Of curves and circles of knowledge: Luigi Berzolari on algebraic curves in the *Encyklopädie der mathematischen Wissenschaften*

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Postprint. January 2026

Abstract

This chapter examines Luigi Berzolari's 1906 article on the theory of higher plane algebraic curves in the *Encyklopädie der mathematischen Wissenschaften*. It shows how Berzolari, by blending mathematical and historical exposition, crafts a particular view of the theory. The study highlights and analyses his choices: omitting certain elementary contributions, juxtaposing old and recent research and forging anachronistic connections between results to strengthen the topic's coherence and embedding in the long term. The chapter also discusses the *Encyklopädie*'s role in establishing algebraic geometry as a distinct domain in the early 20th century.

The genesis of the *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* is well known. In 1894, with the aim of demonstrating the unity of mathematics and pushing its links with other sciences to the fore, Felix Klein, Wilhelm Franz Meyer and Heinrich Weber decided to publish a large survey of the mathematical knowledge of the past century [Rowe 1989; Tobies 1994].

The original plan, conceived by Meyer, was to write a dictionary which would present the main concepts of mathematics. "Although the focus was mainly on newer concepts", Meyer explained, "older and even obsolete terms should also be mentioned in order to preserve them, as in a museum".[1] The dictionary was then supposed to expose the historical development of the concepts from their first appearance to the present time.

Shortly afterwards, however, the persons involved in the project realised that such a dictionary was not an appropriate way to record and display

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[1] "War dabei hauptsächlich an die neueren Begriffe gedacht, so sollten immerhin auch die alten und sogar auch die veralteten Kunstausdrücke Erwähnung finden, um sie wie in einem Museum zu konservieren." [Dyck 1904, p. V].

the mathematical knowledge of the 19th century, in particular because the alphabetic ordering would cause a fragmentation of the content and many repetitions. Instead, Walther Dyck suggested another form of exposition:

> Thus, at Dyck's request, the decision was made [...] to abandon the idea of an actual *lexicon* and to replace the *artificial* system of alphabetical order with the *natural* system of a purely factual arrangement and presentation of the mathematical domains of knowledge.[2] [Dyck 1904, p. VIII]

The idea of an encyclopaedia organised thematically was thus adopted.[3] Six volumes were planned, devoted to arithmetic and algebra, analysis, geometry, mechanics, physics, and geodesy, geophysics and astronomy, respectively.[4] Each of them would then be made of articles whose purpose was to provide the main results associated with a specific topic and "to document the historical development of the mathematical methods since the beginning of the 19th century through careful references to literature."[5] In total, 209 articles, written by 109 authors and amounting to nearly 20,000 pages, were issued between 1898 and 1935.[6]

In the past years, several of these articles have been seized as sources by scholars who investigated historical issues related to their themes: the evolution of topology [Epple 1999, pp. 229–233], the place devoted to Belgian engineer Junius Massau in the history of graphical calculus [Tournès 2003, pp. 237–239], the narratives opposing analytic and synthetic geometries [Lorenat 2015, pp. 45–51], etc.[7] However, it appears that the *Encyklopädie*'s articles have not yet been the objects of more systematic studies of their content and composition. In particular, the general questions raised in the present book, about how encyclopaedias participate to the *patrimonialisation* of mathematics, remain largely open for the *Encyklopädie*.

The aim of this chapter is to go into this direction by focusing on one article. Authored by Luigi Berzolari (1863–1945), this article is contained in

[2] "So kam in Leipzig auf Antrag von Dyck der Beschluss zu Stande, die Idee eines eigentlichen *Lexikons* fallen zu lassen und an Stelle des *künstlichen* Systems einer alphabetischen das *natürliche* System einer rein sachlichen Anordnung und Darlegung der mathematischen Wissensgebiete zu setzen."

[3] As is well known, tensions between alphabetical and thematic orderings in encyclopaedias had been debated since the 18th century. See for instance [Yeo 1991; Becq 1995].

[4] Before becoming the object of the last three volumes, applied mathematics was supposed to be organised differently, in two volumes. Moreover, a volume on the history, philosophy and didactics of mathematics was initially planned but was never published. See [Tobies 1994, pp. 56–69].

[5] "[...] durch sorgfältige Litteraturangaben die geschichtliche Entwicklung der mathematischen Methoden seit dem Beginn des 19. Jahrhundert nachzuweisen." [Dyck 1904, p. IX].

[6] The French version of the *Encyklopädie* was published at the beginning of the 20th century, before World War I interrupted the enterprise [Gispert 1999]. Moreover, 14 chapters were added between 1939 and 1967, as parts of a new German edition.

[7] Other examples of study and use of articles of the *Encyklopädie* include [Gauthier 2007, pp. 167–170; Bottazzini and Gray 2013, pp. 745–749; Lê 2016, pp. 276–282, 2018]. Moreover, the geometric volume has been described as a whole in [Monteiro de Siqueira 2015].

the part on algebraic geometry (which is included in the geometric volume) and is entitled “General theory of the higher plane algebraic curves” [Berzolari 1906].[8] Three main issues will be investigated: the themes, periods, authors and works that are described or, on the contrary, forgotten by Berzolari; the way in which the present and the past coexist in the article; and the way of writing the article, which combines historical and mathematical features.[9] Before I specify these issues and explain how I intend to address them, I first provide an overview of Berzolari's article.

1. An overview on Berzolari's article

The article is organised by mathematical topics, as reflects its division into five sections entitled “I. Generalities”, “II. Singular points”, “III. Reality issues and metric properties”, “IV. The geometry on a curve” and “V. Linear systems of curves”. These sections are in turn divided into 38 thematic subsections whose abridged, translated titles are given in table 1.

Let me consider the first subsection, called “Algebraic plane curves; their real representation”. It begins with the following sentences:

> A plane algebraic curve C^n of the order n^1) is the locus of the – real and imaginary – points, whose homogeneous projective coordinates (in particular homogeneous Cartesian coordinates, for instance) x_1, x_2, x_3 satisfy an equation $f(x) = 0$, where f is a ternary form of order n with constant, real or complex coefficients[2]). If one divides by x_3^n and set $x = x_1 : x_3$, $y = x_2 : x_3$, then the equation $f = 0$ takes the form $F(x, y) = 0$, of degree $m \leqq n$ in y, and $m' \leqq n$ in x.
>
> C^n is called *simple* (*irreducible*) or *reducible*, depending on whether f

[8]The original title is “Allgemeine Theorie der höheren algebraischen Curven”. Berzolari held a chair in projective and descriptive geometry at the University of Turin between 1893 and 1899, when he became a professor at the University of Pavia. Given the chronology of the *Encyklopädie*'s development, it is therefore likely that Berzolari was recruited to write his article on curves when he was in Turin. Although the sources I have consulted do not reveal the precise reasons why the editorial committee chose Berzolari, it is possible that his proximity to Corrado Segre, whose links with Klein are well known, played a part in this decision [Luciano and Roero 2012]. It should be added that by 1893, Berzolari had already published a dozen articles on algebraic curves and surfaces. Finally, let me note that the quality of Berzolari's article was later recognised by the editors of the *Encyklopädie* and other scholars such as Guido Castelnuovo, so much so that he was entrusted with another article, on algebraic transformations and correspondences [Berzolari 1932; Luciano and Roero 2012, pp. 62, 209, 212]. Berzolari also took over the article on space curves and developable surfaces that Karl Rohn had begun to write before passing away [Rohn and Berzolari 1928]. For more details on Berzolari's career, see [Brusotti 1950], especially on pp. 9–11 for his works related to various encyclopaedic projects.

