{pmatrix} 1 & -2 & 4 & -2 & 5 & -4 \\ 0 & 1 & \frac{5}{2} & -\frac{7}{2} & 2 & -\frac{5}{2} \\ 1 & -\frac{5}{2} & 2 & -\frac{1}{2} & \frac{5}{2} & -3 \\ 0 & -1 & \frac{9}{2} & -\frac{7}{2} & 3 & -\frac{7}{2} \\ 0 & 0 & 2 & -2 & 3 & -1 \\ 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

```
A = PolynomialRing(QQ, 'X')
A
```

$\mathbf{Q}[X]$

```
X = A.gen()
```

```
chi = M.characteristic_polynomial(X)
chi
```

$X^6 - 4X^5 + 16X^3 - 12X^2 - 16X + 16$

```
P = M.minimal_polynomial(X)
P
```

$X^5 - 2X^4 - 4X^3 + 8X^2 + 4X - 8$

```
parent(P)
```

$\mathbf{Q}[X]$

```
chi.quo_rem(P)
```

$(X - 2, 0)$

```
MSA = MatrixSpace(A,n,n)
view(MSA)
```

$\mathrm{Mat}_{6\times 6}(\mathbf{Q}[X])$

```
C = matrix(A,6,6,X)-M
C
```

$$\begin{pmatrix} X-1 & 2 & -4 & 2 & -5 & 4 \\ 0 & X-1 & -\frac{5}{2} & \frac{7}{2} & -2 & \frac{5}{2} \\ -1 & \frac{5}{2} & X-2 & \frac{1}{2} & -\frac{5}{2} & 3 \\ 0 & 1 & -\frac{9}{2} & X+\frac{7}{2} & -3 & \frac{7}{2} \\ 0 & 0 & -2 & 2 & X-3 & 1 \\ -1 & \frac{3}{2} & \frac{1}{2} & -1 & -\frac{3}{2} & X-\frac{1}{2} \end{pmatrix}$$

```
C.determinant()
```

$X^6 - 4X^5 + 16X^3 - 12X^2 - 16X + 16$

```
S, U, V = C.smith_form()
```

```
S == U*C*V
```

True

```
S
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & X-2 & 0 \\ 0 & 0 & 0 & 0 & 0 & X^5 - 2X^4 - 4X^3 + 8X^2 + 4X - 8 \end{pmatrix}$$