[9]This way of examining Berzolari's article thus echoes, while adapting it to my case, the approach proposed by Olivier Bruneau for studying articles on fluxions in various encyclopaedias [Bruneau 2022].

is [I B 1 b, Nr. 5, *Netto*; I B 2,[405]) *Meyer*; I B 3 b, Nr. 26, *Vahlen*][3]).[10]
[Berzolari 1906, pp. 316–317]

As can be seen, these sentences are entirely focused on mathematical explanations on objects and concepts of algebraic curve theory. Reflecting the *Encyklopädie*'s specialisation and intended readership, these explanations are not self-contained: for example, homogeneous coordinates, imaginary points and the very concept of locus are not defined. The same applies to the concept of reducibility of a polynomial or a form f, for which, contrary to the previous examples, references to other articles of the *Encyklopädie* are provided.

The three footnotes marked by the superscript numbers 1), 2), 3) present digressions of different kinds. The third one provides a list of works where the reducibility of a curve is tackled by invariant theory. The second one informs of the possibility to represent an algebraic curve with other coordinate systems than Cartesian coordinates, and cites various publications where such systems have been used with success. As for the first footnote, in addition with two thematic detours (on the difference between algebraic and transcendent curves, and on the link between the study of curve intersections and the solution of algebraic equations), it indicates the authorship of the concept dealt with in the body of the text. Specifically, it credits René Descartes and Pierre de Fermat for having first established the link between plane curves and equations in two unknowns.

More generally, the attribution of authorship of objects, concepts or theorems is constant throughout the article. As in the previous extract, such attributions can occur in the footnotes, but they are also often made directly in the body of the text, which then takes on a more historical tone. This is the case in the following lines, which open the second section of the article:

> The idea of resolving singular points through a purely algebraic process is attributed to L. Kronecker[160]) [...]. Independently of Kronecker, M. Noether[161]) established that every plane algebraic [...] curve f [...] can be transformed into a curve that only has ordinary singularities[162]).[11]
> [Berzolari 1906, pp. 362–363]

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[10] "Eine ebene algebraische Kurve C^n von der Ordnung n[1]) ist der Ort der – reellen und imaginären – Punkte, deren homogene projektive Koordinaten (im besondern z. B. homogene Cartesische Koordinaten) x_1, x_2, x_3 einer Gleichung $f(x) = 0$ genügen, unter f eine ternäre Form der Ordnung n, mit konstanten, reellen oder komplexen Koeffizienten verstanden[2]). Dividiert man mit x_3^n und setzt $x = x_1 : x_3$, $y = x_2 : x_3$, so nimmt die Gleichung $f = 0$ die Gestalt $F(x, y) = 0$ an, von einem Grade $m \leqq n$ in y, und $m' \leqq n$ in x. C^n heisst *einfach* (*irreduzibel*) oder aber *reduzibel*, je nachdem er f ist [I B 1 b, Nr. 5, *Netto*; I B 2,[405]) *Meyer*; I B 3 b, Nr. 26, *Vahlen*][3])."

[11] "Den Gedanken, die singulären Punkte durch einen rein algebraischen Prozess aufzulösen, verdankt man L. Kronecker[160]) [...]. Unabhängig von Kronecker hat M. Noether[161]) festgestellt, dass sich jede ebene algebraische [...] Kurve f [...] in eine Kurve überführen lässt, die nur mit gewöhnlichen Singularitäten behaftet ist[162])." The ellipses in this quote conceal technical descriptions which I omit for the sake of brevity.

Subsections (abridged titles)	Min	Max	Mean	Median
1. Algebraic curves; real representation	1637	1905	1837	1879
2. Definitions and elementary properties	1668	1905	1817	1849
3. Continuation; linear curve systems	1692	1901	1855	1868
4. The genus; Riemann's theorem	1841	1903	1878	1878
5. Polar properties	1704	1906	1864	1874
6. The Jacobian curve of three curves	1851	1900	1880	1881
7. Covariant curves of a base curve	1844	1903	1871	1872
8. The Plücker formulas	1818	1903	1861	1858
9. Algebraic ∞^1 curve systems	1857	1902	1875	1875
10. Curve generations	1687	1901	1846	1870
11. Purely geometric investigations	1847	1906	1883	1886
12. Resolution of the singular points	1857	1906	1889	1893
13. Branches (complete and partial)	1676	1905	1862	1881
14. Applications; intersection multiplicity	1864	1905	1887	1889
15. The genus for any singular curve	1865	1904	1886	1886
16. Characteristic numbers of a branch	1874	1899	1885	1885
17. Formulas of Halphen, Smith, Zeuthen	1872	1896	1878	1875
18. Plücker's equivalents	1865	1899	1880	1877
19. Real branches	1704	1905	1864	1877
20. Klein–Riemann surfaces	1874	1900	1886	1887
21. Asymptotic lines, diameters, etc.	1704	1906	1853	1866
22. Evolutes and other derived curves	1692	1905	1862	1870
23. Noether's fundamental theorem	1870	1906	1891	1892
24. The linear pencils of groups of points	1857	1906	1889	1893
25. Remainder theorem	1882	1897	1889	1889
26. Applications of elementary operations	1889	1902	1894	1894
27. Special and non-special pencils	1857	1905	1887	1866
28. The problem of special groups	1857	1904	1886	1887
29. Normal curves	1857	1905	1886	1888
30. The modules of a class of curves	1851	1903	1877	1875
31. Extensions	1879	1879	1879	1879
32. Reducible base curves	1882	1886	1884	1885
33. Applications. Intersection theorems	1720	1905	1863	1873
34. Other enumerative questions	1844	1906	1879	1880
35. Systems determined by base points	1871	1905	1888	1889
36. Properties of invariant linear systems	1872	1897	1888	1889
37. Classification of linear systems	1870	1902	1887	1888
38. Specific investigations	1848	1906	1882	1884

Table 1: The subsections of Berzolari's article with the publication year data of the references cited therein. The horizontal lines delimit the five sections of the chapter.

It should be noted that these sentences include descriptive terms (such as "algebraic" and "independently") which situate the works described in relation to one another and thus contribute to the structure of the narrative. Moreover, as before, the three footnotes comment on the research evoked in the text, digress and provide bibliographic data.

Footnotes can be very extensive in Berzolari's article: while our first quote represent 10 lines in the article, the footnotes 1), 2), 3) are made of 56 lines and contain no less than 38 bibliographic references, alongside with three references to other articles of the *Encyklopädie*.

The result of all this is a very dense and erudite work in which mathematical exposition is interspersed with historical remarks and thematic digressions, many bibliographical references being provided to the reader: in all, the 38 subsections of the article are spread over 143 pages, with 449 footnotes and 1,380 bibliographic references, which are cited 2,456 times.[12]

Given such a vast amount of information, the *patrimonialisation* issues mentioned above will be addressed by combining analyses carried out at different scales. After comparing the topics covered by Berzolari with those found in several contemporary books, I will quantitatively study all of his bibliographical references to identify, at the level of the entire article, the authors, works and time periods that are favoured or left out. This study, which will use the *Catalogue of Scientific Papers* as a point of comparison and will be supplemented by a more detailed reading of certain references, will reveal that Berzolari omitted many contributions published in intermediate journals or having similar characteristics, with the effect of partially overseeing some author nationalities.

The quantitative data related to Berzolari's references will also evidence that very recent research was included in every subsection. A finer-grained analysis will display more precisely how the present and the past coexist in the article. In particular, it will show that Berzolari fashioned different forms of topicality for almost every mathematical subject, thus conveying a lively image and a long-term impression of algebraic curve theory.

I will then focus on a few specific passages to describe characteristics of Berzolari's historical-mathematical writing and their effect on the produced narrative. I will show that the qualification of research as belonging to geometry (for instance), the emphasis of certain works at the expense of others, and the act of bringing together publications that were unknown to each other at the time tend to smooth out rough edges of the actual historical development and

[12]To these references can be added 36 other *Encyklopädie*'s articles, which are cited 155 times by Berzolari. For comparison, the corresponding data for the neighbouring articles by Hieronymous Zeuthen [1905], Gustav Kohn [1908] and Gino Loria [1914] are the following: 56 pages, 244 footnotes, 357 references and 472 citations for Zeuthen; 114 pages, 461 footnotes, 894 references and 1,369 citations for Kohn; 64 pages, 309 footnotes, 409 references and 497 citations for Loria. Berzolari's article thus appears as particularly dense in terms of numbers of references and citations per page.

thus to reinforce the idea of a long-term nature of the subject.

In the conclusion, I will briefly refer to the article on curves that Arthur Cayley wrote for the ninth edition of the *Encyclopædia Britannica* [Cayley 1877]. The comparison of this article with Berzolari's will help me to summarise the characteristics of the latter, and to offer some thoughts on the beginnings of the widespread recognition of the domain known as "algebraic geometry", which I maintain is closely linked to the publication of the *Encyklopädie*.

2. The present and the absent

2.1 A thematic comparison with books

To assess the presence or the absence of topics in Berzolari's article, a first avenue is to compare the content of the latter with that of contemporary books on algebraic curves.

In fact, the comparison cannot be made that immediately, as I cannot find any book whose overall subject would be the general theory of algebraic curves (i.e. the theory of curves of any order n), which is the subject of Berzolari's article. For instance, Heinrich Wieleitner's *Theorie der ebenen algebraischen Kurven höherer Ordnung* and Harold Hilton's *Plane Algebraic Curves* both contain chapters on curves of order 3 and 4 alongside chapters dealing with general curves [Wieleitner 1905; Hilton 1920]. This is also true for books whose subject matter goes beyond algebraic curve theory, such as Georges Salmon's famous *Treatise on Higher Plane Curves*, which covers transcendental curves [Salmon 1873], or Alfred Clebsch's *Vorlesungen über Geometrie*, edited by Ferdinand Lindemann, which includes a chapter on "connexes"[13] [Clebsch and Lindemann 1876].

That said, the general themes addressed by Berzolari, as identified from the titles of his sections and subsections, are all present in the chapters on general algebraic curves of these books. There are therefore no original topics that Berzolari would have singled out.[14]

Conversely, most of the topics tackled in these chapters are covered in Berzolari's article, the notable exceptions being the principle of correspondence and the systematic study of birational transformations and correspondences.[15] Actually, these two subjects do appear in the *Encyklopädie*, in two articles

[13] A connex is an object defined by an homogeneous equation $f(x_1, x_2, x_3, u_1, u_2, u_3) = 0$, where the x_i designate the point coordinates in a plane and the u_i designate the line coordinates in the same plane.

[14] As already alluded to, Berzolari included many recent developments on algebraic curve theory in his account, some of which are not in the mentioned books for mere reasons of chronology. Nevertheless, these developments mainly concern specific subjects that are extensions of themes which are indeed discussed in these books.

[15] Roughly speaking, birational transformations and correspondences are some sort of functions defined on a curve, and the principle of correspondence is a theorem dealing with the number of elements fixed by a correspondence. On this theorem, see [Michel 2021].

which belong to the part on algebraic geometry: that by Hieronymous Zeuthen on enumerative methods and that by Berzolari on algebraic transformations and correspondences [Zeuthen 1905; Berzolari 1932].

In other words, the apparent absence of certain themes in Berzolari's article results from the organisation of the knowledge on algebraic geometry at the scale of the part on algebraic geometry, an organisation which does not coincide with that of contemporary books. – Similarly, separate articles of the *Encyklopädie* deal with special curves, with Gustav Kohn's article on curves of order 3 and 4, and Gino Loria's article on curves of order higher than 4 [Kohn 1908; Loria 1914].

While the main points of knowledge on algebraic curves are thus present in the *Encyklopädie*, it remains possible that other selection phenomena exist, being visible at a different observation level than that adopted thus far.

2.2 Disciplinary classifications

To detect such phenomena, a way to proceed is to study the set of the 1,380 bibliographic references cited throughout Berzolari's article. For this purpose, I decided to compare these references with the *Catalogue of Scientific Papers*, which appeared to me as an adequate source since its goal consisted in listing papers published during the 19th century in a large number of mathematical journals, and in indexing them in a thematic classification [Beaver 1972; Wagner-Döbler and Berg 1996]. Being particularly intrigued by what Berzolari may have left out, I decided to study the papers he did not cite, all the while being present in relevant sections of the *Catalogue*. In order to do so, it is important first to understand the *Catalogue*'s disciplinary classification and how the articles cited by Berzolari are distributed within it.

The *Catalogue* classification echoes to some extent the organisation of the *Encyklopädie* described above. Indeed, the geometry part contains a chapter on higher algebraic curves and surfaces, which includes a section (numbered 7610) that deals with the "metrical and projective properties of algebraic plane curves of degree higher than the second". This chapter thus seems to correspond nicely to Berzolari's article. Furthermore, the next chapter is devoted to "Transformations and general methods for algebraic configurations", and includes a section (8030) on "groups of points on an algebraic curve; [the] genus of curves; [the] principle of correspondence": this mirrors, on the one hand, the exclusion of the principle of correspondence and of the systematic study of transformations and correspondences from Berzolari's article, and, on the other hand, the existence of a separate article on the latter objects [Berzolari 1932]. To complete the picture, let me note that the same *Catalogue* chapter includes a section (8070) devoted to "enumerative geometry", which echoes Zeuthen's *Encyklopädie* article [Zeuthen 1905], as well as a section (8090) on "systems of curves and surfaces", which refers directly to Berzolari's section V.

Let me now look at how Berzolari's bibliographic references are classified in the *Catalogue*. Of the 1,046 papers published between 1800 and 1900 that are cited by Berzolari, I managed to locate 909 in the *Catalogue* index.[16]

As one would expect, the *Catalogue* section with the most entries is the 7610 one, but it actually represents only 24% of the 909 papers. It is followed by the sections 8030 (13%), 8070 (9%), 8090 (8%) and the section 8430 on the curvature of curves, which belongs to the chapter on infinitesimal geometry (5%). Beyond the inevitable variations due to personal appreciation in the assessment of the belonging of a given paper to a disciplinary section, this scattering obviously reflects the difference in nature of the *Encyklopädie* and the *Catalogue*, the writing an encyclopaedic article being by no means the same enterprise as the disciplinary classification of a given series of papers. In particular, the numerous thematic digressions that Berzolari made contribute to the involvement of a large number of *Catalogue* sections.[17]

Reciprocally, the set of all the papers listed in the 81 different sections represented in Berzolari's article obviously do not coincide with the set of the references cited therein, some of these papers being for instance cited in other articles of the *Encyklopädie*.

Now, to understand what is left out by Berzolari, one might want to investigate the papers that are listed in the 7610 section all the while being absent from his article. But the classificatory scattering described above and the differences of organisation between the *Catalogue* and the *Encyklopädie* invite us to be more cautious, inasmuch as the absence of a paper in Berzolari may just mean that its theme makes it cited elsewhere. Thus I will consider the papers from the 7610 section that are cited neither in Berzolari's article, nor in the other (parts of) articles dealing with algebraic curves that I identified above: those by Zeuthen, Kohn and Loria, to which is added the section of Berzolari's 1932 article on algebraic transformations and correspondences that deals specifically with curves. In what follows these papers will be called the *Catalogue* orphans.[18]

[16]Berzolari also cited 105 books, 39 doctoral dissertations and 9 habilitation dissertations and programs written by German teachers at the occasion of their appointment to an institution. The other 1,227 references consists of papers published in 166 different journals between 1684 and 1906.

[17]The many bridges that exist between algebraic curves, invariants, algebraic functions, Riemann surfaces, etc., also explain why a non-negligible number of papers cited by Berzolari are classified in sections related to algebra or analysis.

[18]I also studied the orphans of the reunion of the sections 7610, 8030, 8070 and 8090, which leads to analogous results. In any case, my point here is that the slight differences in the thematic classifications of the *Catalogue* and the *Encyklopädie* must be carefully taken into account for the inquiry.

2.3 The Catalogue *orphans*

Among the 891 papers inventoried in the 7610 section, 460 are orphans. First of all, it should be noted that the vast majority of these orphans seem to have all the reasons to appear in the mentioned *Encyklopädie* articles, in the sense that their status as orphans do not seem to be a consequence of an initial absurd classification in the *Catalogue*. For example, Georges Fouret's paper entitled "On a new geometric definition of the curves of order n with a multiple point of order $n-1$" has no reason *a priori* not to be cited in Berzolari's article [Fouret 1875], and Arthur Cayley's paper "On the cubic curves inscribed in a given pencil of six lines" could well figure among the references of Kohn's article on cubic and quartic curves [Cayley 1868].

It is thus not obvious to understand why such or such paper, considered individually, is not part of the *Encyklopädie*. At this stage of the investigation, I therefore adopted a more quantitative point of view on the 460 orphans, comparing different data of this population of papers with the global corresponding data of the 7610 section.

The situation is striking, first, when looking at the nationality of the authors. In the whole 7610 section, the French contributions are the most numerous, with 216 papers, and are followed by the Germans (174 papers), the British (148), the Austro-Hungarians (97) and the Italians (87).[19] Now, while the orphan ratio is 52% at the level of the whole section, it equals 58% for the French and the Austro-Hungarians, 57% for the British, 48% for the Italians and only 29% for the Germans. Hence the Germans and, to a lesser extent, the Italians are relatively spared by the omission phenomenon in the *Encyklopädie*, while the other nations are more affected. Further, for some other nations which are less important from the numerical point of view, the orphan ratio can reach a very high level: this is especially true for the United States of America, since 21 papers of the 29 that are listed in the 7610 section are orphans, which represents a ratio of 72%.

These numbers can be partially understood when looking at the journals in which the *Catalogue* papers were published. The French contributions are first and foremost papers from *Nouvelles Annales de mathématiques*, with 81 papers – while the second most represented journal is the *Comptes rendus hebdomadaires de l'Académie des sciences*, which counts 41 items. However it turns out that 77% of the papers published in *Nouvelles Annales* are orphans,[20] which explains why the French have a higher orphan ratio than average. More generally, intermediate journals are the publication loci having the highest

[19] Following the decreasing order, the next nations present a markedly lower number of papers, with 34 papers for Denmark, 29 for the Netherlands, 29 for the USA and 25 for Belgium. In all, 17 nations appear in the counts.

[20] Considering the number of orphans within the French contributions to *Nouvelles Annales* leads to the same result, inasmuch as the vast majority of the contributors to this journal were French.

orphan ratios: in particular, 90% of the 20 papers from *Archiv der Mathematik und Physik*, 68% of the 34 papers from the *Quarterly Journal of Pure and Applied Mathematics*[21] and 58% of the 19 from *Giornale di matematiche* do not appear in the *Encyklopädie*. That said, the weight of these journals in the totals for each nationality does not systematically explain the orphan ratios observed above. This is the case for the French and the British, which respectively count 45% and 42% of papers published in intermediate journals, but not for the Austro-Hungarians, with 16% of such papers.

In such cases, the absence of papers from the *Encyklopädie* can be related to their content, which can be described as less at the edge of research. For the USA, a direct inspection of the orphans reveals that many of them, all the while being published in journals such as the *American Journal of Mathematics*, can be qualified as being quite elementary in the sense that they are very short notes, or papers that do not seem to stick to the most advanced topics of their time. For instance, Fabian Franklin wrote a 1-page-long note in the *American Journal of Mathematics* in 1880 with the aim of proving an extension of one result on curve intersections stated in Salmon's *Treatise* [Franklin 1880], and William E. Story published a slightly longer paper on "A new method in Analytic Geometry" [Story 1887], which does not seem to have been taken up by other mathematicians. By contrast, the few American authors who are cited in the *Encyklopädie* include Charlotte Angas Scott, who contributed with two papers on the theory of higher singularities [Scott 1892, 1893].[22] In other words, the significant exclusion of American authors seems to be related to the fact that most of their contributions on algebraic curves were quite elementary, similar to what can be found in intermediate journals.[23]

This conclusion is corroborated when looking at the 228 mathematicians who appear with only one paper in the 7610 section. While 72% of them are orphans, their names, indeed, correspond to a great extent to profiles of mathematicians who published in intermediate journals, typically being

[21] Although the *Quaterly Journal* cannot be qualified as an intermediary journal in the strict sense of the term, its mixed nature, especially from 1865 on (the year the *Proceedings of the London Mathematical Society* began to be published) is explained in [Crilly 2004].

[22] On these works of Scott, see [Lorenat 2020].

[23] It does not seem relevant to connect this exclusion to the question of the accessibility of the *Encyklopädie* contributors to American journals, as many of the orphans were published in volumes which contain articles that were cited in the *Encyklopädie*. Moreover, although mathematicians from Austria-Hungary often wrote in German, linguistic reasons prevented me from reading all the papers they published. Nevertheless I suspect a conclusion similar to the Amercian case, with papers of a less advanced content although being published in non-intermediary journals, such as the *Sitzungsberichte* of the Academy of Prag, which counts 81% of orphans out of 21 papers. The combination of factors such as the language, the loci of publication and the mathematical content may explain why mathematicians such as the Czech Karel Zahradnik are almost completely forgotten by the *Encyklopädie* all the while having a non-negligible number of papers listed in the *Catalogue*. On Zahradnik, see [Bečvářová and Čižmár 2011], especially the pages 379–393, which contain an English summary of the book.

teachers, engineers or students.[24]

To put these results into perspective, let me finally note that the journals that are commonly identified as the loci where research is at the front are largely represented in the *Encyklopädie*. For example, the orphan ratios of the French *Comptes rendus* (47 papers in all), of Crelle's journal (72 papers) and of *Mathematische Annalen* (63 papers) are only equal to 17%, 13% and 6%, respectively. In particular, the high weight of the latter two journals in the German contributions to the 7610 section is to be linked to the fact that the Germans present an orphan rate lower than average.

The parts of the *Encyklopädie* devoted to algebraic curves therefore appear to be places where the most advanced contributions on the subject were favoured and those of a more elementary level were excluded, with the effect of obscuring certain nationalities to a greater or lesser extent.

If the orphan papers are thus distinguished by their content, it must be added that nothing conclusive can be said on their dates of publication. In particular, there seem to be no effect of selection of old or recent papers, the orphan ratio for each year from 1800 to 1900 being approximately stable during the whole period. This leads me to the question of how the present and the past are articulated in Berzolari's article.

3. The present and the past

As shown in table 1, no fewer than 36 of the 38 subsections of Berzolari's article contain at least one reference dated after 1895.[25] In fact, 30 of them refer to research published after 1899, and 18 even cite works published in 1905 or 1906, which is the year when Berzolari finished writing the article. Further, as suggest the means and the medians given in table 1, and as confirms a direct inspection of the publication dates, the recent works count for a large part of the cited references, being thus far from marginal in the representation of the topics. If this certainly reflects the general increase of the mathematical production in the course of the 19th century, it also evidences that Berzolari drew a lively image of the theory of algebraic curves in accounting for the latest developments in almost all the topics addressed in the article.[26]

Moreover, the juxtaposition of the most recent references with others dating back to the beginning of the 19th century or before implicitly creates a form

[24]Of course, some mathematicians having only one publication in the 7610 section are prolific authors in view of their whole production (the most extreme examples being David Hilbert and Émile Borel). Such cases are far from being the majority.

[25]The two exceptional subsections (numbered 31 and 32) contain only three references each, a very low number compared to the other subsections. These references are dated between 1879 and 1886, which cannot be qualified as very old.

[26]While the vast majority of the recent references are research mathematical texts, a very restricted number of them are of a historical nature. I will comment on them in the next subsection.

of long-term history for the subject of algebraic curves. To better understand how this juxtaposition works, let me consider a few examples.

3.1 Bridging the present and the past

As expounded above, at the very beginning of the article, Berzolari explained that an algebraic curve of order n is a locus of points defined by a polynomial equation of degree n, and he immediately added in the first footnote that the idea of representing curves by equations between two unknowns is due to Pierre de Fermat and to René Descartes. Regarding the latter, Berzolari not only referred to *La Géométrie* as the third appendix of the 1637 *Discours de la méthode* [Descartes 1637], but also as a part of Descartes' collected works edited by Victor Cousin [Descartes 1824], and he additionally cited the new 1886 edition and the 1894 German translation by Ludwig Schlesinger [Descartes 1886, 1894]. Similar situations can be observed for other mathematicians, although the time gaps between the original versions and their editions are smaller than in Descartes' extreme case: Gottfried Wilhelm Leibniz's research from the very end of the 17th century is cited together with the 1858 *Mathematische Schriften* [Leibniz 1684a,b, 1686, 1858], Julius Plücker's papers, all originally published in the 1830s, are also cited as parts of the 1895 *Gesammelte mathematische Abhandlungen* [Plücker 1895], etc.

Berzolari also referred to recent texts of a historical nature. Among them, Alexander Brill's and Max Noether's report on the development of the theory of algebraic functions and Ernst Kötter's report on the history of synthetic geometry were the most frequently cited [Brill and Noether 1894; Kötter 1901]. Most of these citations were intended to steer the reader towards sources where they could find more details on specific topics. In Kötter's case, these topics were rather old, extending from the beginning of the 18th century to the beginning of the 1830s. As for the report by Brill and Noether, it was invoked both for works from the 18th century and for much more recent research, developed in the last quarter of the 19th century.

In any case, by referring to recent historical sources and editions of older texts, that is, to recent works which evidence by their very nature an interest in past mathematics, Berzolari created a first form of topicality of certain subjects.

Nonetheless such a topicality was first and foremost engendered by the citation of recent works which were related to the themes discussed by Berzolari by their mathematical content. Several types of such relations can be observed.

Berzolari sometimes reported on topics that he presented as quite new, in the sense that he rooted them in very recent research. This is illustrated by the theory of general linear curve systems, for which Guido Castelnuovo was depicted as having first introduced, in a 1892 memoir, the concepts that allowed its development [Castelnuovo 1892; Berzolari 1906, pp. 438–439]. In such cases, the existence of recent works in an article starting with Descartes' and Fermat's works hence provided, even if implicitly, a form of long-term view on the topic of algebraic curves, seen as a whole.

Berzolari also frequently showed that specific subjects, even those rooted in works dating back several decades or more, enjoyed vitality until the early 20th century. For instance, when dealing with the so-called Plücker formulas, Berzolari first referred to Julius Plücker's works where these formulas were first stated and proved [Plücker 1834, 1835]. After having evoked a related technical question, he then provided a list of 12 references which contain other proofs of the formulas and were published between 1866 and 1901 [Berzolari 1906, p. 343].

An analogous situation can be seen around the notion of the reducibility of an algebraic curve, which, as we saw, Berzolari defined at the beginning of the article, just after having explained what a curve of order n is. Contrary to the case of the Plücker formulas, Berzolari did not cite any references which would be singled out as the first ones to contain a definition of this notion. Instead, he inserted a large footnote where he enumerated a number of works, most of them very recent, which tackled the issue from the viewpoint of invariant theory: in the case of the reducibility of a curve into straight lines, the given references were papers by Friedrich Junker [1894], Alexander Brill [1893, 1898], Paul Gordan [1894] and Jacques Hadamard [1899]. Hence the subject of reducibility, all the while being mathematically elementary at first sight, was given a current relevance by the activation of viewpoints provided by recent rapprochements with the neighbouring mathematical domain of invariant theory.

3.2 Chronological orders

To complete our view on how the present and the past are articulated in Berzolari's article, let me close this section with some comments on the chronologies. As all the preceding examples may suggest, Berzolari's article was not written by following a strict global chronological order.

To begin with, the thematic division that the five main sections incarnate only corresponds partially to a chronological division. On the one hand, the oldest works that are cited in each section, and which can thus be seen as founding their main topic, are indeed chronologically ordered (see table 1). On the other hand, as indicated above, each section contains references published at the beginning of the 20th century. Therefore it cannot be claimed that the content of one of these sections precedes or follows that of another.[27]

Further, the progression of the subsections within a section does not correspond to the chronological order either, even if one focuses on the works that

[27] That said, sections IV and V appear to display a larger number of recent works than sections I to III, which reflects the fact that their topic were developed mainly in the late 19th century.

are cited as grounding the subject of each subsection. For instance, subsection 10, devoted to curve generations, begins with the fundamental contributions of Isaac Newton published at the very beginning of the 18th century [Newton 1704], while the preceding subsection 9, on algebraic infinite curve systems and to the theory of characteristics, is explicitly rooted in a series of works by Ernest De Jonquières and Michel Chasles from the 1860s.

Chronological orders can be detected more locally, at the scale of specific topics within the subsections. To take an example, let me consider again subsection 10 on curve generations and focus on the works described in the body of the text, to avoid the problem of footnotes digressions. The progression of these works is first chronological, with a sequence of works on the so-called "projective generations" which begins with Newton [1704] and then includes publications by Colin MacLaurin [1720], William Braikenridge [1733], Jacob Steiner [1848], Hermann Grassmann [1851a,b], Chasles [1853a,b,c, 1854, 1855a,b], De Jonquières [1862], Karl Bobek [1885] and finally Karl Küpper [1897]. Two temporal steps backwards then occur as Berzolari moved on to describing other types of curve generation: first those developed by Gustav von Escherich [1877, 1882a,b, 1884], then others that were researched by Grassmann [1844]. The breaks in the chronologies are thus induced by the changes of mathematical (sub)topics. This phenomenon is also evident in the inclusion of footnotes containing digressions, which were themselves also organised chronologically by subject.

This example echoes that of the Plücker formulas, which I used to show how certain subjects were endowed with a form of topicality. Thus, through these numerous local chronological progressions on a wide range of very specific subjects, Berzolari created many threads of continuity, many of which spanned several decades and, aided by the other characteristics noted above, contributed to creating a form of general long-term history of algebraic curve theory. Such temporal continuities were reinforced as coherent sequences of works thanks to other devices, which touch the way certain topics were presented and which I analyse in the next section.

4. The mathematical-historical writing

The general editorial guidelines of the *Encyklopädie* included an item specifying that while the contributors should not use meliorative terms such as "epoch-making", "genial" or "classical", they were allowed to qualify mathematical results as "new" or "based on a more rigorous basis", so as to indicate "in which direction the progress goes".[28] Berzolari did observe this rule: while no

[28] "In ihrer Allgemeinheit nichtssagende *epitheta ornantia*, wie epochemachend, genial, grossartig, klassisch u. s. w. werden zu vermeiden sein. Dagegen wird angegeben, in welcher Richtung jedesmal der Fortschritt liegt: ob in Auffindung *neuer Resultate* – oder in *strenger Begründung* vorher nur vermutungsweise aufgestellter oder ungenügend bewiesener Sätze

adjective of the first kind can be found in the chapter, indications of the degree of rigour of past works are frequent – see for instance [Berzolari 1906, pp. 325, 359, 424]. Other terms act in a similar way by characterising some works as being simple, general, systematic, erroneous, as lacking of exactness (pp. 435, 348, 354, 355, 356), or as depending on certain disciplinary domains such as algebra, geometry and arithmetic (pp. 410, 364, 329), the qualification being sometimes made more precise, as illustrate the phrases "purely algebraic" and "more analytic" (pp. 325, 410). As may be expected, these descriptions were sometimes taken from the comments of the mathematicians whose research Berzolari analysed, while in other cases they were the product of the latter's own point of view.

Characterising works by such qualifiers is not the only way how Berzolari left his mark on the narrative. In particular, at several occasions he privileged certain ways to present and explain mathematical results with the effect of smoothen the actual historical progress. He also sometimes asserted that some research dealt with a given theorem whereas it was not stated so by its author, thus creating mathematical links between publications which were independent at their time.

To illustrate these phenomena, let me consider some passages of the fourth subsection of the article, entitled "The genus; Riemann's theorem on its conservation by birational transformations; Zeuthen's extension" [Berzolari 1906, pp. 329–332].

Berzolari began with the statement that Bernhard Riemann, in his 1857 memoir on Abelian functions, grouped in classes the irreducible algebraic curves that can be transformed birationally one into another [Riemann 1857]. In doing so, he ascribed to Riemann some results on algebraic curves although the latter did not actually deal with these objects.[29] This interpretation, which was common at the time, therefore conveyed a distorted disciplinary conception of the past and contributed to confirming a classic narrative which included Riemann in the development of the subject as if interpreting geometrically his works was not an issue.

Still during his depiction of Riemann's work, Berzolari went on to explain that it is possible to associate a Riemann surface with any algebraic curve. This surface, seen in turn as a usual surface in the real three-dimensional space, can be characterised by its connectivity order, a number which is necessarily of the form $2p+1$.[30] The number p, Berzolari continued, is then called the genus of both the algebraic curve and the Riemann surface. Although Riemann did not use the term "genus", this is indeed how the number p appeared in his

[...]" [Dyck 1904, p. XV].

[29] All the statements in this subsection about the absence of algebraic curves in Riemann's work and the differences between his approach and Clebsch's are based on [Lê 2020].

[30] A surface is said to be of connectivity order $n+1$ if it can be disconnected into two pieces by the effect of n cuts.

1857 memoir.

A few lines later, Berzolari expounded on what he called "Clebsch's formula". First, he indicated that Riemann proved that p is equal to the number of linearly independent integrals of the first kind associated with the curve.[31] Then he added:[32]

> In the case of an irreducible [curve of order n] whose point-singularities consist only of d double points and r cusps, A. Clebsch and P. Gordan brought this theorem in a geometric form, where p denotes the number of linearly independent curves of order $n-3$ that pass through those $d+r$ points[45]), from which Clebsch's formula[46]) follows:
>
> $$p = \frac{1}{2}(n-1)(n-2) - d - r.$$

The first part of this quote refers to Alfred Clebsch and Paul Gordan's book *Theorie der Abelschen Functionen* [Clebsch and Gordan 1866], where the two authors did describe p as a number of curves alongside as a number of integrals. Nonetheless, even if Clebsch and Gordan made great use of algebraic curves in their approach of Abelian functions, they did not explicitly present this facet of p as "geometric": Berzolari thus operated here to another disciplinary interpretation.

Further, as evidence the words "from which" and "follows", the second part of the quote deals with a mathematical proof. As very often in the article, the wording is too vague to know whether Berzolari's sentence was intended to be a demonstration which should be understandable by the reader (even if the latter had to complete the argument on their own) or whether it was only a sketch of the demonstration as presented by Clebsch and Gordan, in which case the reader was meant to refer to their book. At any rate, it is true that Clebsch and Gordan proved the formula for p on the basis of its interpretation as a number of curves [Clebsch and Gordan 1866, p. 15].

At the same time, in the footnote numbered 46 and inserted just after the phrase "Clebsch's formula", Berzolari cited two papers by Clebsch, both anterior to the 1866 book, without commenting why. The first one was published in 1865, and if the formula for p was stated therein, it was not proved [Clebsch 1865]. The citation of this paper hence appears as a justification of the appellation of the formula. The second citation, which was introduced by a vague "cf.", refers to Clebsch's famous 1864 paper on the application of Abelian functions to geometry [Clebsch 1864]. This paper contained the statement and the proof

[31] From a current point of view, these integrals are integrals of holomorphic differentials on the curve.

[32] "Dieses Theorem haben für den Fall einer irreduzibeln C^n, deren Punktsingularitäten nur aus d Doppelpunkten und r Sptizen bestehen, A. Clebsch und P. Gordan[44]) in geometrische Form gebracht, indem p die Anzahl der linear unabhängigen Kurven von der Ordnung $n-3$ bedeutet, die durch jene $d+r$ Punkte einfach hindurchgehen[45]), woraus die Clebsch'sche Formel[46]) folgt: $p = \frac{1}{2}(n-1)(n-2) - d - r$." [Berzolari 1906, p. 330].

of the formula for p, but only in the particular case where $r = 0$, and in a way that did not involve the interpretation of p as a number of curves.

Hence what could appear as a historical presentation of the origins of the concept of genus and its development to the proof of Clebsch's formula turns out to be a sort of mixed narrative. Riemann's inclusion in this narrative was made all the more obvious by the fact that his work was described using the vocabulary of algebraic curves, thereby erasing Clebsch's originality of having interpreted Riemann with the help of such curves in his 1864 memoir. Further, the proof of the formula for p which can be found in this memoir was overshadowed by the later proof by Clebsch and Gordan, a proof which was more suited to the progression of Berzolari's narrative inasmuch as it was in direct relation with Riemann's interpretation of p as a number of integrals. On the other hand, such arrangements were not meant to bypass completely a historical presentation. Indeed, Berzolari did not choose what could be an optimal mathematical exposition, independent from any historical consideration and aiming at the most general technical framework possible: a more general formula for the genus, applying to curves with singularities other than double points and cusps and proved by Noether in 1874, was presented much later in the article, in 15th subsection.

After the explanations on Clebsch's formula for the genus p and other developments, Berzolari passed to another formula, due to Hieronymous Zeuthen [1871] and linking together the genera of two curves between which there exists an algebraic correspondence. After some explanations, Berzolari stated and proved a corollary of this formula, for which Heinrich Weber and Johannes Thomae were cited [Weber 1873; Thomae 1873, 1889].

In his paper, Weber did prove a result which can be identified with the corollary, provided the language of algebraic curves is adopted: in the image of Riemann's research, Weber only dealt with algebraic functions defined by equations and never talked about algebraic curves. Moreover, Weber mentioned neither the name of Zeuthen nor his formula. If the link between this formula and Weber's result was thus not the latter's product, it was not Berzolari's own creation either, since it was already mentioned in the *Vorlesungen über Geometrie* by Clebsch and Lindemann [1876, p. 459], a book that Berzolari cited a great deal throughout his article – but not for the link between Zeuthen and Weber. As for Thomae, his 1873 paper also dealt with algebraic functions only and ignored Zeuthen, but contrary to Weber's, it was not cited by Clebsch and Lindemann: the relation between this paper and Zeuthen's result thus appears as being Berzolari's product. Finally, while Thomae's 1889 article did bear on algebraic curves, it mentioned neither Weber nor Zeuthen. By citing these two authors simultaneously, Berzolari hence created scientific links between contributions which seemed to be independent from one another at their time, with the effect of making the narrative more coherent.

Berzolari's article was therefore ambivalent in terms of history and mathematics, in the sense that although the narrative appeared at first sight to follow a historical progression, it was marbled with numerous interventions of the author interpreting anachronistically certain results as relating to algebraic curve theory, favouring certain presentations of proofs and bringing together texts which were isolated in their time. These interventions thus contributed to shape an overall consistence that smoothed out some of the rough edges of the past and helped to reinforce a long-term impression of the subjects addressed.

5. Towards the constitution of algebraic geometry

This way of accounting for algebraic curve theory was not entirely characteristic of Berzolari. As a point of comparison, let me consider the article entitled "Curve", written by Cayley for the ninth edition of the *Encyclopaedia Britannica* [Cayley 1877].

Like Berzolari, Cayley organised his narrative chronologically in a loosely way, with occasional backtracks when changing mathematical topics. Very recent results were also described for several of these topics, as illustrates the mention of Zeuthen's 1874 works on the shape of quartic curves [Zeuthen 1874]. The fifty or so references that Cayley cited did not include any articles published in intermediate journals, with the exception of one, which appeared in the *Nouvelles Annales* as a French translation of Zeuthen's Danish doctoral dissertation [Zeuthen 1865, 1866]. And anachronistic interpretations, such as that consisting in attributing results on algebraic curves to Riemann, also helped to solidify the coherence of the narrative and emphasise the anchoring of mathematics in the long term. This last feature, however, was much less pronounced than in Berzolari's work, as the making of the topicality of subjects through the citation of very recent research was limited to a much smaller number of topics. For example, no reference was given about the question of the reducibility of curves, the numerous proofs of Plücker's formulas were not mentioned, and nor were the most recent developments around the notion of the genus of algebraic curves.

It is obvious that these differences are largely due to the different formats of Berzolari's and Cayley's articles and of the encyclopaedias in which they appeared. The specialisation of the *Encyklopädie* and the space available, indeed, allowed Berzolari to go into much greater detail on his subject and thus to endow almost every topic with a high degree of topicality.

But there is another aspect that clearly differentiates the two articles. As recalled in the introduction of the present chapter, the architecture of the *Encyklopädie* was thematic rather than alphabetic – contrary to the *Encyclopædia Britannica* – and this highlighted the existence of mathematical domains, whose components were incarnated by series of thematic articles.

Berzolari's article was included in the part on algebraic geometry, which was one of the three divisions of geometry in the *Encyklopädie*, the other two being devoted to the pure geometric theories and the basis of the application of algebra and analysis to geometry, and to differential geometry. However, it turns out that the phrase "algebraic geometry" was still rare at the time of the *Encyklopädie*'s publication.

Indeed, I fortuitously found this phrase only in a handful of texts of the 19th century, for instance in a paper by Cayley [1849] or in the titles of textbooks by Dionysus Lardner [1831] and George Hale Puckle [1854]. According to the *Jahrbuch über die Fortschritte der Mathematik*, the first publications whose title contains the expression "algebraic geometry" are dated from the early 20th century. These publications begin with the 1903 re-edition of Puckle's book just mentioned, which is followed by a series of texts which appeared regularly from 1905 on, among which a book chapter by Hermann Schubert called *Ganzzahligkeit in der algebraischen Geometrie* [Schubert 1905], two (elementary) textbooks by William Meath Baker [1906, 1907], or Francesco Severi's famous (and much more advanced) *Lezioni di geometria algebrica* [Severi 1908].

The *Jahrbuch* itself began to distinguish algebraic geometry as a subject only in 1925, while the *Catalogue* and the *Répertoire bibliographique des sciences mathématiques* never did. However, in 1898, at the occasion of the 30th anniversary and the 50th volume of *Mathematische Annalen*, a retrospective register of the journal was published. One part of this register was devoted to a thematic classification of the papers from the 50 first volumes, a classification which included a part on algebraic geometry. Arnold Sommerfeld, who edited this work, explained that "at the request of the editors of the *Annalen*, [he] based the subject index essentially on the classification envisaged for the forthcoming *Encyklopädie der Mathematischen Wissenschaften*" [Sommerfeld 1898, v].[33] Accordingly, the geometry was divided into the three same parts as in the *Encyklopädie*.

These clues suggest that the appearance and use at a great scale of the phrase "algebraic geometry" to designate a certain part of geometry is strongly tied to the very creation of the *Encyklopädie* – and to Felix Klein. These observations hence go in the same direction as those of historians who described encyclopaedias as media that contribute to the constitution and the consolidation of scientific disciplines.[34]

It is still beyond my grasp to fully understand what early 20th-century mathematicians gradually came to understand as being circumscribed by

[33]"Bei der Bearbeitung des Sachregisters habe ich auf Wunsch der Annalen-Redaction im Wesentlichen diejenige Eintheilung zu Grunde gelegt, welche für die im Erscheinen begriffene Encyclopädie der Mathematischen Wissenschaften [...] in Aussicht genommen ist."

[34]Interestingly, Richard Yeo, who dealt mainly with the *Britannica* and the *Metropolitana* in the 18th century and the beginning of the 19th, associated this phenomenon with the choice of organising an encyclopaedia alphabetically [Yeo 1991]. See also [Stichweh 1984; Falconer 2021], as well as [Goldstein and Schappacher 2007, pp. 90–97] for the links of the *Encyklopädie* (and other catalogues) with the process of disciplinarisation of number theory at the end of the 19th century.

algebraic geometry, taking into account the multiple personal interpretations that may, for example, stem from the vague definition of the domain as the study "algebraic constructs",[35] a definition which does not take mathematical methods into account and says nothing about the possible social and institutional boundaries of the domain.[36]

But if one accepts that the *Encyklopädie* did play a role in establishing algebraic geometry as a domain in its own right, one should also bear in mind that this process was certainly not reduced to a mere enumeration of topics, as would incarnate the succession of articles, sections and subsections encompassed in the corresponding part. The concentric circles of mathematical knowledge that are these articles, sections and subsections are not mere lists of theorems. They are historical-mathematical narratives which highlight some aspects of a subject at the expense of others and undertake countless reinterpretations of the past to offer a certain vision of what their topic is.

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[35]The original phrase is *algebraische Gebilde*, which refers to objects that can be defined by polynomial equations, like algebraic curves and surfaces. This definition of algebraic geometry is the one proposed by Wilhelm Franz Meyer and Hans Mohrmann, the main editors of the geometric volumes of the *Encyklopädie* [Meyer and Mohrmann 1923, v].

[36]To assess the influence of Berzolari's article on the development of algebraic geometry is beyond the scope of the present chapter. On this issue, I will simply note that this article was cited in various works from the early 20th century as a bibliographical or historical source on the theory of algebraic curves or on certain more specific elements. See for instance [Severi 1926, p. 16; Coolidge 1931, pp. iv, 99; H. F. Baker 1933, p. 90].

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